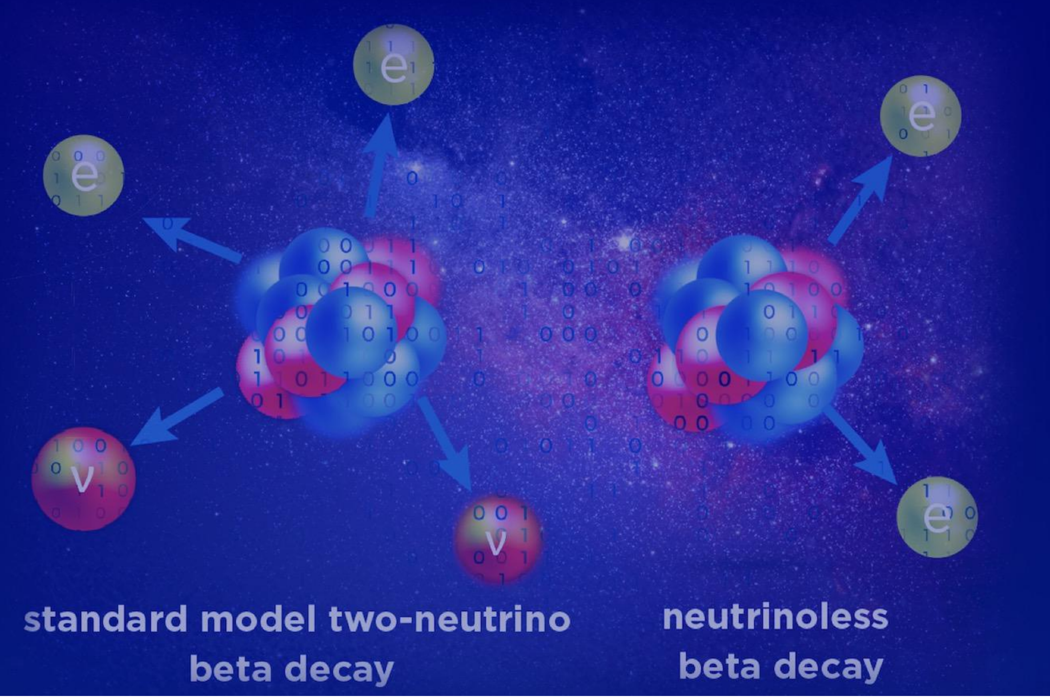
Hamiltonian-Based Generator Coordinate Method for Neutrinoless Double-β Decay

Changfeng Jiao (焦长峰)

Shool of Physics and Astronomy

Sun Yat-Sen University



Workshop on Neutrinoless double-beta decay, May 22nd @ Zhuhai, Guangdong

Baryogenesis through Leptogenesis

- Neutrino oscillation experiments → neutrinos have masses.
  Beyond the standard model.

- Neutrino masses are much less than charged leptons and quarks.
  Dirac masses from Higgs mechanism? Not likely.

- One solution: if neutrinos are ***Majorana fermions***, i.e., their own anti-particles, **the seesaw mechanism** can introduce right-handed neutrinos with large Majorana masses.

- They decay into either leptons or anti-leptons via Yukawa couplings. The CP asymmetries of these decays result in lepton number asymmetry in the universe.

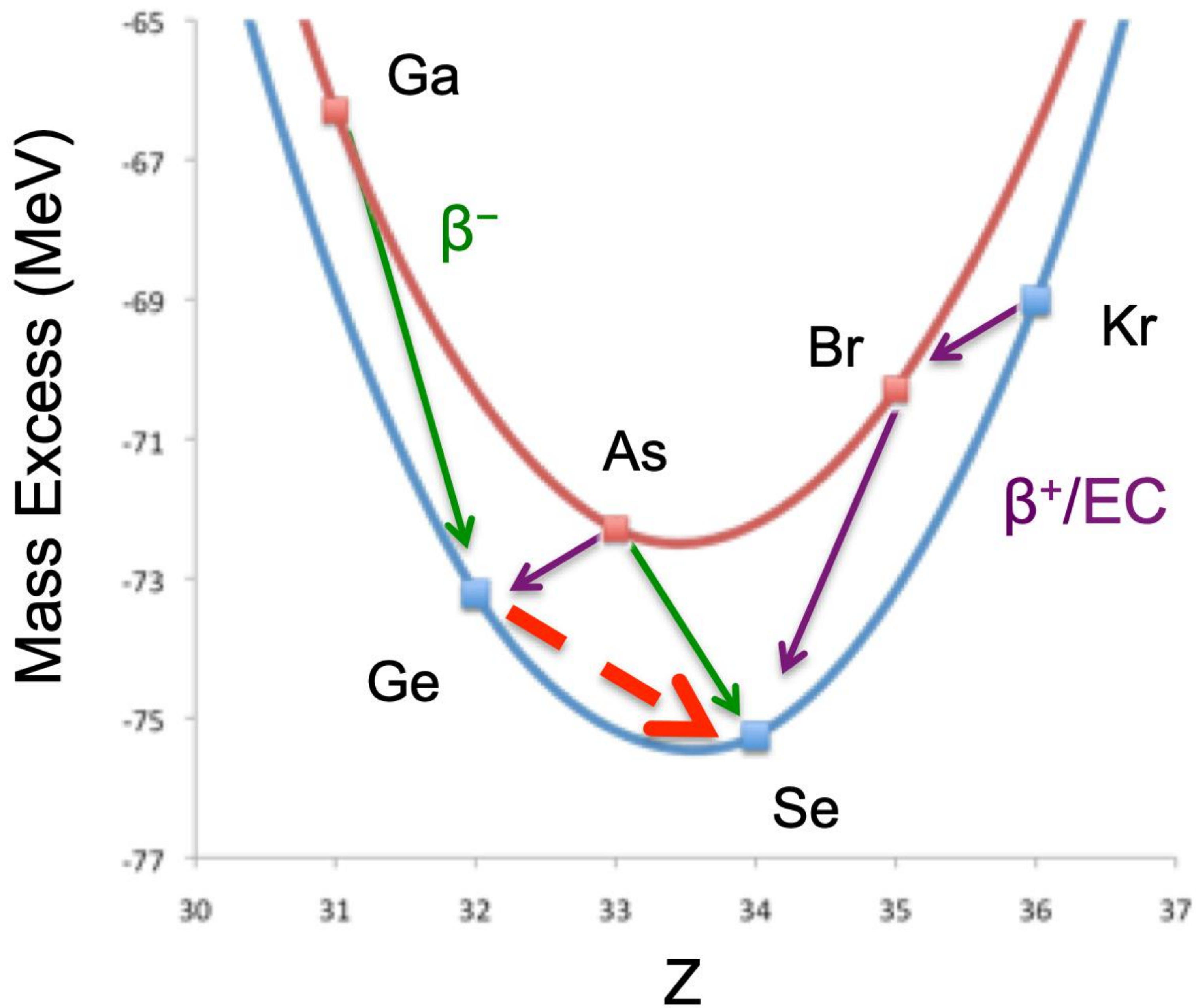
Lepton number asymmetry $\xrightarrow{\text{sphaleron process}}$ **Baryon number asymmetry**

If this is true, neutrinos should be Majorana fermions. But how do we know that?

Probes: Neutrinoless Double-β Decay

In certain even-even nuclei, β-decay is energetically forbidden, because m(Z, A) < m(Z+1, A), while double-β decay, from a nucleus of (Z, A) to (Z+2, A), is energetically allowed.

Isotope	$Q_{\beta\beta}$ (MeV)
^{76}Ge	2.039
^{82}Se	2.992
^{100}Mo	3.034
^{130}Te	2.528
^{136}Xe	2.468
^{150}Nd	3.368

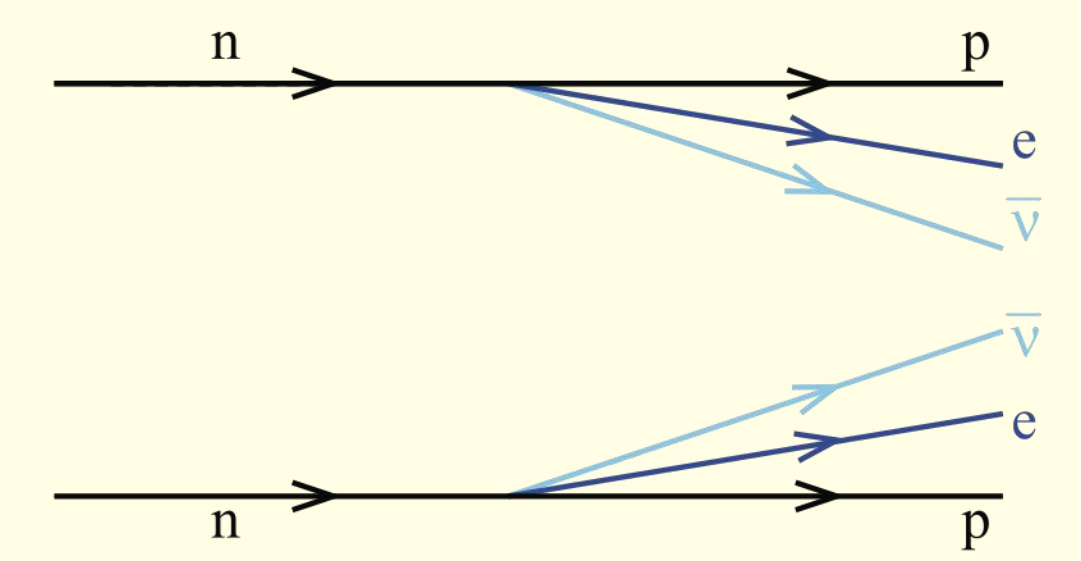


Double-β Decay Modes

2ν double-β decay (2νββ): $(Z, A) \rightarrow (Z+2, A) + 2e^- + 2\bar{\nu}_e$

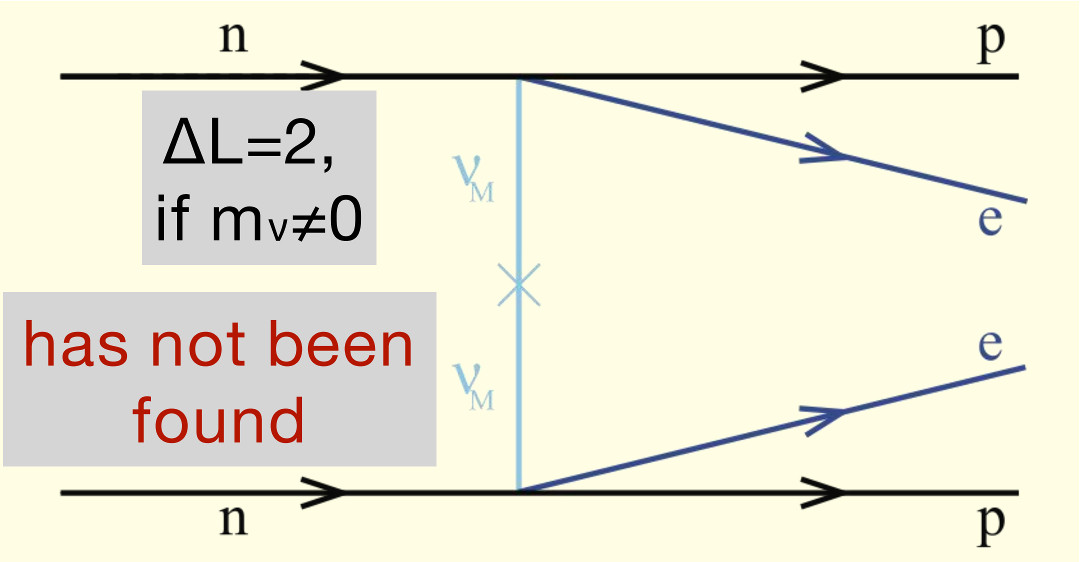
Allowed second-order weak process
Maria Goeppert-Mayer (1935)

2νββ observed for
^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo,
^{116}Cd, ^{128}Te, ^{130}Te, ^{136}Xe, ^{150}Nd



0ν double-β decay (0νββ): $(Z, A) \rightarrow (Z+2, A) + 2e^-$

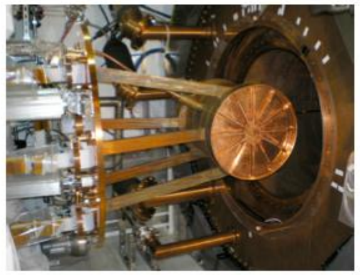
- Tests lepton number conservation.
- The practical technique to determine if neutrinos might be Majorana particles.
- A method for determining the overall absolute neutrino mass scale



0νββ Decay Experiments

CUORE

EXO200

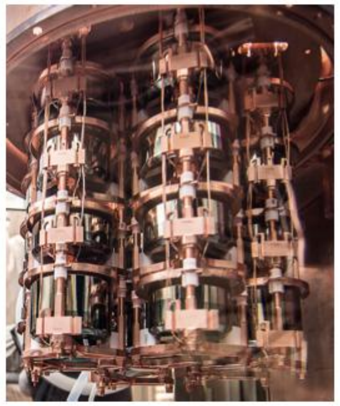


KamLAND Zen

Collaboration	Isotope	Technique	mass (0νββ isotope)	Status
CANDLES	Ca-48	305 kg CaF_2 crystals - liq. scint	0.3 kg	Construction
CARVEL	Ca-48	$^{48}CaWO_4$ crystal scint.	~ ton	R&D
GERDA I	Ge-76	Ge diodes in LAr	15 kg	Complete
GERDA II	Ge-76	Point contact Ge in LAr	31	Operating
MAJORANA DEMONSTRATOR	Ge-76	Point contact Ge	25 kg	Operating
LEGEND	Ge-76	Point contact	**~ ton**	R&D
NEMO3	Mo-100 Se-82	Foils with tracking	6.9 kg 0.9 kg	Complete
SuperNEMO Demonstrator	Se-82	Foils with tracking	7 kg	Construction
SuperNEMO	Se-82	Foils with tracking	100 kg	R&D
LUCIFER (CUPID)	Se-82	ZnSe scint. bolometer	18 kg	R&D
AMoRE	Mo-100	$CaMoO_4$ scint. bolometer	1.5 - 200 kg	R&D
LUMINEU (CUPID)	Mo-100	$ZnMoO_4$ / Li_2MoO_4 scint. bolometer	1.5 - 5 kg	R&D
COBRA	Cd-114,116	CdZnTe detectors	10 kg	R&D
CUORICINO, CUORE-0	Te-130	TeO_2 Bolometer	10 kg, 11 kg	Complete
CUORE	Te-130	TeO_2 Bolometer	**206 kg**	Operating
CUPID	Te-130	TeO_2 Bolometer & scint.	**~ ton**	R&D
SNO+	Te-130	0.3% ^{nat}Te suspended in Scint	**160 kg**	Construction
EXO200	Xe-136	Xe liquid TPC	79 kg	Operating
nEXO	Xe-136	Xe liquid TPC	**~ ton**	R&D
KamLAND-Zen (I, II)	Xe-136	2.7% in liquid scint.	**380 kg**	Complete
KamLAND2-Zen	Xe-136	2.7% in liquid scint.	**750 kg**	Upgrade
NEXT-NEW	Xe-136	High pressure Xe TPC	5 kg	Operating
NEXT	Xe-136	High pressure Xe TPC	100 kg - **ton**	R&D
PandaX - 1k	Xe-136	High pressure Xe TPC	~ **ton**	R&D
DCBA	Nd-150	Nd foils & tracking chambers	20 kg	R&D

GERDA

MAJORANA



SNO+

Taken from J. F. Wilkerson's slides

Neutrino Mass Hierarchy

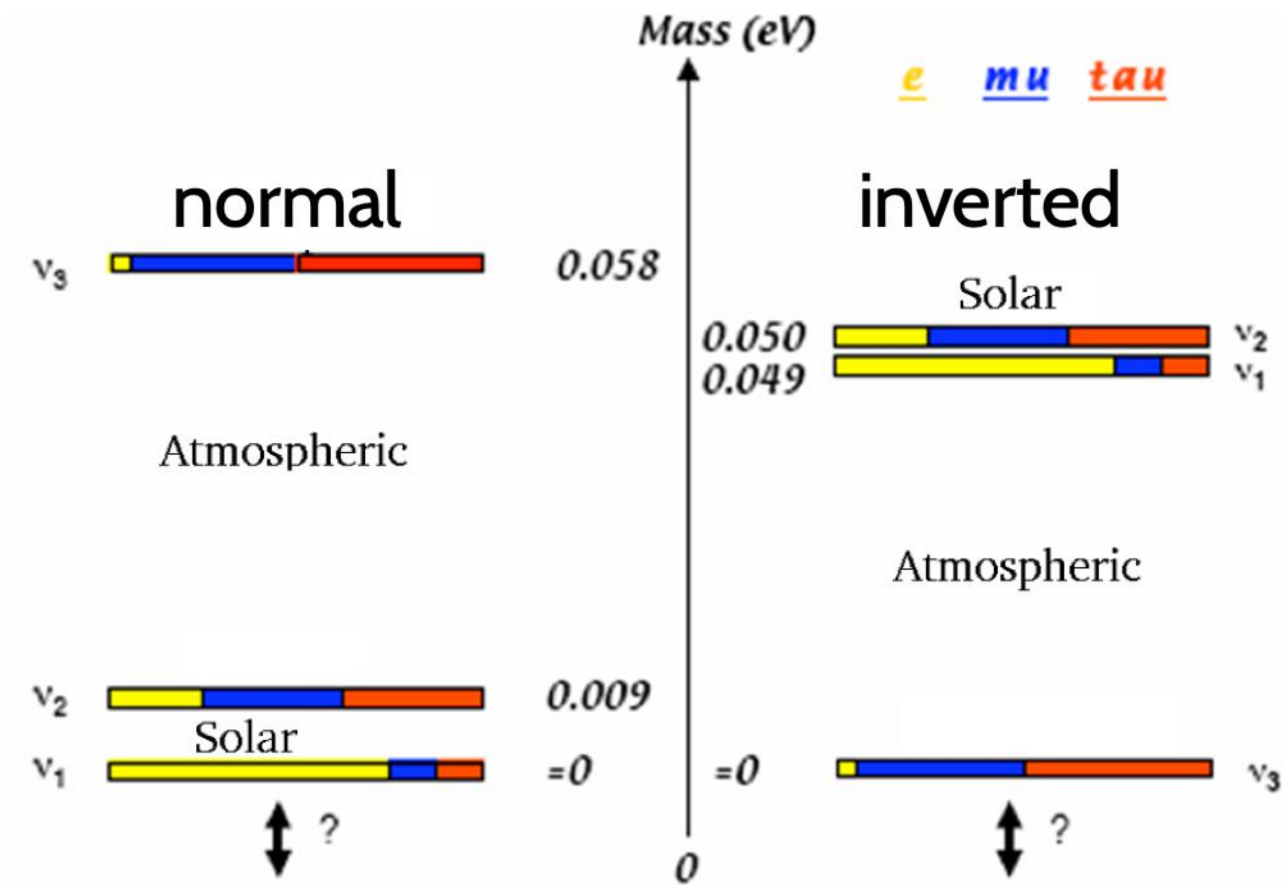
From neutrino oscillations we know

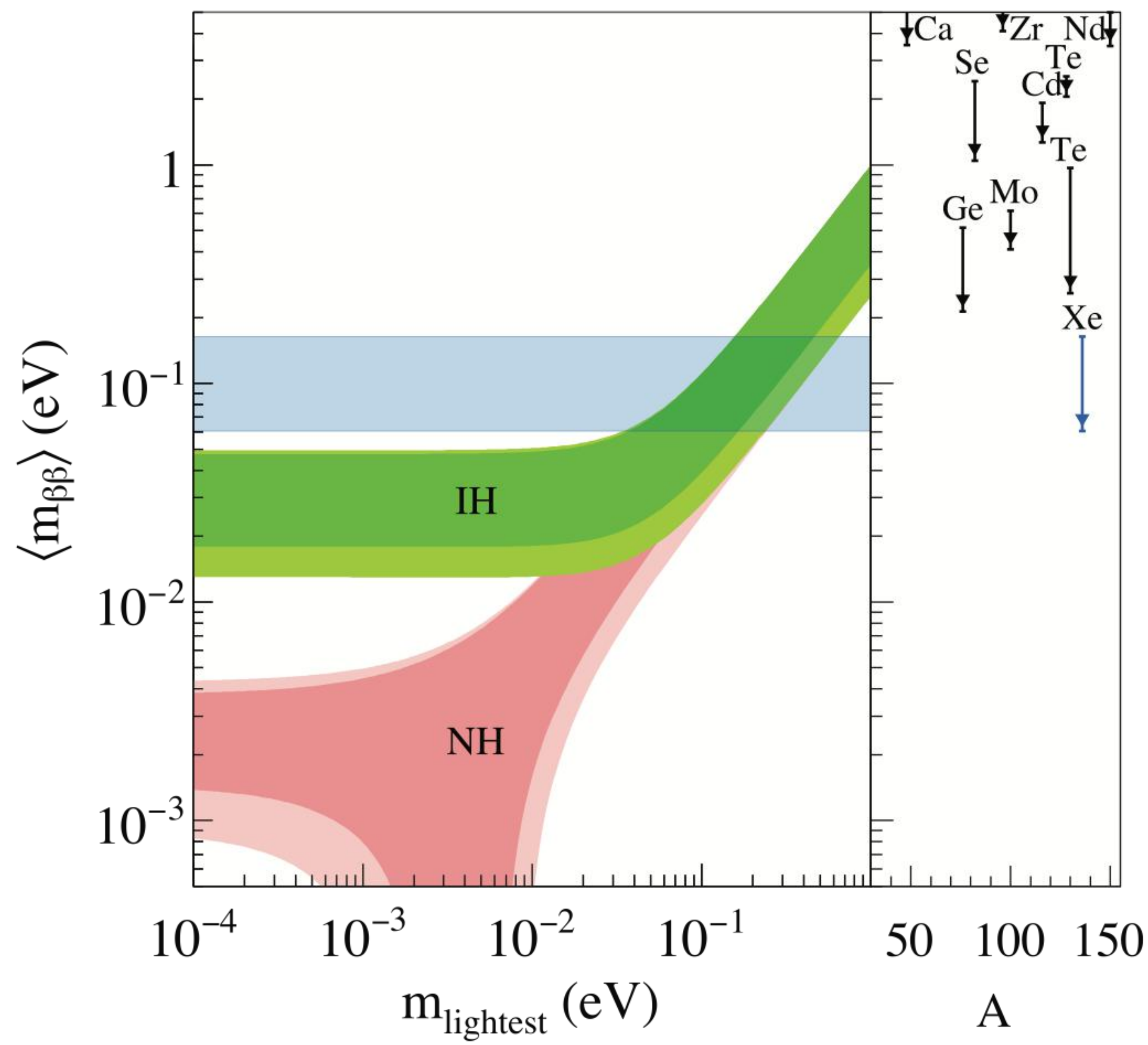
$$\Delta m^2_{\text{sun}} \simeq 75\ \text{meV}^2 \qquad \Delta m^2_{\text{atm}} \simeq 2400\ \text{meV}^2$$

We also know the mixing angles that specify the linear combinations of flavor eigenstates

$$m_{\beta\beta} \equiv \left| \sum_k m_k U^2_{ek} \right|$$

But we don't know the mass hierarchy.





A. Gando et al. PRL **117**, 082503 (2016)

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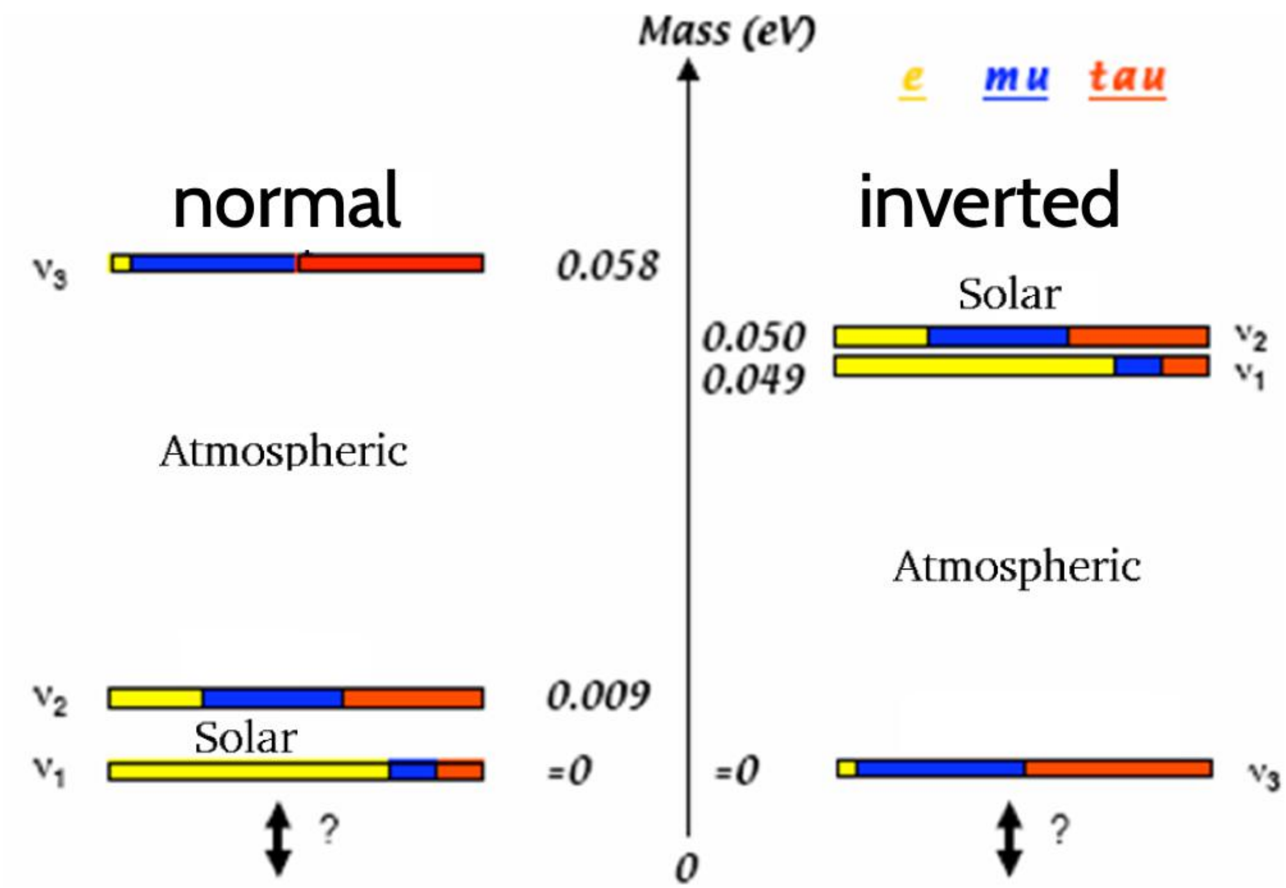
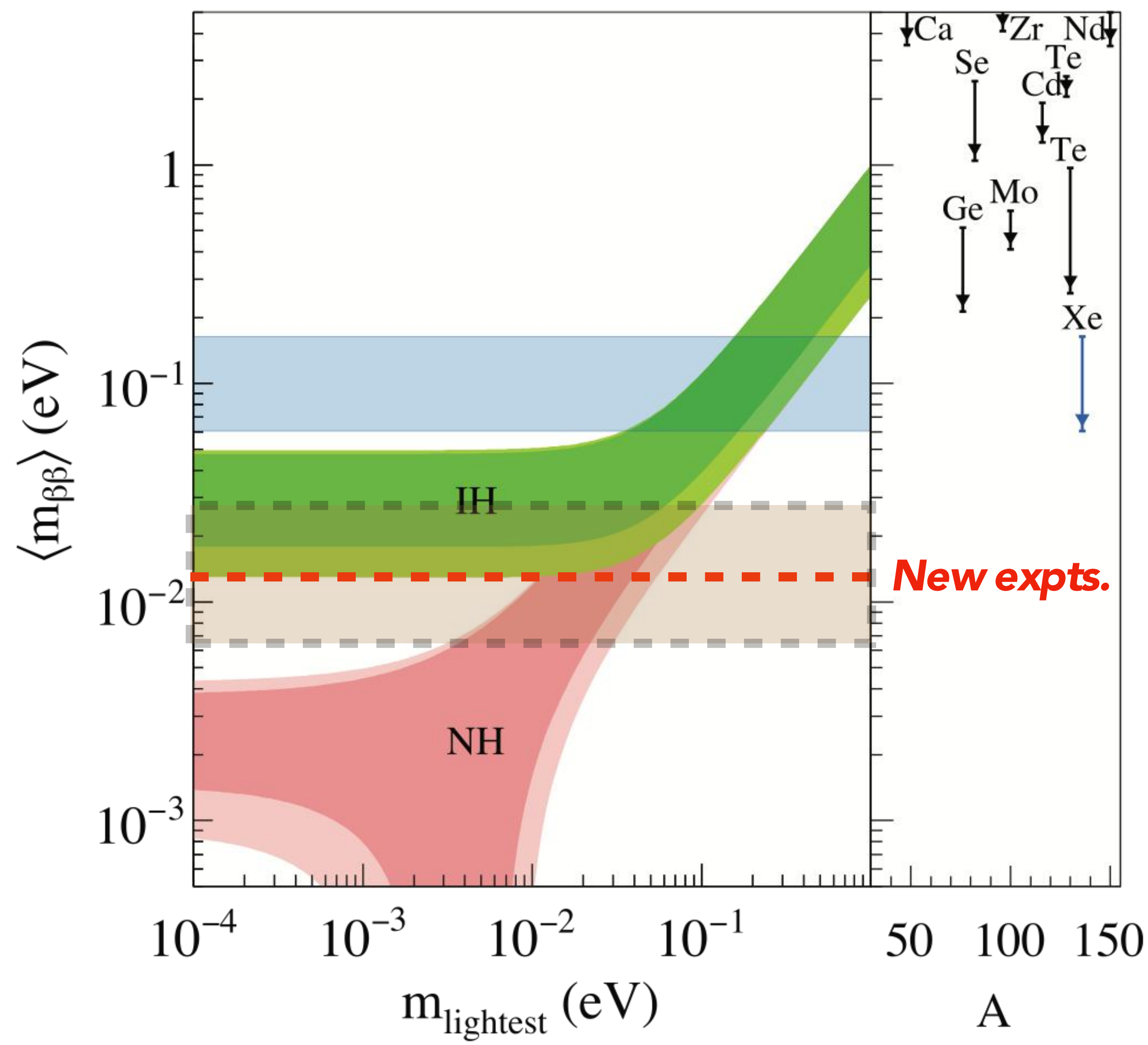




A. Gando et al. PRL **117**, 082503 (2016)

$0\nu\beta\beta$ Decay Rates and Relevant Terms

In case of process induced by light exchange, mass mechanism

$$[T_{1/2}^{0\nu}]^{-1} = G_{0\nu}(Q, Z)|M_{0\nu}|^2\langle m_{\beta\beta}\rangle^2$$

$0\nu\beta\beta$ rate

Phase space

Nuclear matrix elements

Effective Majorana mass

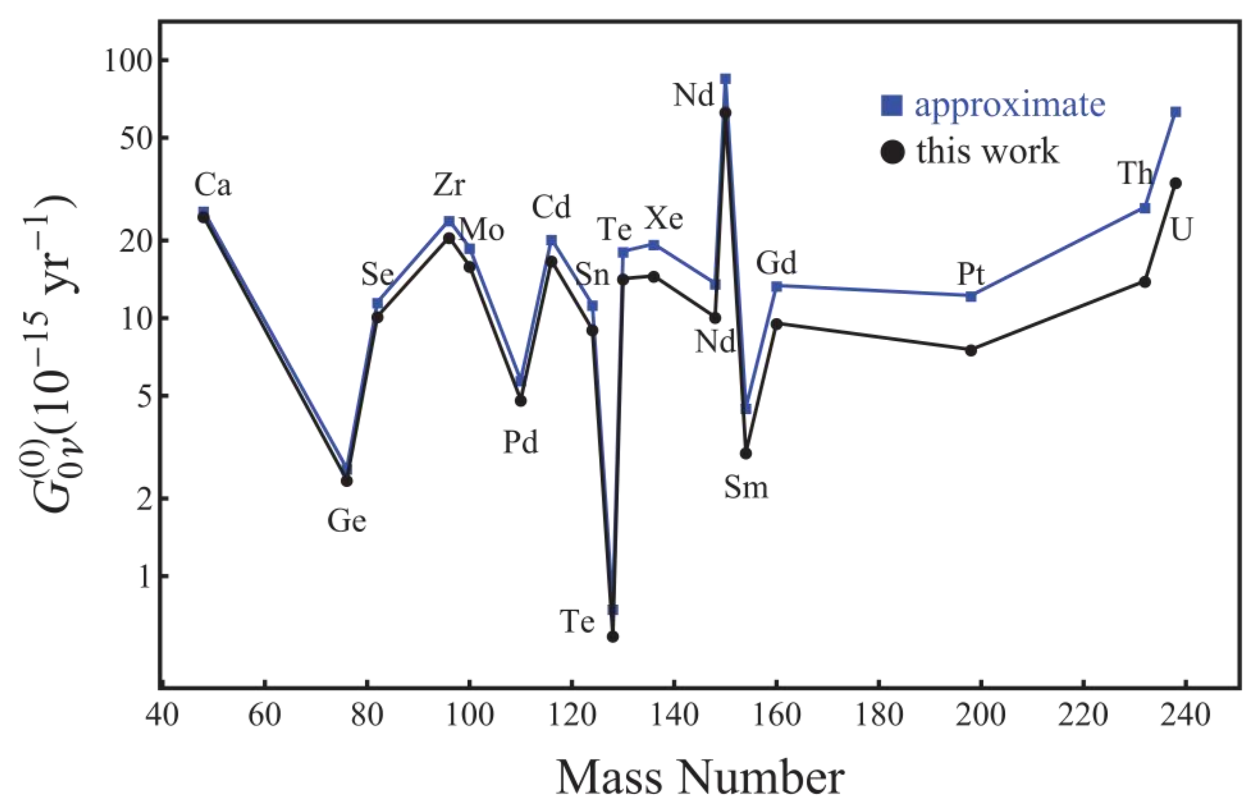
Phase space factor:
can be accurately calculated

Uncertainty from R: ~7% for $0\nu\beta\beta$ *decay*

Uncertainty from Q:

TABLE V. The uncertainty on the PSF due to the uncertainty of the Q value.

$Q_{\beta\beta}$ (keV)	$G^{(0)}_{2\nu\,\mathrm{SSD}}$ (yr^{-1})	$G^{(0)}_{0\nu}$ (yr^{-1})
2004.00(1133)[a]	$1.386(67) \times 10^{-19}$	$4.707(86) \times 10^{-15}$
2017.85(64)[b]	$1.469(05) \times 10^{-19}$	$4.815(06) \times 10^{-15}$



J. Kotila and F. Iachello, PRC **85**, 034316 (2012)

0νββ Decay Rates and Relevant Terms

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0νββ rate

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Phase

can b

Uncerta

Uncerta

TABLE V.

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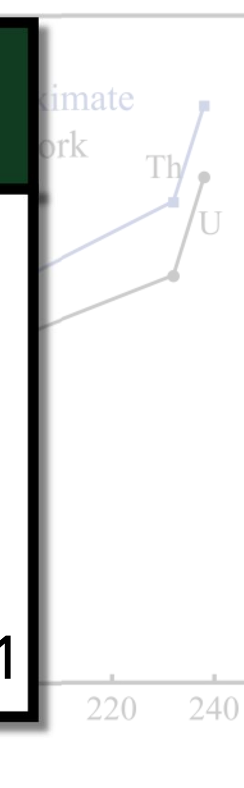
But the rate also depends on a nuclear matrix element.

"An uncertainty of a factor of three in the matrix element thus corresponds to nearly an order of magnitude uncertainty in the amount of material required…"

J. Engel and J. Menendez, Rep. Prog. Phys. 80 (2017) 046301



J. Kotila and F. Iachello, PRC **85**, 034316 (2012)

$0\nu\beta\beta$ Decay Nuclear Matrix Element

$$[T^{0\nu}_{1/2}]^{-1} = G_{0\nu}(Q,Z)|M_{0\nu}|^2\langle m_{\beta\beta}\rangle^2$$

Must be calculated by nuclear physics!

$$\boxed{M^{0\nu} = M^{0\nu}_{\mathrm{GT}} - \frac{g_V^2}{g_A^2}M^{0\nu}_{\mathrm{F}} + M^{0\nu}_{\mathrm{T}}}$$

with

$$M^{0\nu}_{\mathrm{GT}} = \frac{2R}{\pi g_A^2}\int_0^\infty |q|d|q|\langle f|\sum_{a,b}\frac{j_0(|q|r_{ab})h_{\mathrm{GT}}(|q|)\vec{\sigma}_a\cdot\vec{\sigma}_b}{|q|+\bar{E}-(E_i+E_f)/2}\tau_a^+\tau_b^+|i\rangle$$

$$M^{0\nu}_{\mathrm{F}} = \frac{2R}{\pi g_A^2}\int_0^\infty |q|d|q|\langle f|\sum_{a,b}\frac{j_0(|q|r_{ab})h_{\mathrm{F}}(|q|)}{|q|+\bar{E}-(E_i+E_f)/2}\tau_a^+\tau_b^+|i\rangle$$

$$M^{0\nu}_{\mathrm{T}} = \frac{2R}{\pi g_A^2}\int_0^\infty |q|d|q|\langle f|\sum_{a,b}\frac{j_2(|q|r_{ab})h_{\mathrm{T}}(|q|)[3\vec{\sigma}_j\cdot\hat{r}_{ab}\vec{\sigma}_k\cdot\hat{r}_{ab}-\vec{\sigma}_a\cdot\vec{\sigma}_b}{|q|+\bar{E}-(E_i+E_f)/2}\tau_a^+\tau_b^+|i\rangle$$

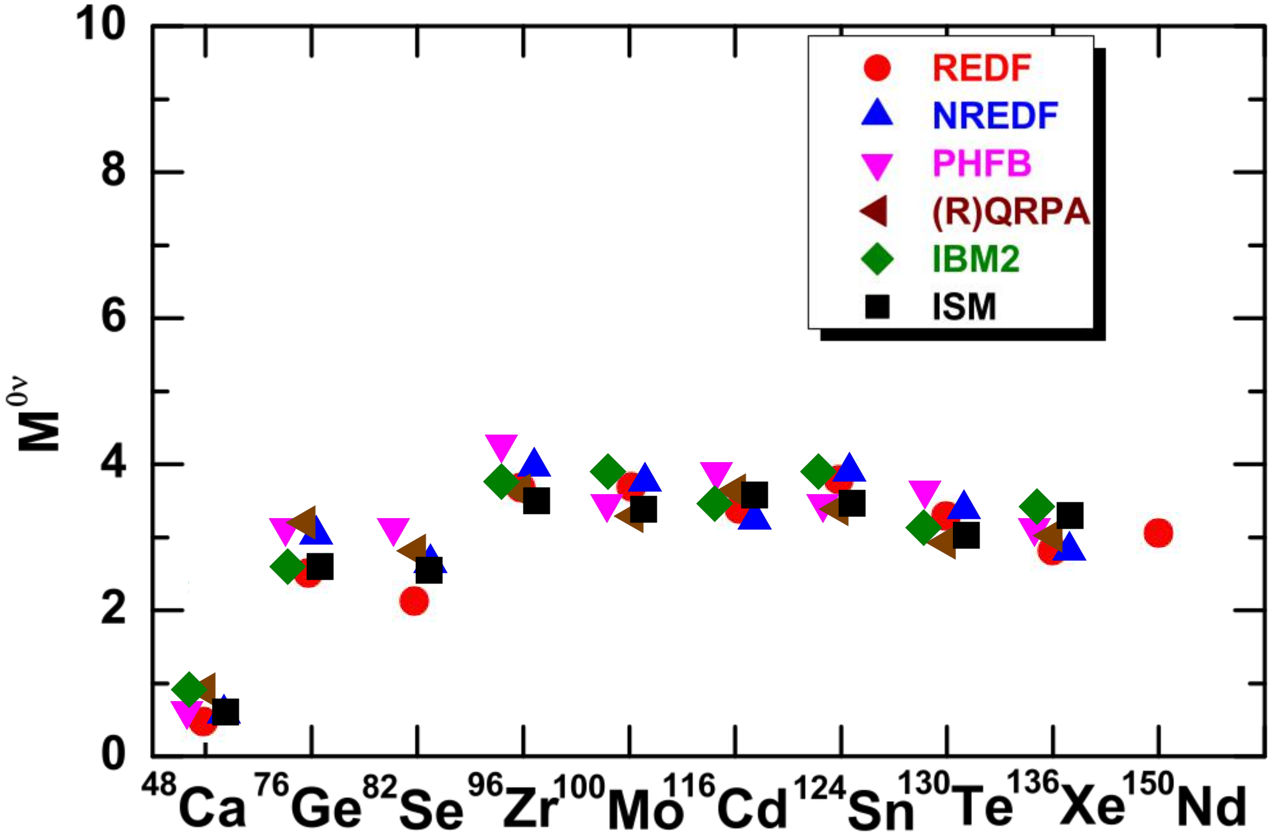
Lines of attack:

- Construct effective operator.
- *Find good initial and final ground-state wave functions: challenge for nuclear physics.*

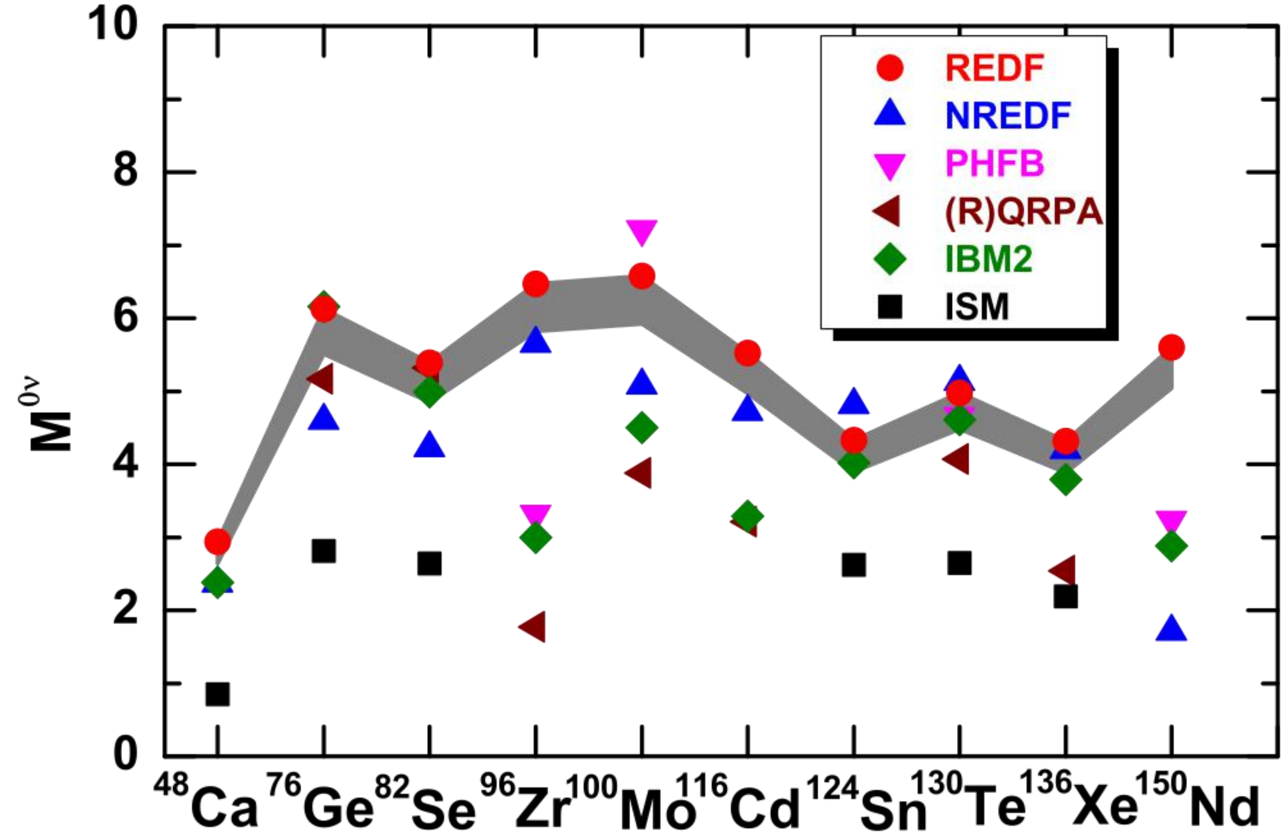
0νββ Decay Nuclear Matrix Element

Nuclear Models: **QRPA, Shell model, GCM, etc.**

What we hope: 😄



What we've got: 😥

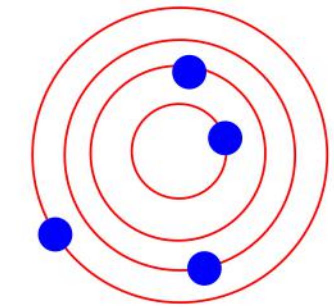


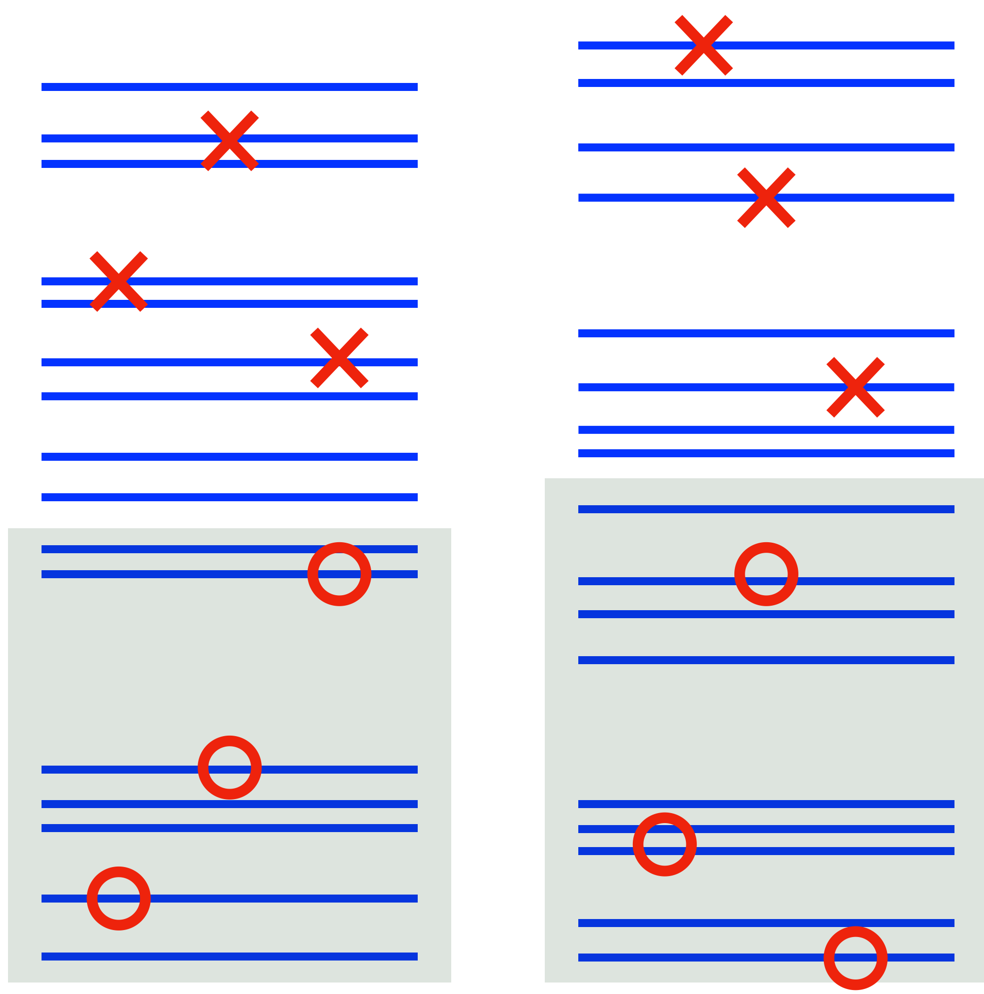
Current situation:

- Significant spread.
- Hard to quantify uncertainty.
- All the models miss important physics: omits correlations, omits single particle levels…

Review of Different Nuclear Models

Some models are built on single independent-particle state.





Protons Neutrons

Starting from one Slater determinant, e.g., the HF state $|\psi_0\rangle$, the ground state

$$|0\rangle = |\psi_0\rangle + \sum_{mi} C^0_{mi} a^\dagger_m a_i |\psi_0\rangle + \frac{1}{4} \sum_{mnij} C^0_{mn,ij} a^\dagger_m a^\dagger_n a_i a_j |\psi_0\rangle + \cdots$$

But exact diagonalization in complete Hilbert space is not solvable.

Review of Different Nuclear Models

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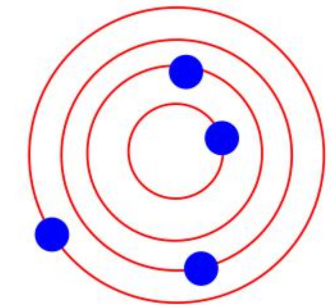
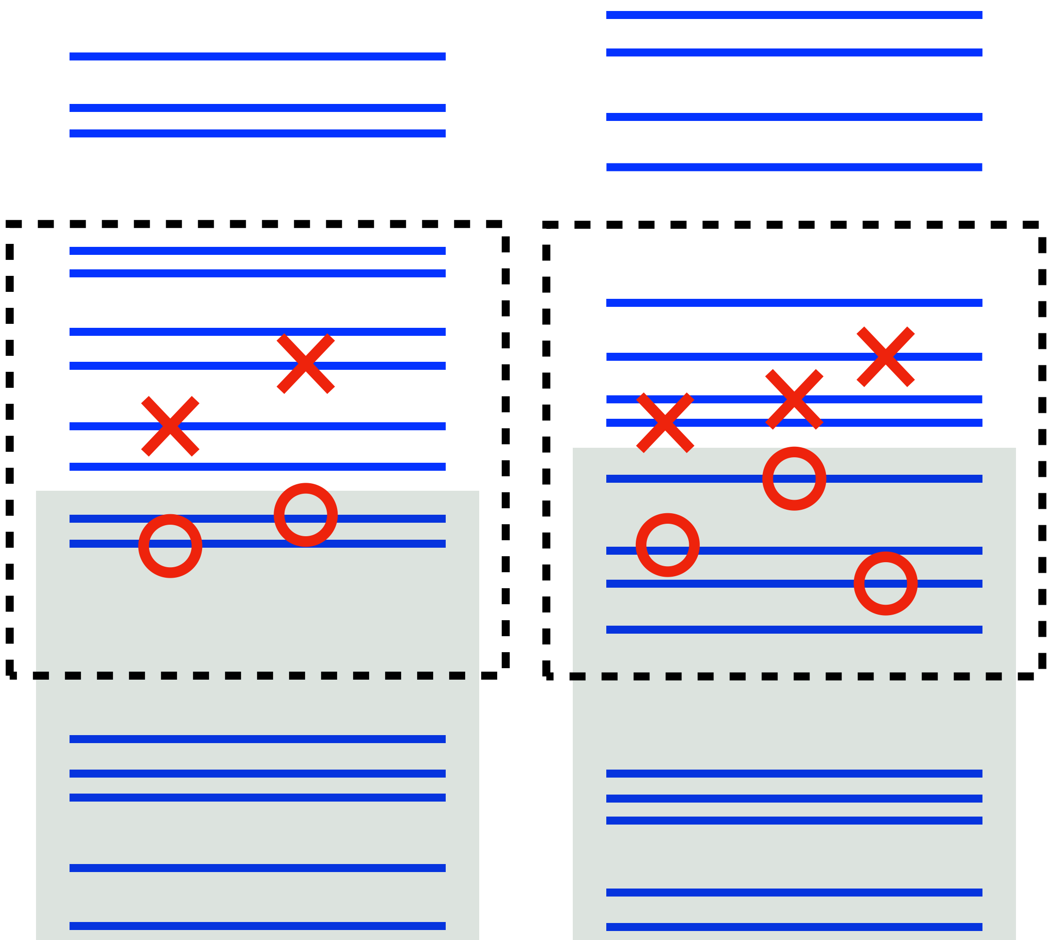


Interacting shell model (ISM)



- Same starting point . $|0\rangle$
- Instead of solving Schrödinger equation in complete Hilbert space, one restricts the dynamics in a configuration space.

$$H|\Phi_i\rangle = E_i|\Phi_i\rangle \rightarrow H_{\text{eff}}|\bar{\Phi}_i\rangle = E_i|\bar{\Phi}_i\rangle$$

Configuration interaction of orthonormal Slater determinants:

$$|\bar{\Phi}_i\rangle = \sum_j c_{ij}|\psi_j\rangle, \quad \langle\psi_j|\psi_k\rangle = \delta_{jk}$$

Diagonalizing the H_{eff} in the orthonormal basis.

Review of Different Nuclear Models

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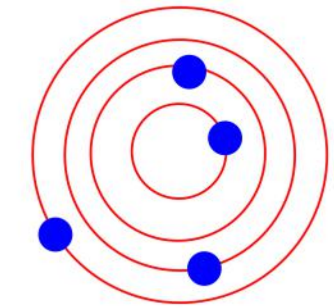

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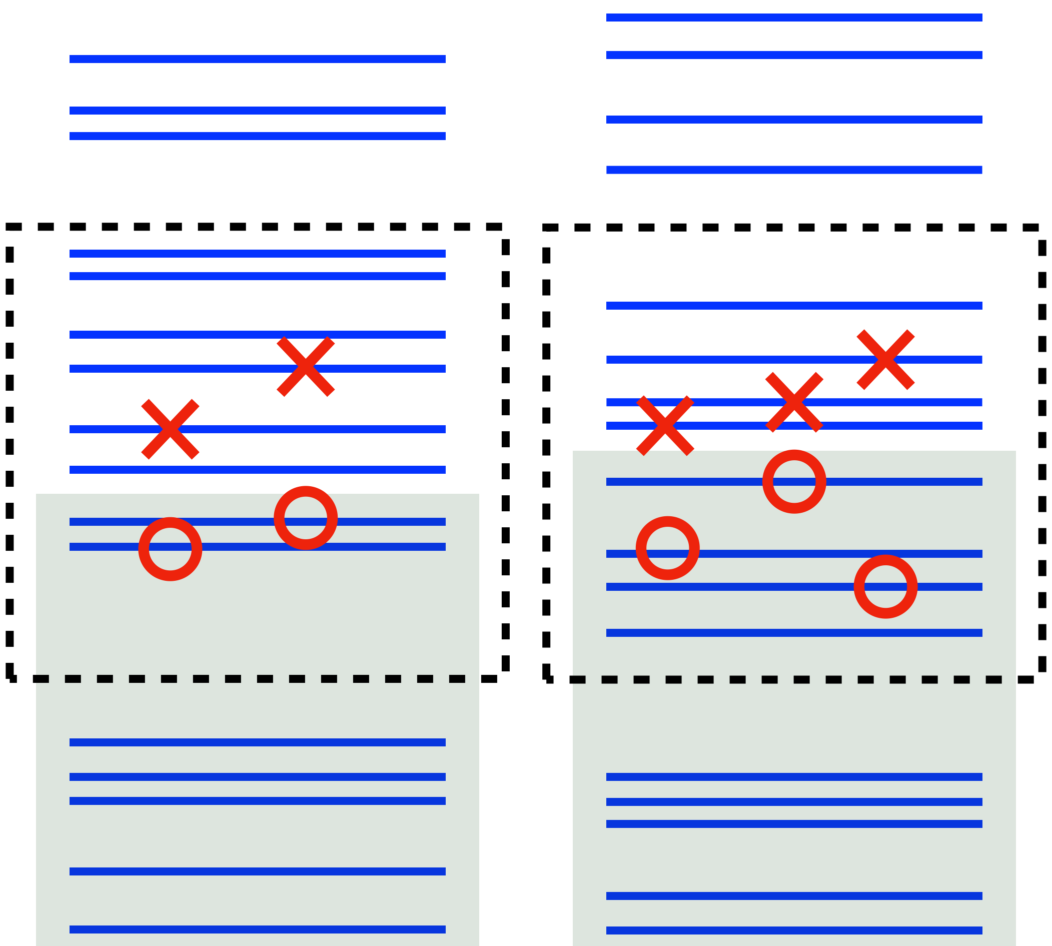


Protons Neutrons

Pros:

❖ Arbitrarily complex correlations within the model space.

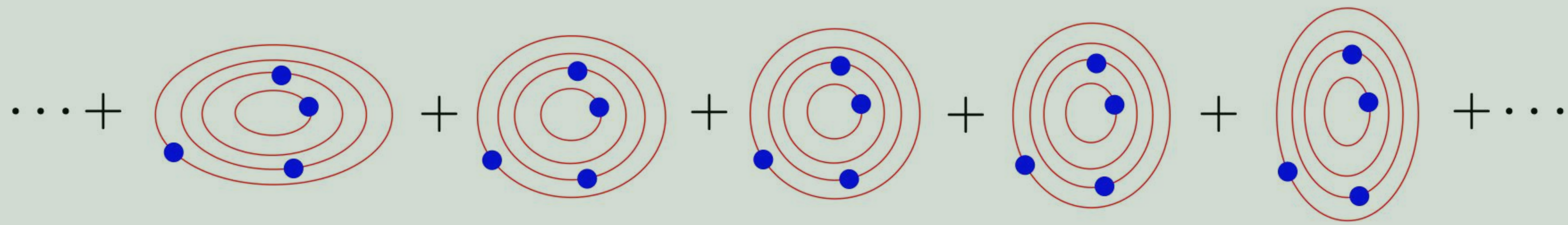
Cons:

❖ Relatively small configuration spaces.
  - *At present most of the $0\nu\beta\beta$ decay NME calculations carried out by SM are limited in one single shell.*

The Other Way Around...

Another way to build many-body states:

Instead of configuration interaction with orthogonal states, one can diagonalize the Hamiltonian in a set of ***non-orthogonal*** basis.



$$|\Phi\rangle = \sum_j c_j |\psi_j\rangle, H_{jk} = \langle j|H|k\rangle$$

$$\sum_k H_{jk} c_k = E \sum_k N_{jk} c_k, N_{jk} = \langle j|k\rangle$$

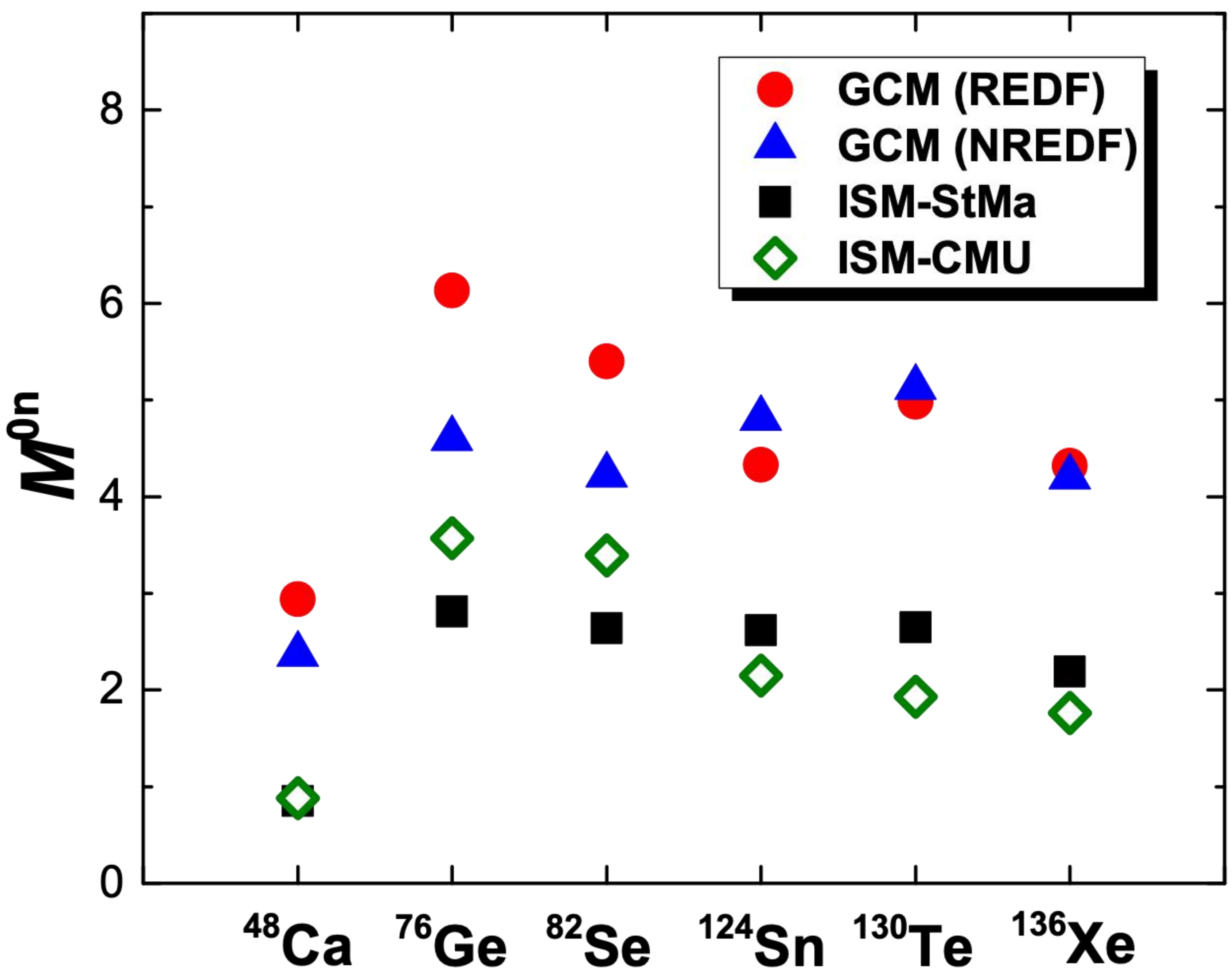
The non-orthogonal states can be ***highly optimized***, and hence reduce the dimension of basis states.

Generator-Coordinate Method (GCM)

Generator Coordinate Method

Generator Coordinate Method (GCM): an approach that treats large-amplitude fluctuations, which is essential for nuclei that cannot be approximated by a single mean field.

However, the results of GCM based on energy density functional look different from the ones given by the SM calculations.



Both the shell model and the EDF-based GCM could be missing important physics.

- The EDF-GCM omits correlations.
- The shell model omits many single-particle levels.

Does the discrepancy come from methods themselves, or the interactions they use?

Generator Coordinate Method

Let's combine the virtues of both frameworks through an idealistic GCM that includes all the important correlations in a large single-particle space!

Sure. My current achievement is the first step in this direction: Developed a Hamiltonian-based GCM in one and two (and possibly more) shells.

More correlations, larger space.

Another way is IM-SRG + GCM (IM-GCM), c.f. Jiangming Yao's talk...

Generator Coordinate Method

- Using a realistic effective Hamiltonian.
- ***We are trying to include all possible collective correlations.***

$$\mathcal{O}_1 = Q_{20}, \quad \mathcal{O}_2 = Q_{22},$$

$$\mathcal{O}_3 = \frac{1}{2}(P_0 + P_0^\dagger), \quad \mathcal{O}_4 = \frac{1}{2}(S_0 + S_0^\dagger),$$

- HFB states $|\Phi(q)\rangle$ with multipole constraints

$$\langle H' \rangle = \langle H_{\text{eff}} \rangle - \lambda_Z(\langle N_Z \rangle - Z) - \lambda_N(\langle N_N \rangle - N) - \sum_i \lambda_i(\langle \mathcal{O}_i \rangle - q_i),$$

- Angular momentum and particle number projection

$$|JMK; NZ; q\rangle = \hat{P}^J_{MK} \hat{P^N} \hat{P^Z} |\Phi(q)\rangle$$

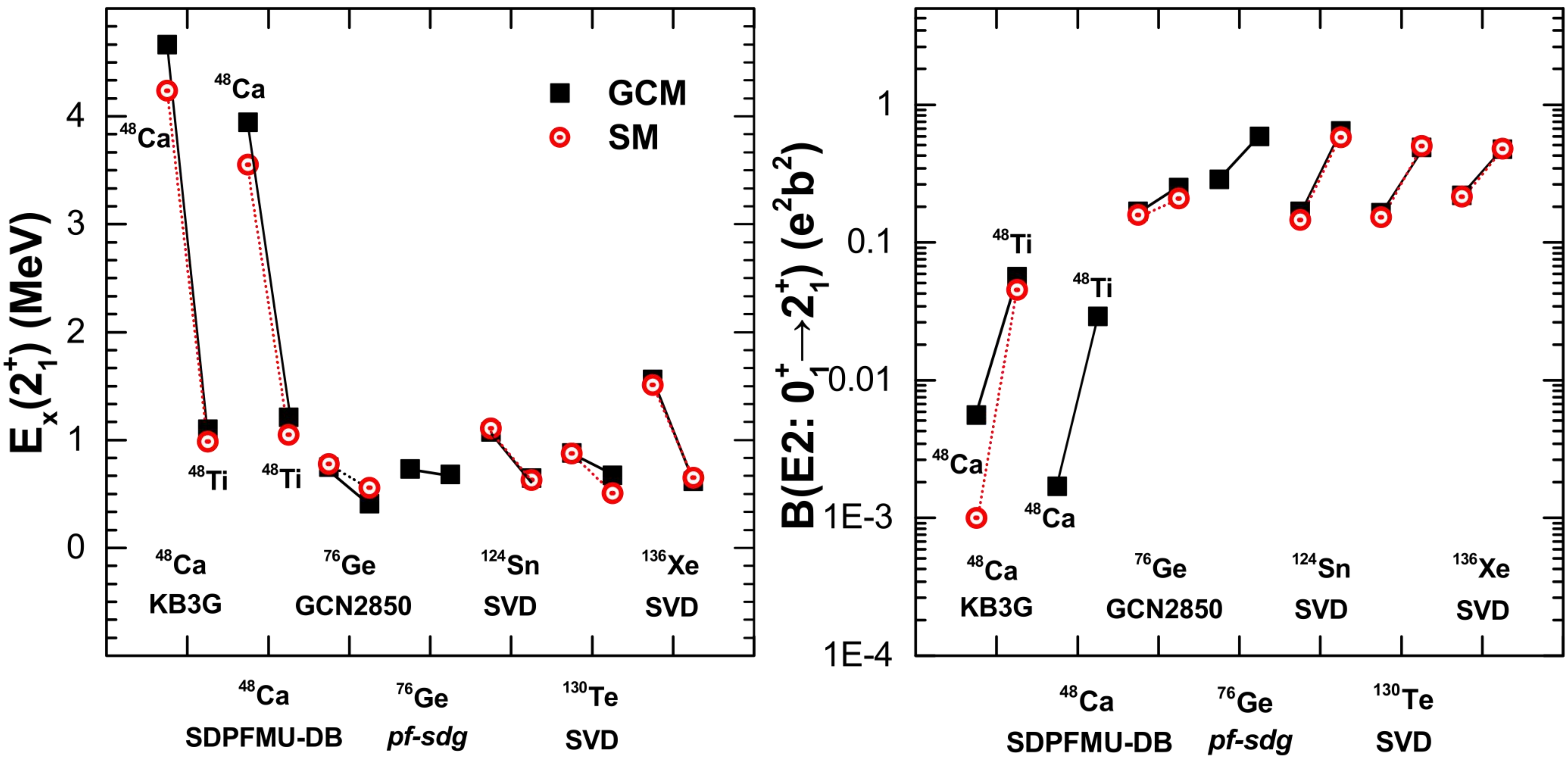
- Configuration mixing within GCM:

GCM wavefunction: $$|\Psi^J_{NZ\sigma}\rangle = \sum_{K,q} f^{JK}_\sigma(q) |JMK; NZ; q\rangle$$

Hill-Wheeler equation: $$\sum_{K',q'} \{\mathcal{H}^J_{KK'}(q;q') - E^J_\sigma \mathcal{N}^J_{KK'}(q;q')\} f^{JK'}_\sigma(q') = 0$$

0vββ NME: $$M^{0\nu\beta\beta}_\xi = \langle \Psi^{J=0}_{N_f Z_f} | \hat{O}^{0\nu\beta\beta}_\xi | \Psi^{J=0}_{N_i Z_i} \rangle$$

Generator Coordinate Method

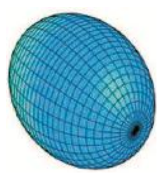


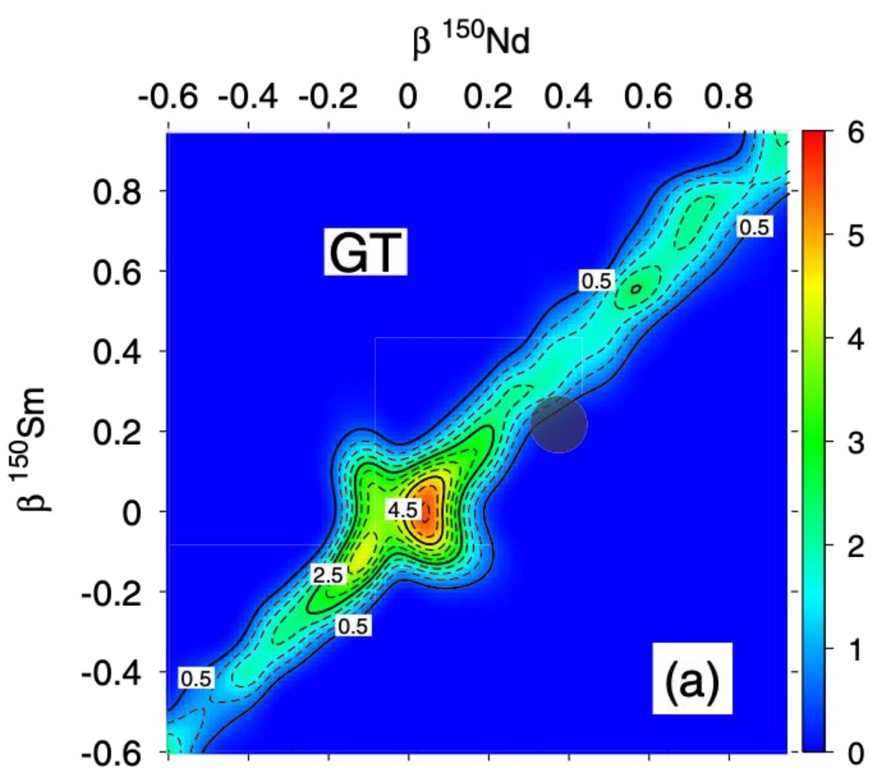
The first 2⁺-state energies and B(E2) given by Hamiltonian-based GCM are in great agreement with SM results.

Generator Coordinate Method

Q1: Which correlations are the most relevant to $0\nu\beta\beta$ NMEs?

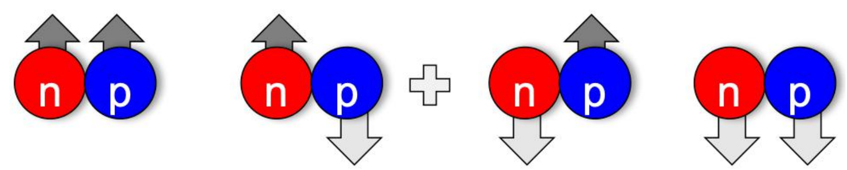
Axial deformation

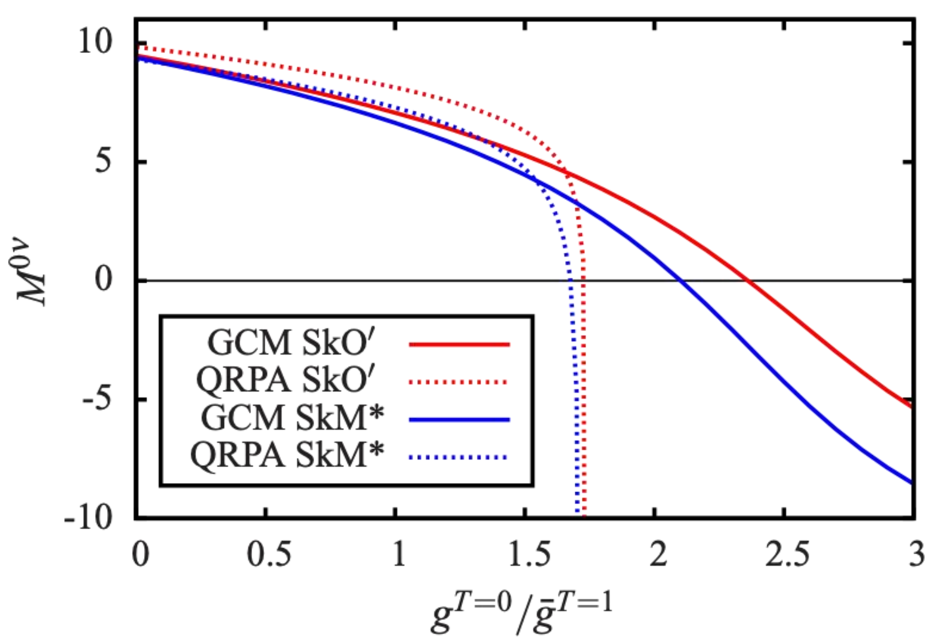
prolate   oblate



T. Rodriguez and G. Martinez-Pinedo
PRL **105**, 252503 (2010)

Proton-neutron pairing





N. Hinohara and J. Engel
PRC **90**, 031301(R) (2014)

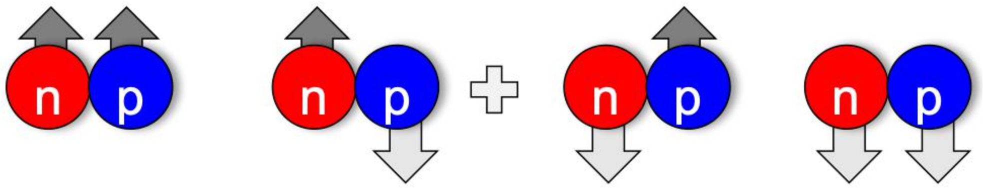
Triaxial deformation

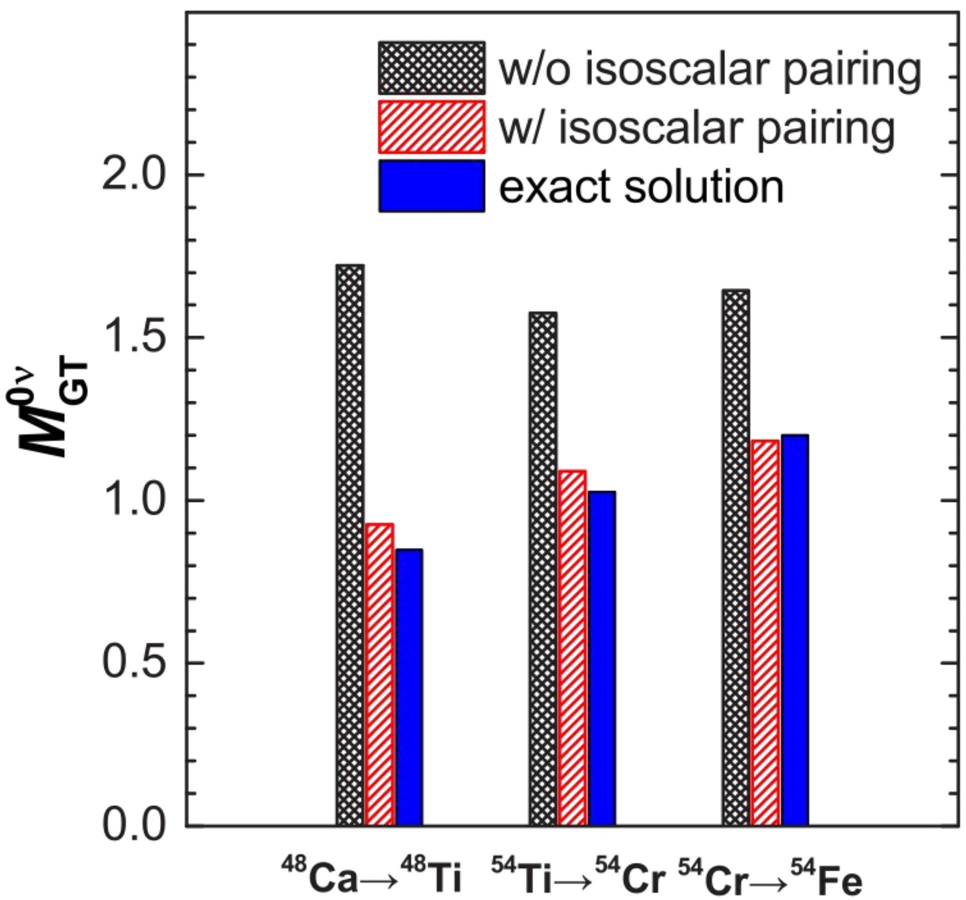
prolate oblate triaxial

Both theory and experiment indicate that ^{76}Ge and ^{76}Se are triaxially deformed, but the effect on $0\nu\beta\beta$ NMEs has never been investigated.

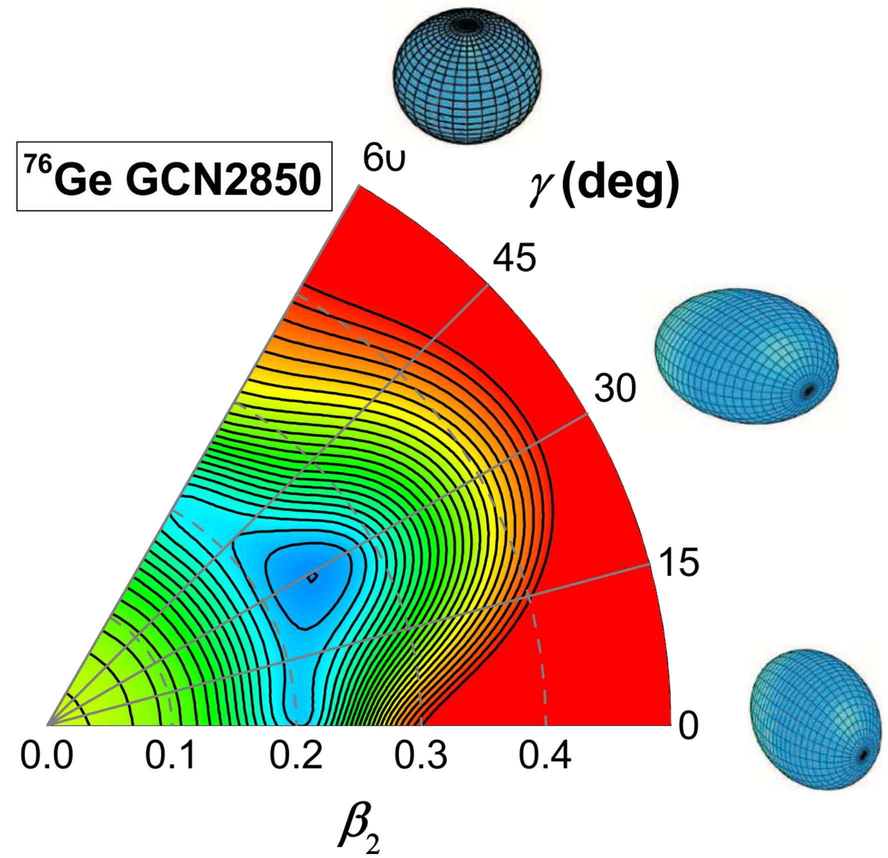
Generator Coordinate Method

Proton-neutron pairing





Triaxial deformation



	GCN2850	JUN45
Axial GCM	$\boxed{2.93}$	$\boxed{3.51}$
Triaxial GCM	$\boxed{2.56}$	$\boxed{3.16}$
Exact	2.81 [6]	3.37 [35]

~10% reduced if triaxial-shape fluctuation is included.

CFJ, J. Engel, and J.D. Holt, PRC 96, 054310 (2017)

Generator Coordinate Method

Q2: What is the effect from enhancement of model space?

*For the first time, we work in the full fp-sdg two-shell space, which is **unreachable** by shell model.*

- The effective fp-sdg-shell interaction calculated by EKK perturbative method.
- The three-body part is reduced to an effective two-body force by summing the third particle over a set of occupied states (^{56}Ni here).

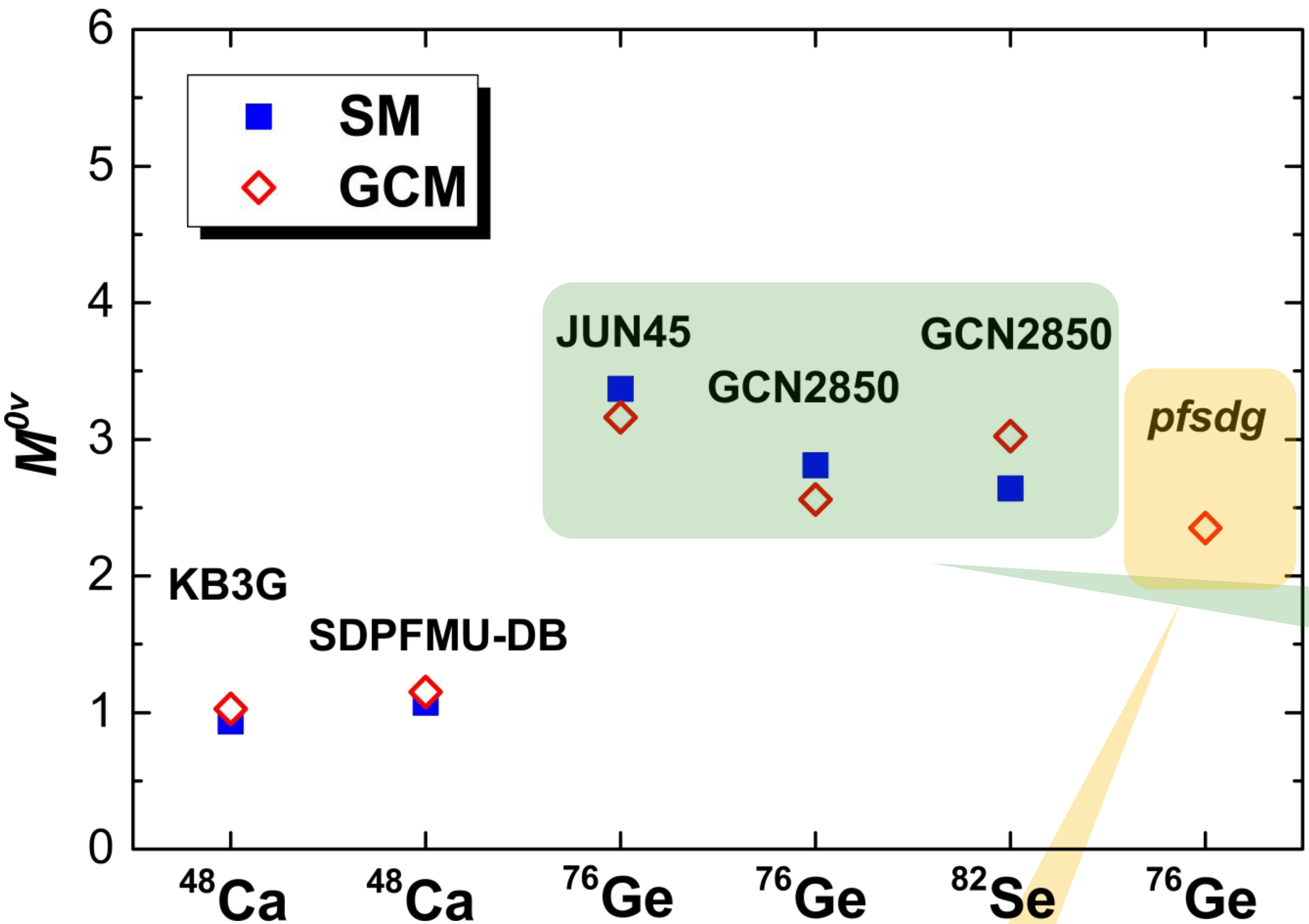
TABLE II. GCM results for the Gamow-Teller ($M^{0\nu}_{\mathrm{GT}}$), Fermi ($M^{0\nu}_{\mathrm{F}}$), and tensor ($M^{0\nu}_{\mathrm{T}}$) $0\nu\beta\beta$ matrix elements for the decay of ^{76}Ge in two shells, without and with triaxial deformation.

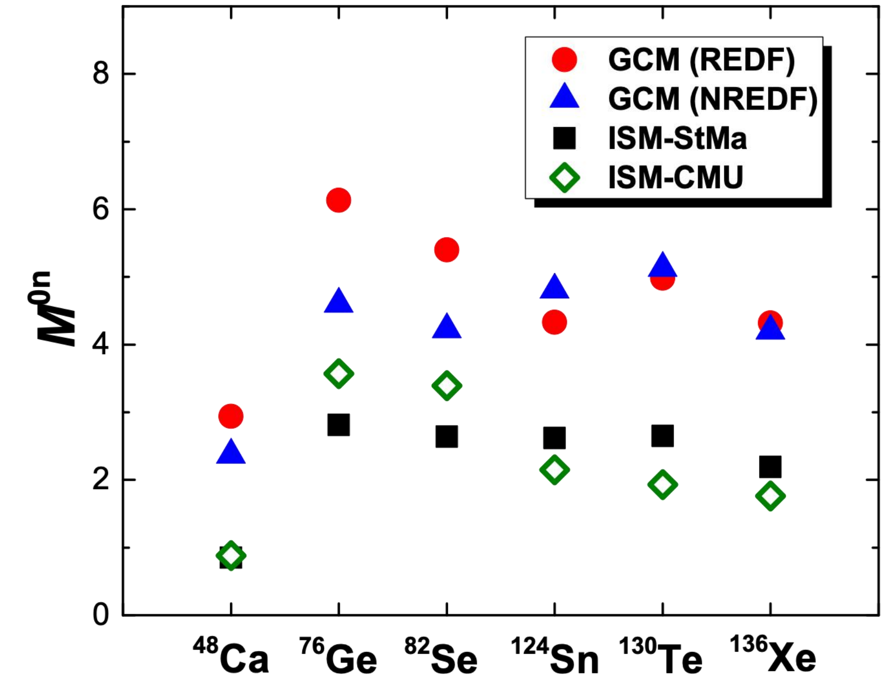
	Axial	Triaxial
$M^{0\nu}_{\mathrm{GT}}$	$\boxed{3.18}$	$\boxed{1.99}$
$-\frac{g_V^2}{g_A^2} M^{0\nu}_{\mathrm{F}}$	0.55	0.38
$M^{0\nu}_{\mathrm{T}}$	−0.01	−0.02
Total $M^{0\nu}$	3.72	2.35

Enhanced axially-deformed result:
Larger space captures more like-particle pairing.

Reduced triaxially-deformed result:
Larger space captures more effect from triaxial deformation.

Generator Coordinate Method





First-of-its-kind GCM calculation including triaxial deformation.

First-of-its-kind two full shell Hamiltonian-based GCM calculation (with triaxial deformation)

CFJ, J. Engel, and J.D. Holt, PRC 96, 054310 (2017)

Generator Coordinate Method

Q3: Is shape + pn pairing correlations good enough?

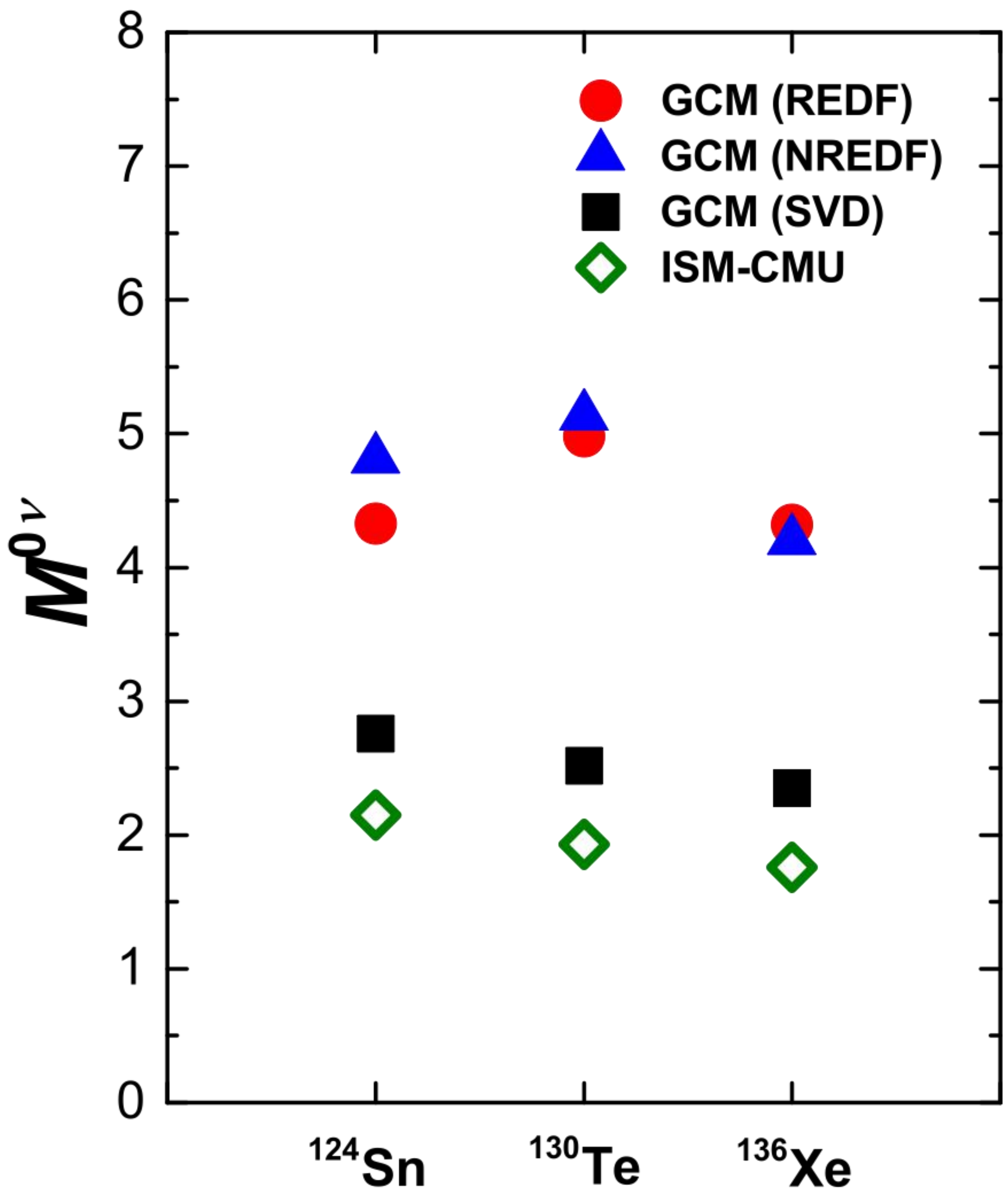


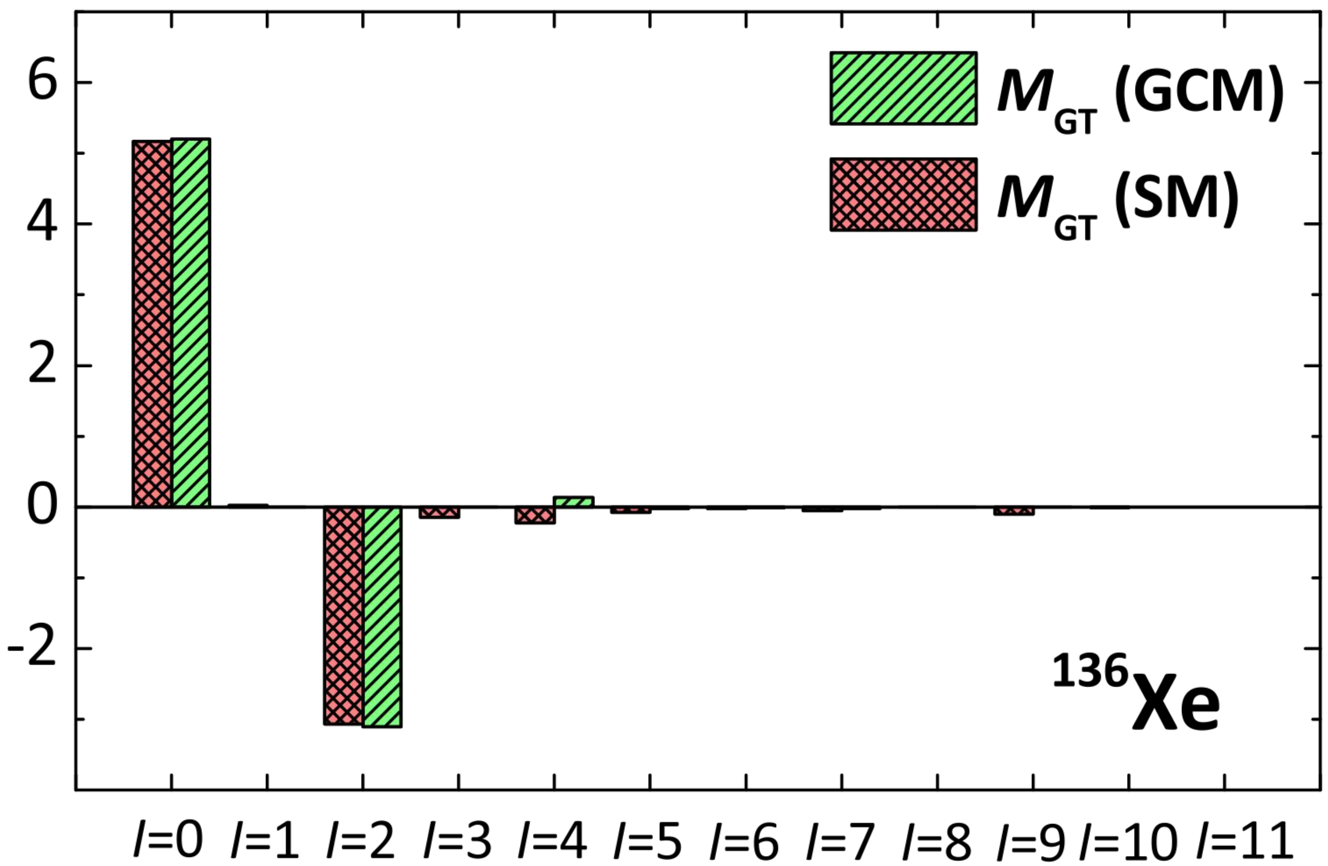
TABLE III. The NMEs obtained with SVD Hamiltonian by using GCM and SM for ^{124}Sn, ^{130}Te, and ^{136}Xe. The SM results are taken from Refs. [9,10]. CD-Bonn SRC parametrization was used.

		$M^{0\nu}_{GT}$	$M^{0\nu}_{F}$	$M^{0\nu}_{T}$	$M^{0\nu}$
^{124}Sn	GCM	2.48	−0.51	−0.03	2.76
	SM	1.85	−0.47	−0.01	2.15
^{130}Te	GCM	2.25	−0.47	−0.02	2.52
	SM	1.66	−0.44	−0.01	1.94
^{136}Xe	GCM	2.17	−0.32	−0.02	2.35
	SM	1.50	−0.40	−0.01	1.76

Fermi part agrees well.
Gamow-Teller part is improved remarkably, but still ~30% overestimated. ***WHY?***

CFJ, M. Horoi, and A. Neacsu, PRC 98, 064324 (2018)

0vββ Decay NME for Sn, Te, and Xe



I-pair decomposition:

Decomposition of the NMEs over the angular momentum I of the proton (or neutron) pairs, that is

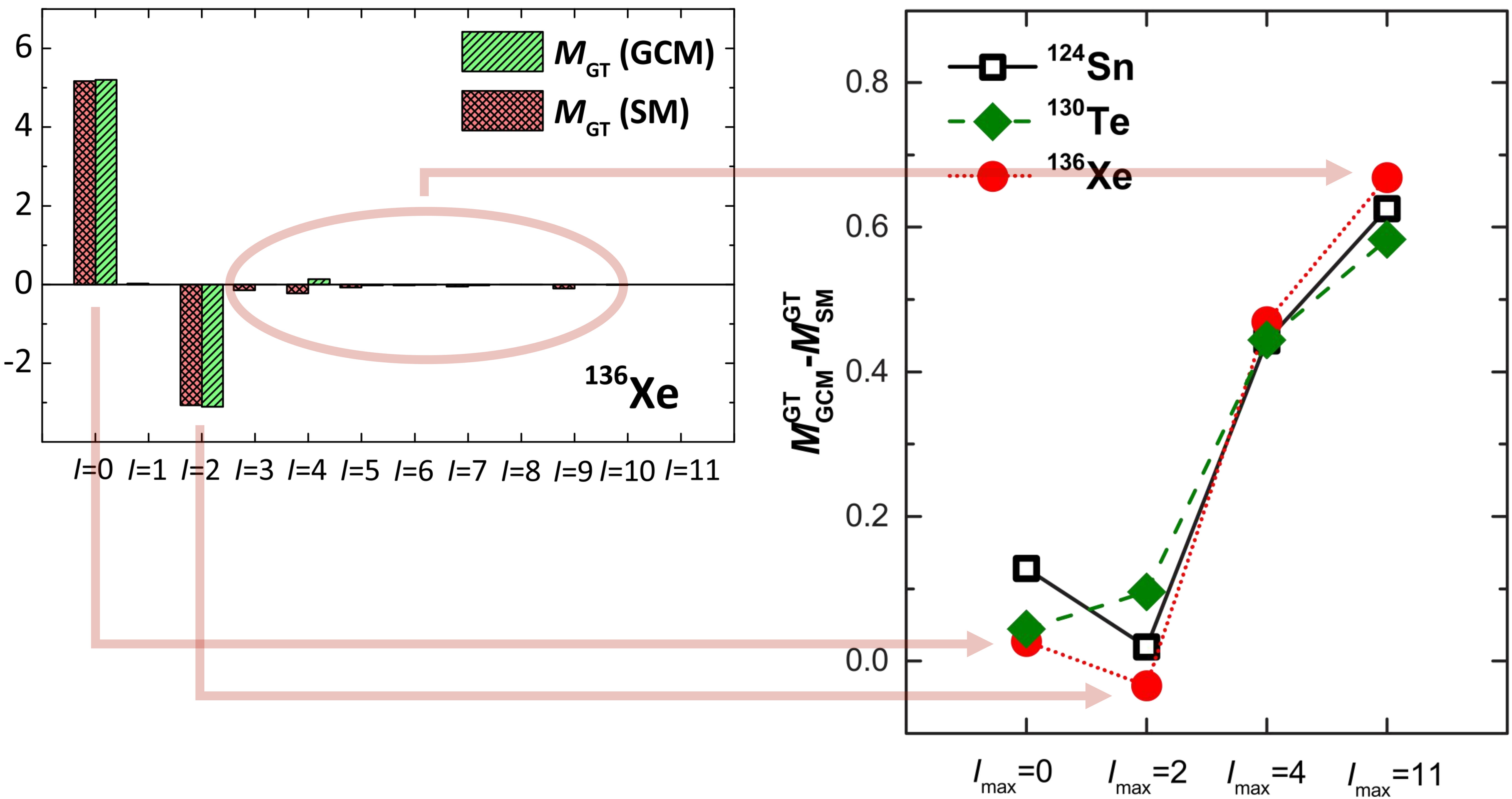
$$M_\alpha^{0\nu} = \sum_I M_\alpha^{0\nu}(I)$$

where $M_\alpha^{0\nu}(I)$ represent the contribution from each pair-spin I to the part of the NME.

- GCM reproduces well the cancellation between the $I = 0$ and $I = 2$ contributions.
- GCM barely produce any contributions with $I \geq 4$.

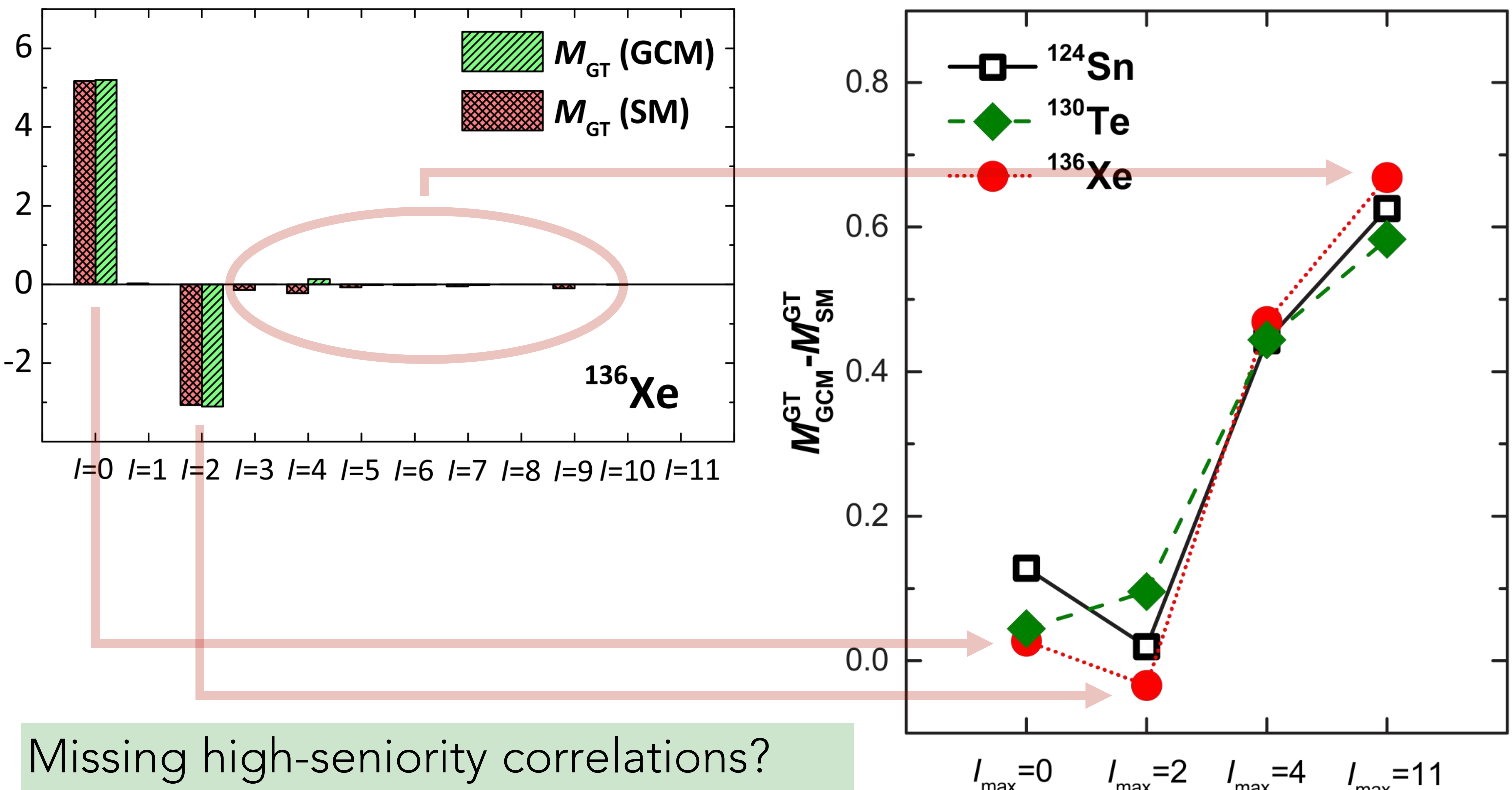
CFJ, M. Horoi, and A. Neacsu, PRC 98, 064324 (2018)

$0\nu\beta\beta$ Decay NME for Sn, Te, and Xe



CFJ, M. Horoi, and A. Neacsu, PRC 98, 064324 (2018)

0νββ Decay NME for Sn, Te, and Xe



Missing high-seniority correlations?
Vibrational, quasiparticle excitation, etc…

CFJ, M. Horoi, and A. Neacsu, PRC 98, 064324 (2018)

QTDA-driven GCM

Q4: So shape + pn pairing correlations is not enough.
How to pin down all the correlations that are relevant?

I proposed a novel idea to incorporate important correlations in GCM.

Starts from the HF minimum.

Apply Thouless evolution to explore the energy landscape

Thouless theorem:

$$\exp(\hat{Z})|\Psi\rangle = |\Psi'\rangle \equiv |\Psi(Z)\rangle$$

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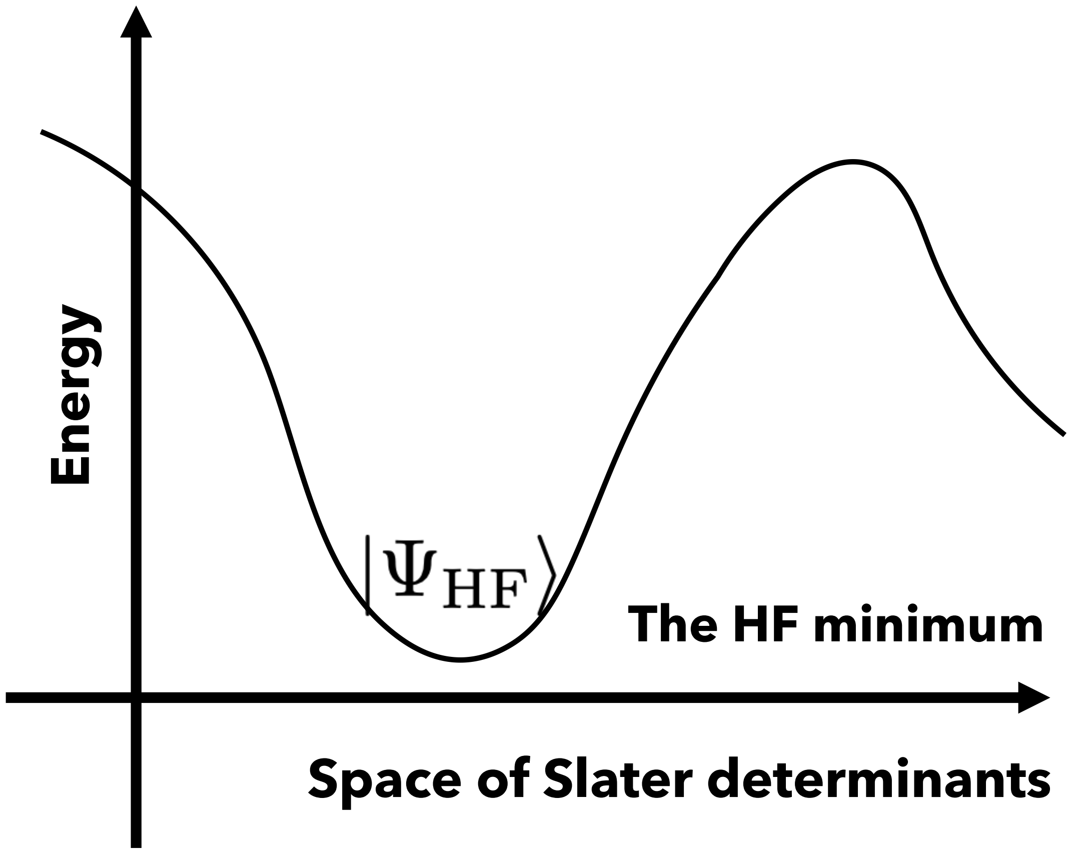

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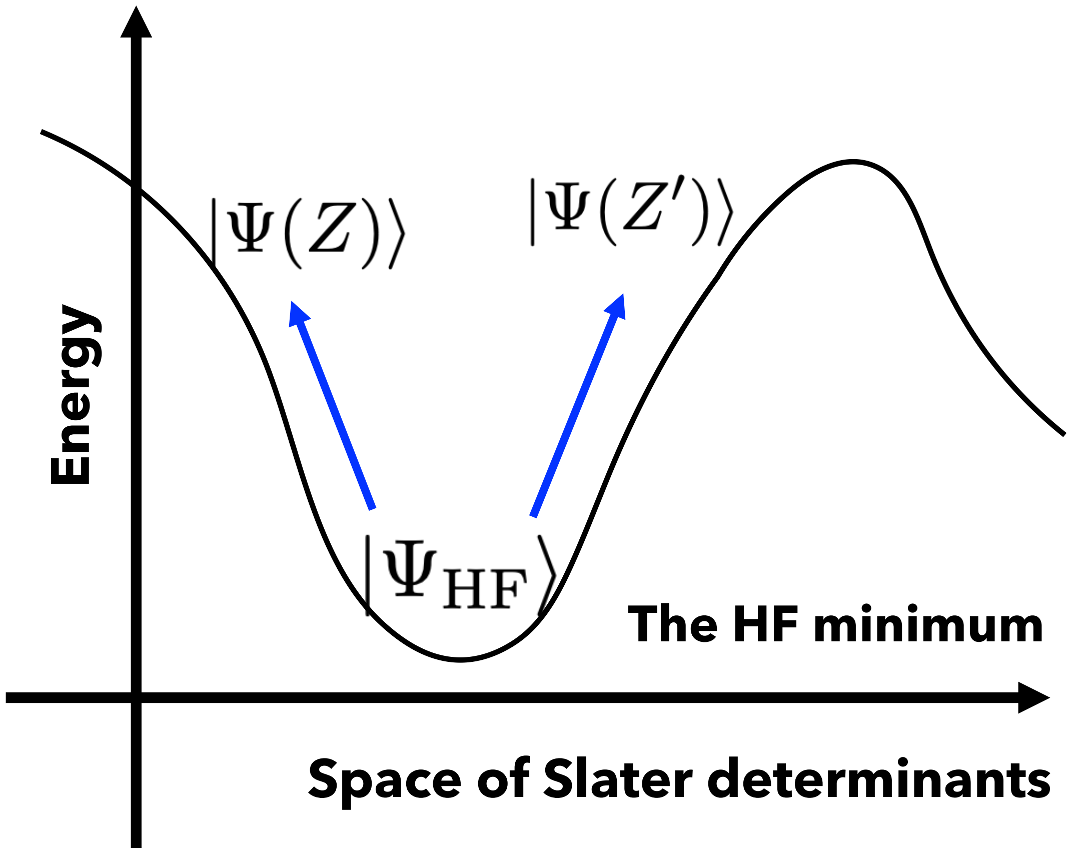

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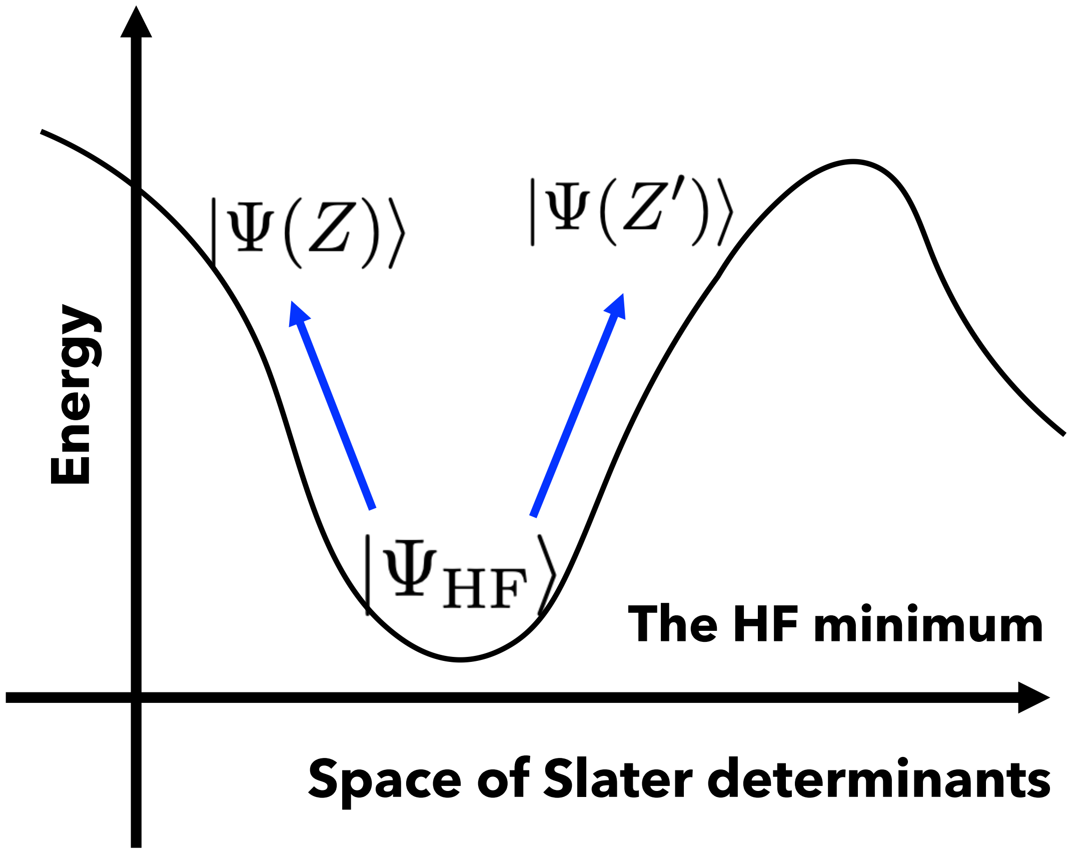


Difine an energy landscape $E(Z) = \langle\Psi(Z)|\hat{H}|\Psi(Z)\rangle$ which can be expanded in Z. Note that the curvature around HF minimum approximates the landscape as a quadratic in Z and thus a multi-dimensional harmonic oscillator, leading to TDA/RPA and their quasiparticle extension.

QTDA-driven GCM

Here we generate non-orthogonal states by applying Thouless evolution with QTDA operators.

Low-lying excited states are approximated as linear combinations of two-quasiparticle excitations, represented by QTDA operator:

$$\hat{Z}_r = \frac{1}{2}\sum_{\alpha\alpha'} Z^r_{\alpha\alpha'}\hat{c}^\dagger_\alpha(0)\hat{c}^\dagger_{\alpha'}(0) \quad \text{where} \quad \hat{c}_\alpha(0) = \sum_\beta \hat{a}_\beta U^*_{\beta\alpha}(0) + \hat{a}^\dagger_\beta V^*_{\beta\alpha}(0)$$

One computes the matrix elements of the Hamiltonian in a basis of two-quasiparticle excited states

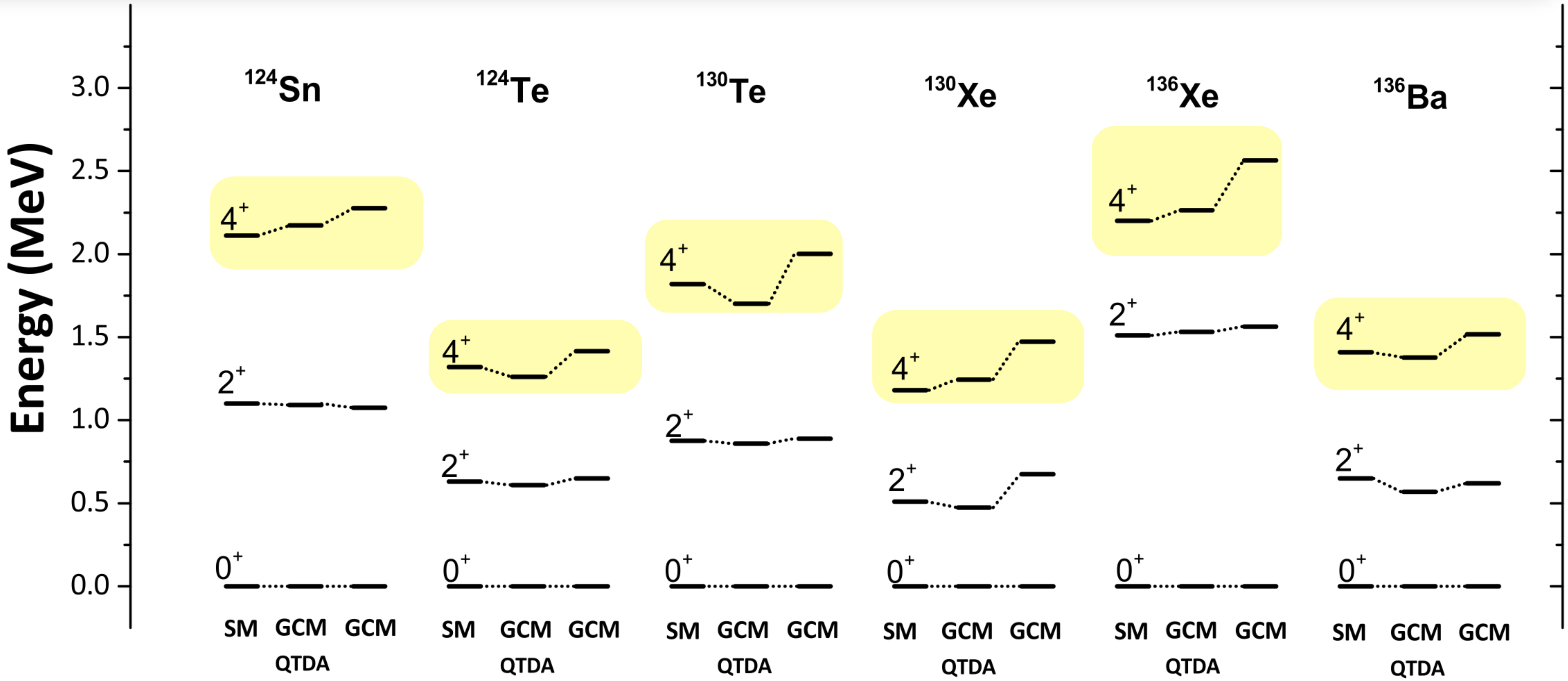
$$A_{\alpha\alpha',\beta\beta'} = \langle\Phi_0|[\hat{c}_{\alpha'}(0)\hat{c}_\alpha(0), [\hat{H}, \hat{c}^\dagger_\beta(0)\hat{c}^\dagger_{\beta'}(0)]]|\Phi_0\rangle$$

We then solve $\sum_{\beta\beta'} A_{\alpha\alpha',\beta\beta'} Z^r_{\beta\beta'} = E^{\mathrm{QTDA}}_r Z^r_{\alpha\alpha'}.$

to find the coefficients $Z^r_{\alpha\alpha'}$ of QTDA operator, and apply Thouless theorem to get a new state

$$|\Phi_r\rangle = \exp\left(\lambda\hat{Z}_r\right)|\Phi_0\rangle$$

QTDA-driven GCM



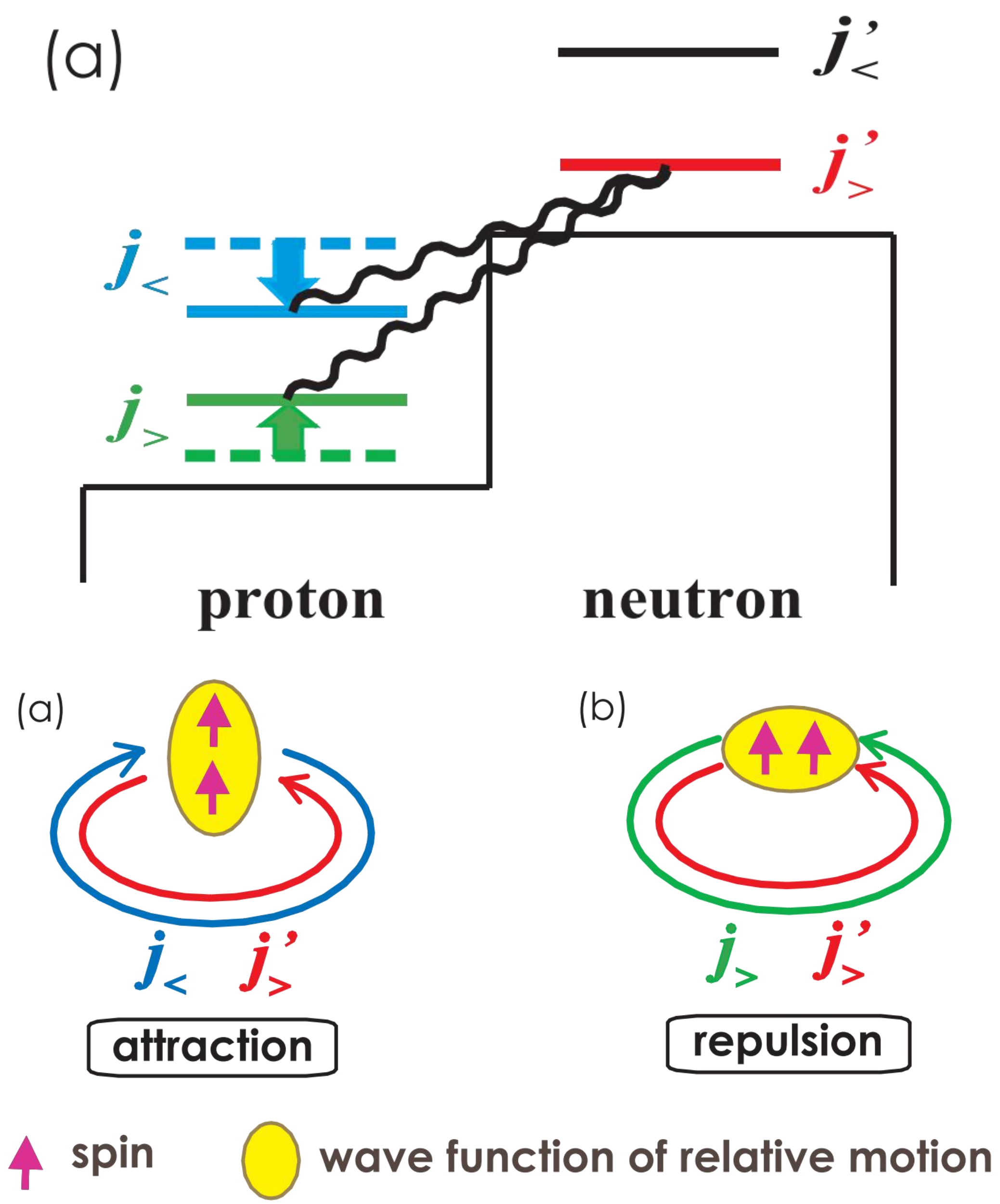
		$M^{0\nu}_{GT}$	$M^{0\nu}_{F}$	$M^{0\nu}_{T}$	$M^{0\nu}$
^{124}Sn	CHFB-GCM	2.48	−0.51	−0.03	2.76
	QTDA-GCM	2.08	−0.73	−0.01	2.53
	SM	1.85	−0.47	−0.01	2.15
^{130}Te	CHFB-GCM	2.25	−0.47	−0.02	2.52
	QTDA-GCM	1.97	−0.69	−0.01	2.39
	SM	1.66	−0.44	−0.01	1.94
^{136}Xe	CHFB-GCM	2.17	−0.32	−0.02	2.35
	QTDA-GCM	1.65	−0.50	−0.01	1.96
	SM	1.50	−0.40	−0.01	1.76

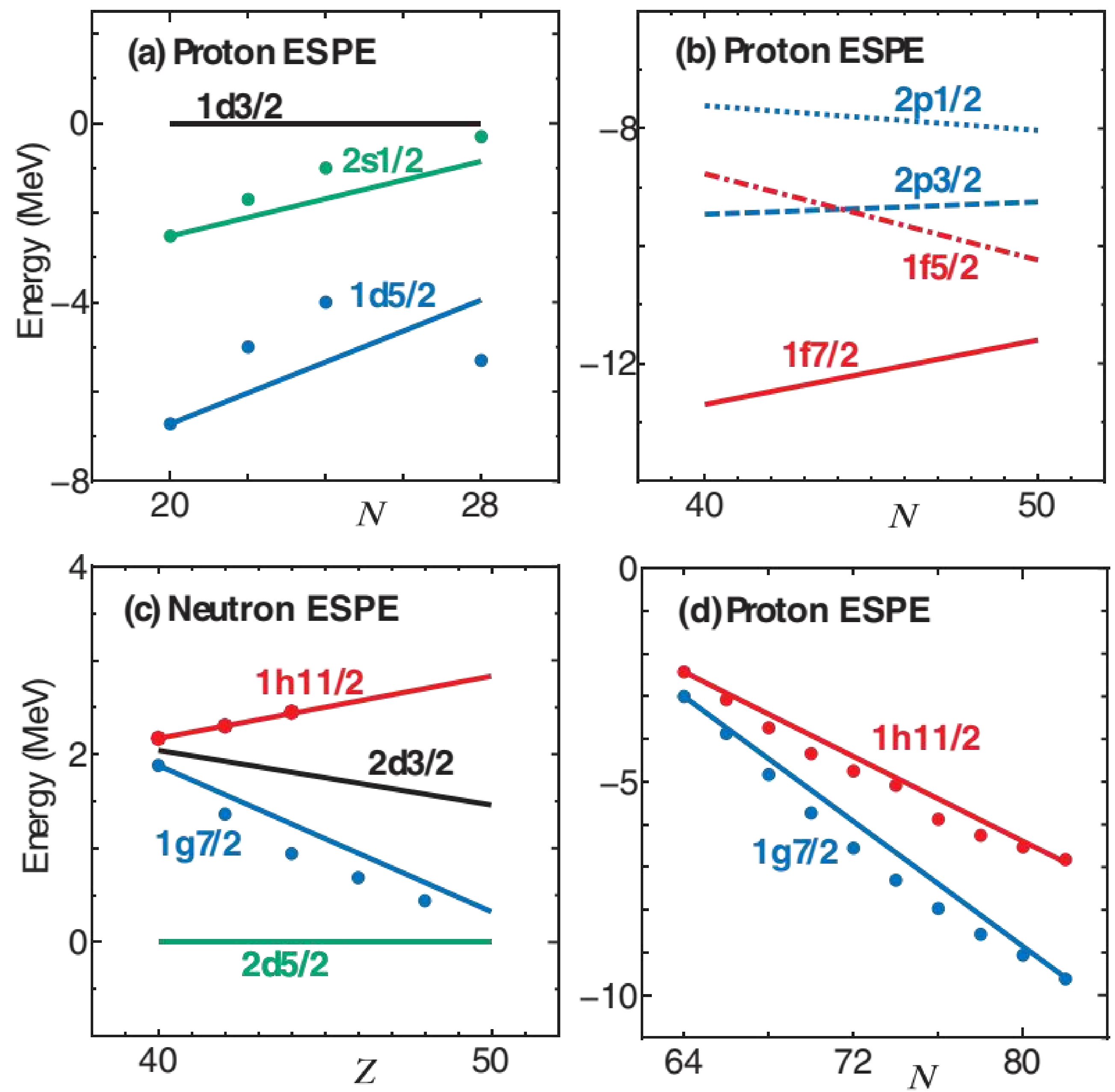
Inclusion of the vibrational motion and two-quasiparticle configurations is important.

CFJ and C.W. Johnson, PRC 100, 031303(R) (2019)

The effect from the tensor force

Q5: How about the effect from the tensor force?
Considering that it has a robust effect on the single-particle energies of nuclei

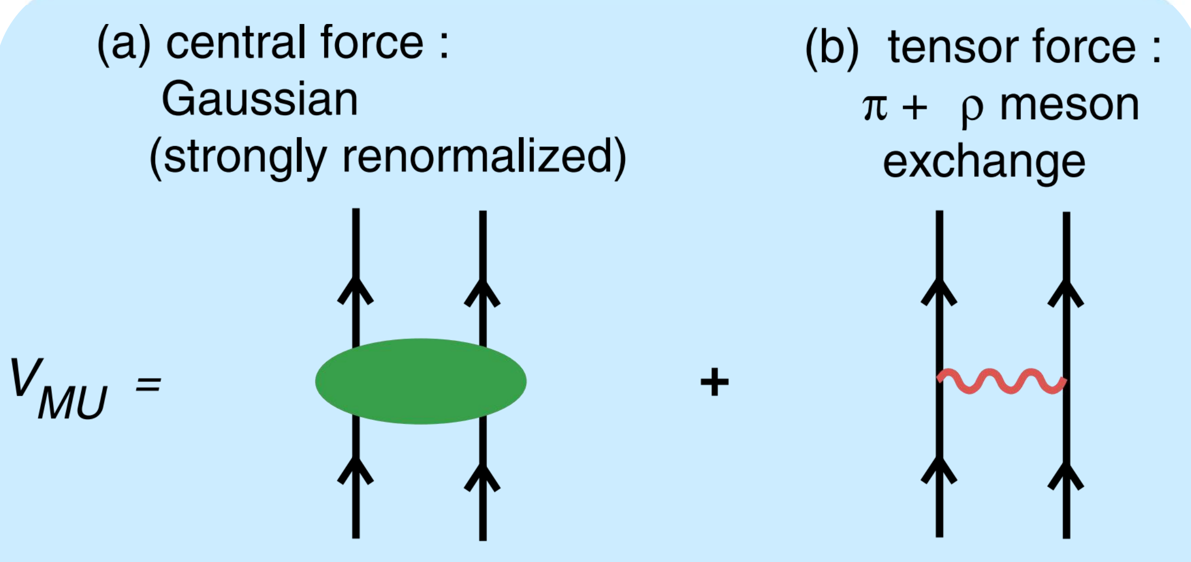




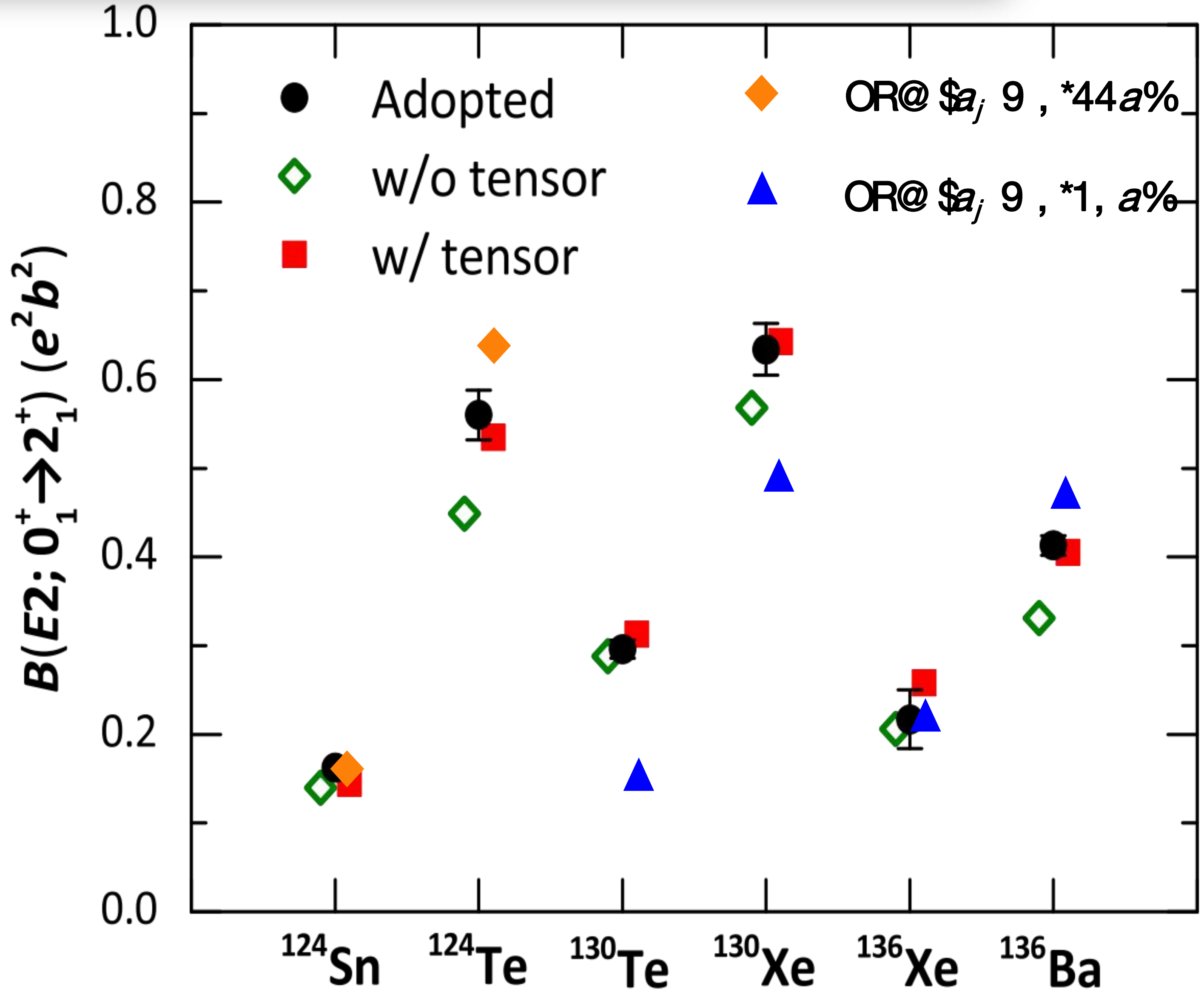
the monopole interaction produced by the tensor force.

T. Otsuka *et al.*, PRL 95, 232502 (2005)
T. Otsuka et al., PRL 105, 012501 (2010)

The effect from the tensor force



Diagrams for the V_{MU} interaction



Calculated $0\nu\beta\beta$ NMEs:

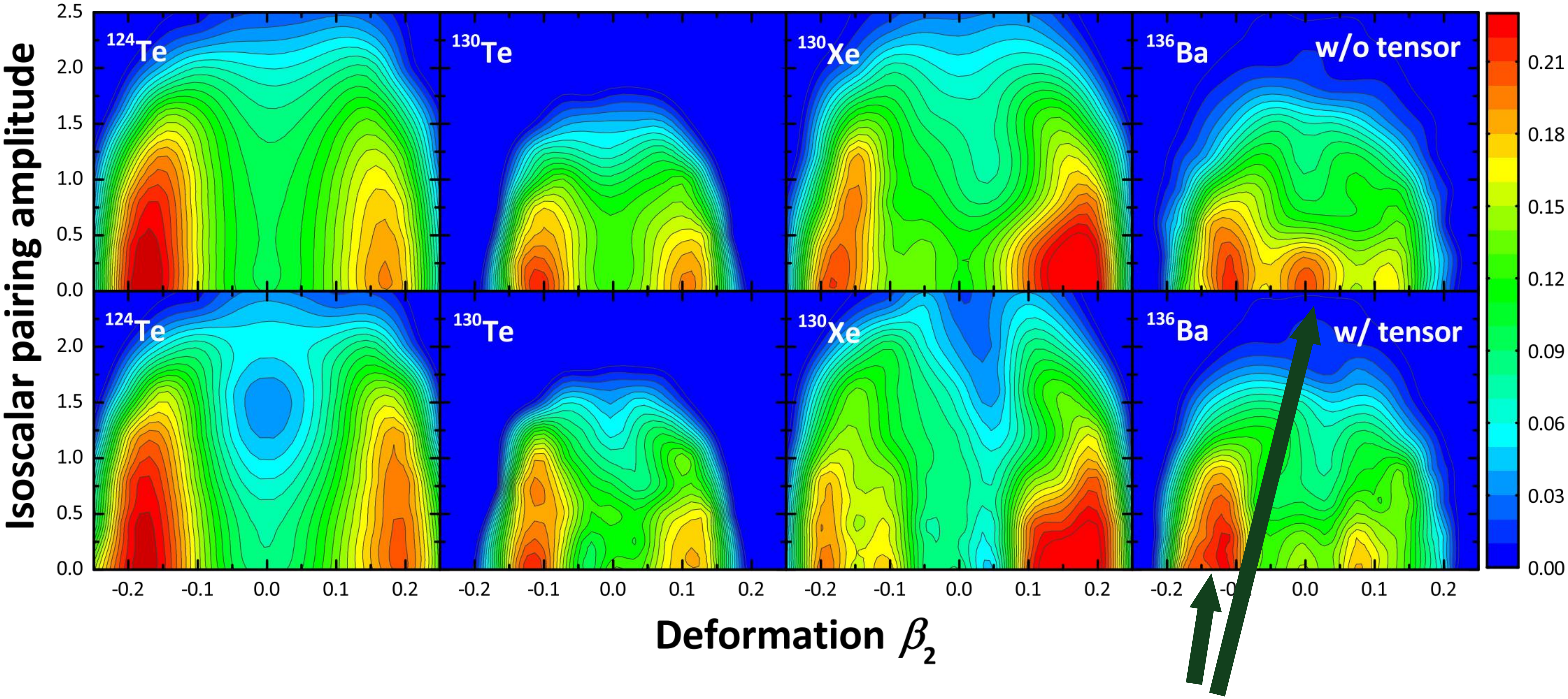
		$M^{0\nu}_{GT}$	$M^{0\nu}_{F}$	$M^{0\nu}_{T}$	$M^{0\nu}$
^{124}Sn	w/o tensor	3.56	−0.64	−0.061	3.91
	w/ tensor	2.65	−0.64	−0.020	3.04
^{130}Te	w/o tensor	4.29	−0.75	−0.064	4.70
	w/ tensor	3.33	−0.65	−0.015	3.73
^{136}Xe	w/o tensor	3.26	−0.44	−0.046	3.49
	w/ tensor	2.17	−0.50	−0.009	2.48

V_{MU} provides a better description of nuclear structure properties of ^{124}Sn/^{124}Te, ^{130}Te/^{130}Xe, and ^{136}Xe/^{136}Ba

CFJ and C. X. Yuan, in preparation.

NMEs are suppressed, ***why?***

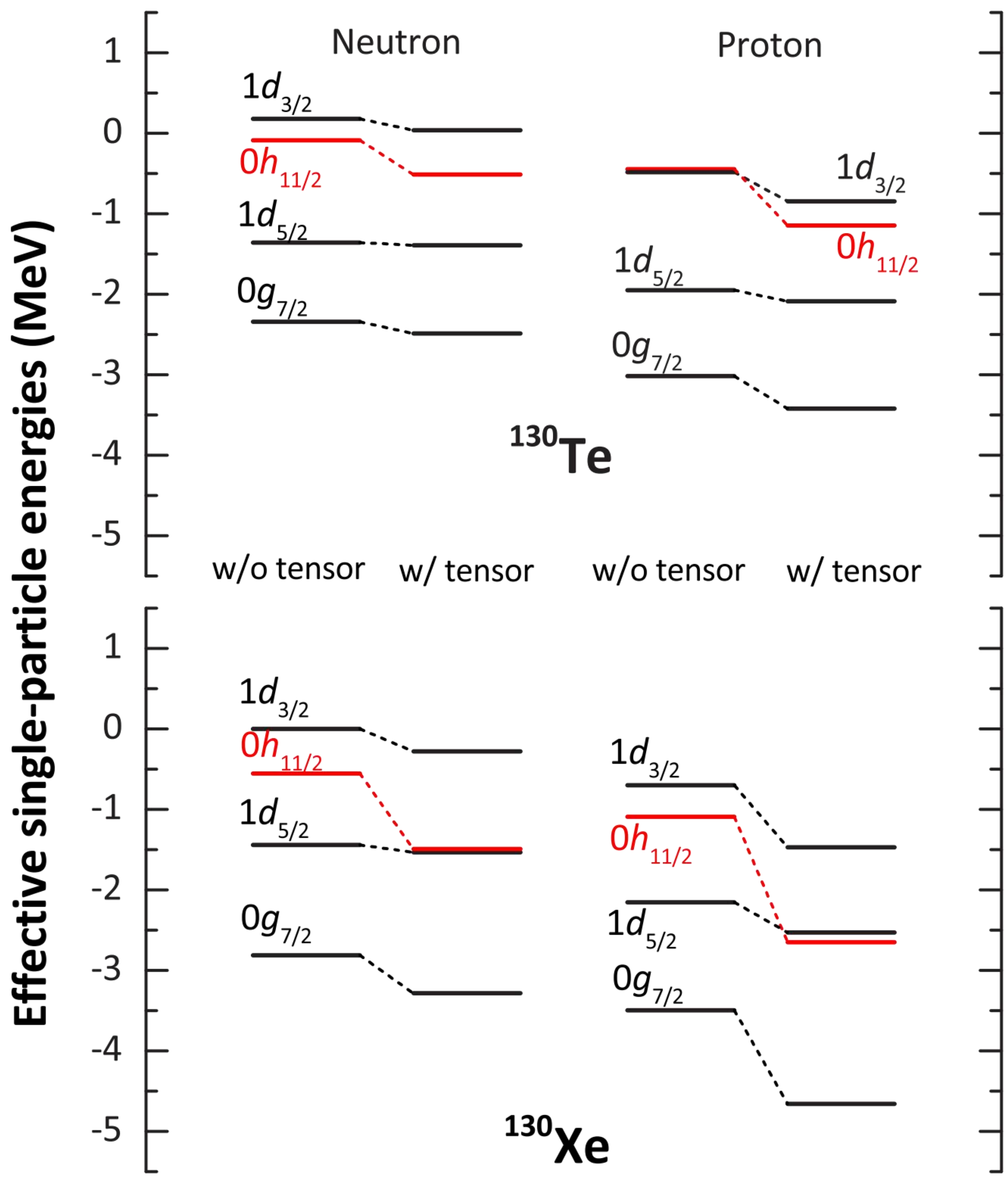
The effect from the tensor force



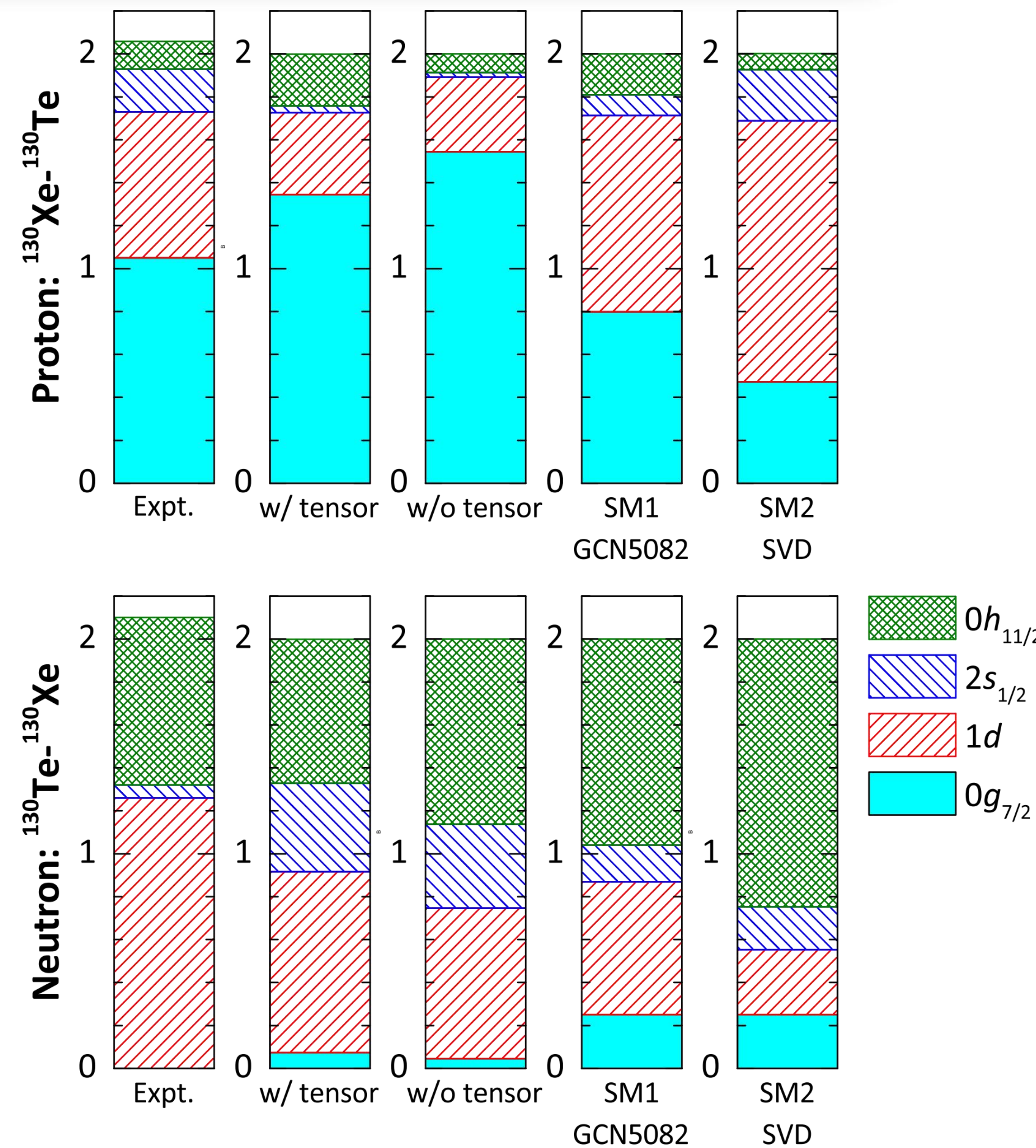
- Enhanced quadrupole deformation.
- Enhanced isoscalar pairing.

CFJ and C. X. Yuan, in preparation.

The effect from the tensor force



Neutron and proton effective single-particle energies at spherical shape relative to $2s_{1/2}$ orbit.



Change in proton occupancies and neutron vacancies

CFJ and C. X. Yuan, in preparation.

Summary

- $0\nu\beta\beta$ decay is crucial probe for determining whether neutrinos are Majorana fermion.
- Hamiltonian-based GCM enables treatment of systems presently **unreachable** by other methods.
- Using vibration modes (e.g. QTDA) to build basis states around HFB shows improvement in nuclear structure aspects and $0\nu\beta\beta$ NMEs.

Next Steps from Here…

- More reference states
  - More QTDA phonons, or combine QTDA evolution with constrained HFB.
- Quasiparticle random phase approximation (QRPA) operators.
- Effective Hamiltonian in larger space, or from *ab initio* non-perturbative method.
  - **Target nuclei: ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{150}Nd…**

In Collaboration with...

- Jiangming Yao, SYSU
- Ning Li, SYSU
- Cenxi Yuan, SYSU
- Jonathan Engel, UNC
- Calvin W. Johnson, SDSU
- Jason D. Holt, TRIUMF
- Mihai Horoi, CMU
- Nobuo Hinohara, U of Tsukuba
- Javier Menendez, U of Barcelona

Thanks for your attention!