Study of $0\nu2\beta$-decay from Lattice QCD

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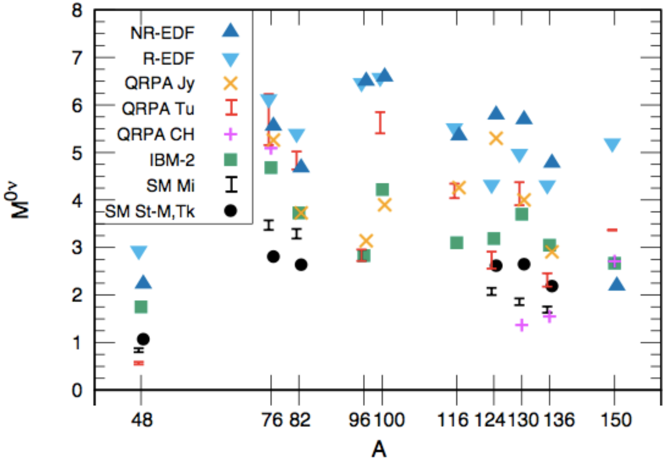
Theoretical prediction of nuclear matrix element

For nuclear matrix element, various models yield O(100%) discrepancies



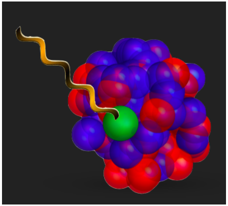
Uncertainties originates from

- Complication of nuclear force
- Short distance correlation in double beta decay

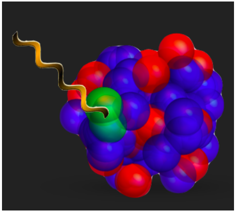
Single β decay of nuclei

Coupling of currents to nuclei in nuclear EFT [W. Detmold, talk at Lat18]

- One body coupling dominates

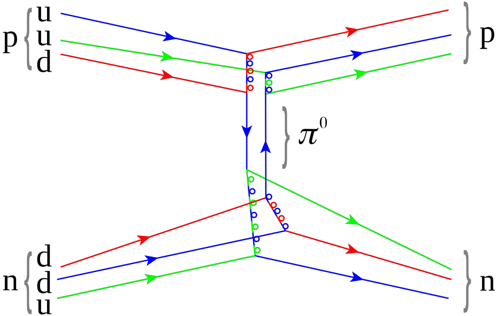


- Two nucleon contributions are subleading but non-negligible



A promising way to provide few-body inputs to ab initio many-body calculations

Nuclear force is mediated by the pion, first proposed by H. Yukawa in 1935

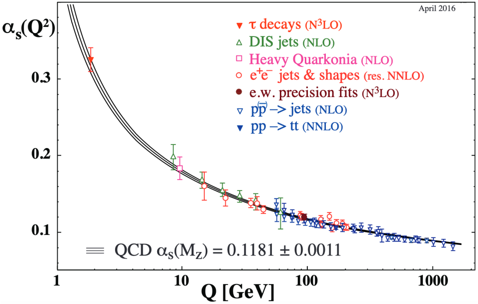


Nuclear force is essentially strong interaction between quarks and gluons

Strong interaction between quarks and gluons is described by fundamental theory $\Leftarrow$ Quantum Chromodynamics (QCD)

QCD's two peculiar properties

- **asymptotic freedom**
  $\Rightarrow$ non-perturbative at low energy scale
- **color confinement**
  $\Rightarrow$ quarks and gluons cannot be isolated singularly



ECT* workshop on $0\nu2\beta$

Progress and Challenges in Neutrinoless Double Beta Decay

ECT* workshop subscription

ECT*, Strada delle Tabarelle, 286, Villazzano, 38123 Trento, Italy

Monday, 15 July 2019 at 08:00 - Friday, 19 July 2019 at 18:00 (CEST)

organized by Menendez, Mereghetti, Nicholson, Pastore, Walker-loud

Summarize on recent advances in

- Lattice QCD
- Chiral effective field theory
- Many-body nuclear theory

Target on

- a seamless connection between the theory at quark and nuclear level
- reliable calculations of the nuclear matrix elements, with robust uncertainty

Introduction to lattice QCD

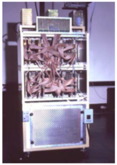
Nearly 50 years for lattice QCD

- Invented by Kenneth G. Wilson in 1973
- 1^{st} numerical implementation by M. Creutz in 1979
- QCD computers 1983 – 2011 [credit by N. Christ]

Matrix Multiplier

1Mflops 1983

16-Node



256 Mflops 1985

64-Node

1.0 Gflops 1987

256-Node

16 Gflops 1989

QCDSP

600 Gflops 1998

QCDOC

20 Tflops 2005

LLNL Sequoia, IBM

20 Pflops 2011

QCD computers start to enter in the Eflops generation, 10^{18} floating point operation per second

Supercomputer in China

Sunway TaihuLight & Tianhe-2A ranked 4th & 5th in 2020 TOP500 list

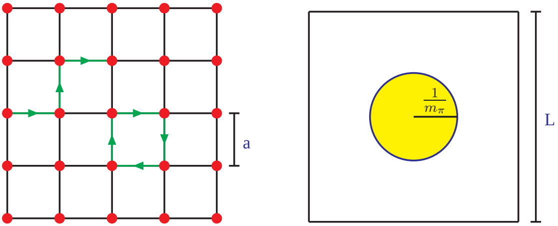
计算速度达到
每秒$0.93 \cdot 10^{17}$
次浮点运算

计算速度达到
每秒$0.34 \cdot 10^{17}$
次浮点运算

PKU group is using Tienhe-3 Proptotype Machine

Lattice discretization

- quark fields live on the lattice sites, $\psi(x)$, $x_\mu = n_\mu a$
- gluons represented as links between lattice sites, $U_\mu(x) = e^{iagA_\mu(x)}$



With finite a and L, quarks and gluons can be simulated on supercomputer

Euclidean path integral:

- Minkowski time replaced by $x_0 \to -it \quad \Rightarrow \quad e^{-iHx_0} \to e^{-Ht} = e^{-S[\psi,\bar{\psi},A]}$
- Same Hamiltonian H for Minkowski space and Euclidean space

$$\langle O \rangle \sim \int [d\psi][d\bar{\psi}][dA]\, O\, e^{-S[\psi,\bar{\psi},A]}$$

Configuration simulation

Integrate out the quark fields using Grassmann Algebra

$$\langle O \rangle \sim \int [dU] O[U] \det(\not{D} + m) e^{-S_g[U]}$$

Importance sampling: generate gauge configurations with probability distribution

$$p[U] \propto \det(\not{D} + m) e^{-S_g[U]}$$

this can be achieved by hybrid Monte Carlo simulation: Monte Carlo + Molecular Dynamics

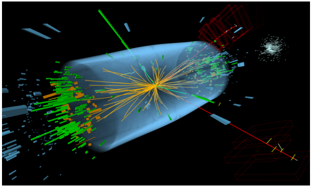
Integration is approximated by average over gauge configurations

$$\int [dU] \det(\not{D} + m) e^{-S_g[U]} \quad \rightarrow \quad \frac{1}{N} \sum_{\{U\}}$$

statistical error is reduced by $1/\sqrt{N}$

HEP Experiment

BEPC collider(Energy、Luminosity)



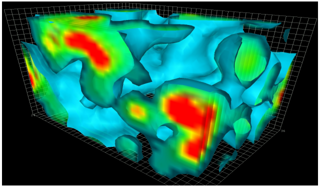
Collision， Events

↓

BES III Detector, measurement

LQCD simulation

Super Computer(Performance、Memory)



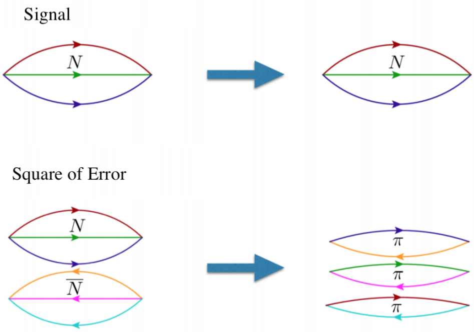
Simulation, QCD vacuum

↓

Lattice QCD calculation

Challenges for Lattice QCD

At present, lattice QCD mainly targets on light nuclei
– because of two exponential difficulties



- For nucleus A: $\frac{\text{signal}}{\text{noise}} \sim \exp\left[-A(M_N - 3/2 m_\pi)t\right] \quad \Rightarrow \quad$ a sign problem!
- Complexity: number of Wick contractions $= N_u!N_d!N_s!$
  - e.g. $^4\mathrm{He} \Rightarrow$ naively 5×10^5 contractions!
    $\Rightarrow$ Smart contraction code designed

Lattice QCD calculation of $0\nu2\beta$ decay

Double β decay of nuclei

Cirigliano, Dekens, Mereghetti, Walker-loud, PRC97 (2018) 065501

Begin with the effective Lagrangian $\mathcal{L}_{\rm eff}$ for the single β decay

$$\mathcal{L}_{\rm eff} = 2\sqrt{2} G_F V_{ud} (\bar{u}_L \gamma_\mu d_L)(\bar{e}_L \gamma_\mu \nu_{eL})$$

Contributions are identified into three regions in EFT

- Hard region: $\Lambda \gg 1$ GeV

$$\int d^4x\, e^{i\Lambda x} \mathcal{L}_{\rm eff}(x) \mathcal{L}_{\rm eff}(0) \sim 8 G_F^2 V_{ud}^2 \frac{m_{\beta\beta}}{\Lambda^2} \underbrace{(\bar{u}_L \gamma_\mu d_L)(\bar{u}_L \gamma_\mu d_L) \bar{e}_L e_L^c}_{\text{dim-9 operator}}.$$

  - Hard neutrino and gluons are integrated out
- Soft region: $O(100$ MeV$)$ - $O(1$ GeV$)$
  - Few-body decay dominates
- Ultrasoft or radiative region: $\Lambda \ll 100$ MeV
  - Neutrinos feel the complete nucleus instead of just the nucleons

Lattice QCD can deal with both hard and soft region

Exploratory lattice QCD studies

Recent review - Lattice QCD Inputs for Nuclear Double Beta Decay

[Cirigliano, Detmold, Nicholson, Shanahan, 2003.08493]

- $2\nu 2\beta$ decay: $nn \to ppee\nu\nu$ @ $m_\pi = 800$ MeV

  [NPLQCD, PRD96 (2017) 054505, PRL119 (2017) 062003]

- $0\nu 2\beta$ decays in the pion sector
  - SD contributions in $\langle \pi^+ | O_i | \pi^- \rangle$, O_i the local four-quark operators

    [A. Nicholson et al., PRL121 (2018) 172501]
  - LD contributions in $\pi^-\pi^- \to ee$

    [XF, L. Jin, X. Tuo, S. Xia, PRL122 (2019) 022001]
  - LD contributions in $\pi^- \to \pi^+ ee$

    [X. Tuo, XF, L. Jin, PRD100 (2019) 094511]
    [W. Detmold, D. Murphy, arXiv:2004.07404]

[NPLQCD, PRD96 (2017) 054505, PRL119 (2017) 062003]

- Not detectable by experiment, as *nn* is not bound
- Theoretically calculable to extract the two-body coupling
- Gamow-Teller matrix elment

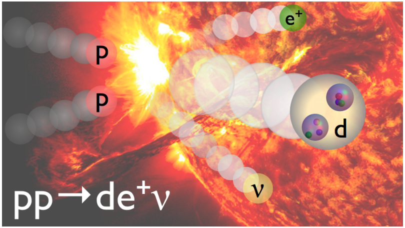
$$M_{GT}^{2\nu} = \int d^4x \langle pp|J_3^+(x)J_3^+(0)|nn\rangle = \sum_\alpha \frac{\langle pp|J_3^+|\alpha\rangle\langle\alpha|J_3^+|nn\rangle}{E_\alpha - (E_{nn}+E_{pp})/2}$$

where $J_3^+ = \bar{q}\gamma_3\frac{1-\gamma_5}{2}\tau^+ q$, τ^+ is isospin Pauli matrix, allowing $n \to p$

- Initial *nn* and final *pp* are all $^1S_0$ states
- Lightest intemediate state is $^3S_1$ pn bound state, namely $\alpha = d$ (deuteron)

$$\langle pp|J_3^+|d\rangle \quad \Rightarrow \quad pp \to de^+\nu$$

$$\langle d|J_3^+|nn\rangle \quad \Rightarrow \quad \bar{\nu}d \to nne^+$$



Involve *pp* fusion (energy production in the Sun) and $\bar{\nu}d$ collsion

[NPLQCD, PRD96 (2017) 054505, PRL119 (2017) 062003]

- Approximation: if neglect the interaction between the two nucleons?

$$\langle pp|J_3^+|d\rangle \quad \to \quad \langle p|J_3^0|p\rangle \quad \to \quad g_A$$

The deuteron contributes as g_A^2/Δ, where $\Delta = (E_{nn} + E_{pp})/2 - E_d$

- Lattice results@$m_\pi = 800$ MeV

$$\frac{\Delta}{g_A^2}\frac{|\langle pp|J_3^+|d\rangle|^2}{\Delta} = 1.00(3)(1)$$

$$\frac{\Delta}{g_A^2}\sum_{\alpha\neq d}\frac{\langle pp|J_3^+|\alpha\rangle\langle\alpha|J_3^+|nn\rangle}{E_\alpha - (E_{nn}+E_{pp})/2} = -0.04(4)(2)$$

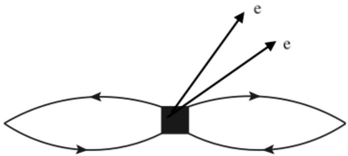
$$\frac{\Delta}{g_A^2}M_{GT}^{2\nu} = -1.04(4)(4)$$

- Total contribution from all excited state amounts to $\sim 4\%$

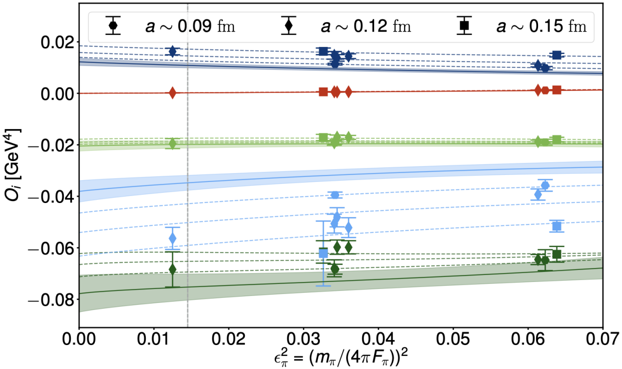
Although small, it is a non-trivial result

$\pi^- \to \pi^+ ee$: short-distance contribution

Short-distance contribution to $\pi^- \to \pi^+ ee$
$\Rightarrow$ described by four-quark-two-lepton dim-9 operators



Lattice results [A. Nicholson et al., PRL121 (2018) 172501]

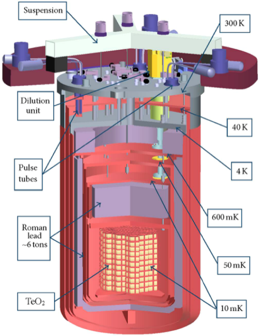


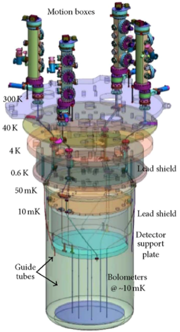
Lattice calculations from PKU group

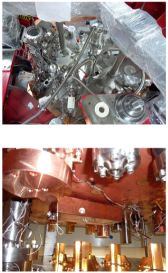
Inspired by a talk given by Prof. Huan-Zhong Huang in 2017

Majorana Neutrinos
&
Recent CUORE Result on Neutrinoless Double Beta Decay

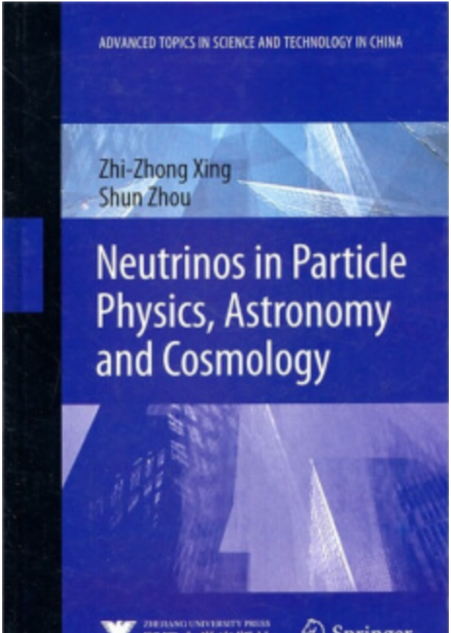
Huan Zhong Huang (黄焕中)





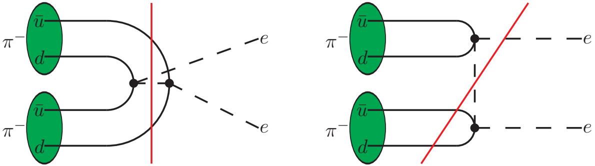


Useful book by Prof. Zhi-Zhong Xing and Shun Zhou



The amplitude $\mathcal{A}$

$$\mathcal{A} = \frac{1}{2!}\int d^4x \, \langle e_1 e_2 | \mathcal{L}_{eff}(x)\mathcal{L}_{eff}(0)|\pi\pi\rangle = \int dt \, \mathcal{M}(t)$$

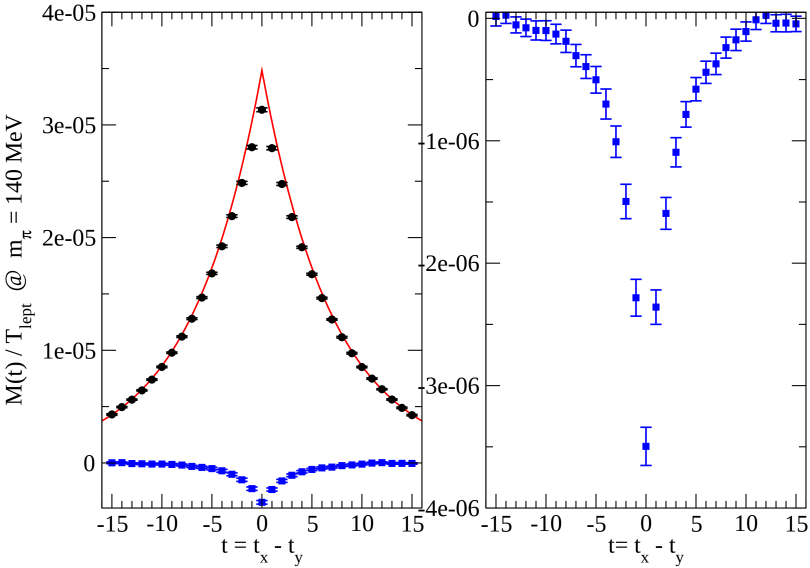


At large $|t|$, $\mathcal{M}(t)$ is saturated by ground intermediate state - $e\bar{\nu}\pi$

$$\mathcal{M}(t) \xrightarrow{|t|\gg 0} -T_{\mathrm{lept}}\frac{1}{V}\frac{2\langle 0|J_{\mu L}|\pi\rangle_{\mathrm{V}}\langle \pi|J_{\mu L}|\pi\pi\rangle_{\mathrm{V}}}{(2m_\pi)(2E_\nu)}e^{-m_\pi|t|}$$

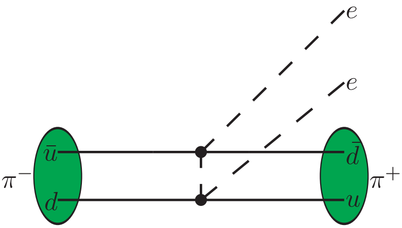
$\pi\pi \to ee$ decay amplitude @ $m_\pi = 140$ MeV

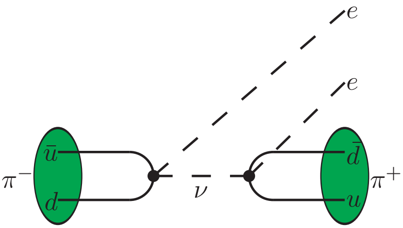
XF, L. Jin, X. Tuo, S. Xia, PRL122 (2019) 022001



$\pi^- \to \pi^+ ee$: long-distance contribution

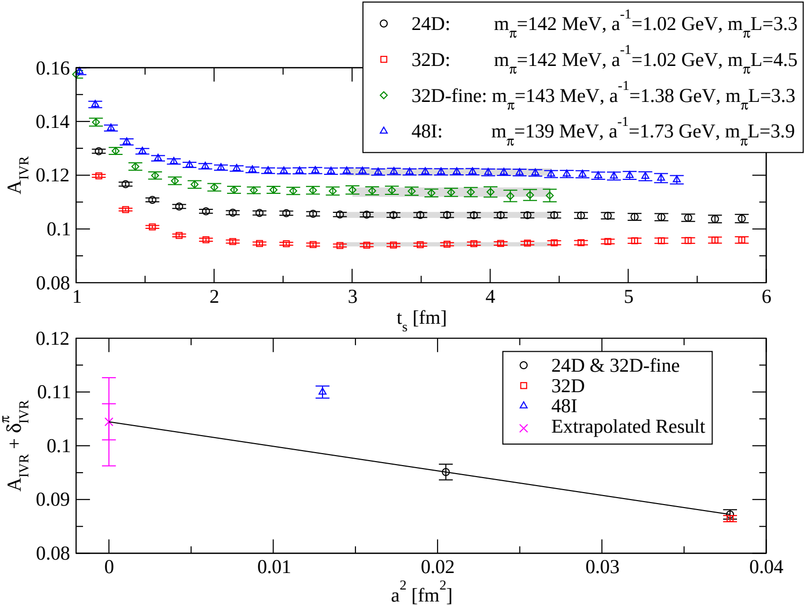
Assume that $0\nu\beta\beta$ is mediated by exchange of light Majorana neutrinos





$\pi^- \to \pi^+ ee$ decay amplitude @ $m_\pi = 140$ MeV

X. Tuo, XF, L. Jin, PRD100 (2019) 094511



Summary of $\pi^-\pi^- \to ee$ and $\pi^- \to \pi^+ ee$

Chiral perturbation theory for $\pi^-\pi^- \to ee$

[Cirigliano, Dekens, Mereghetti, Walker-Loud, PRC97 (2018) 065501]

$$\frac{\mathcal{A}(\pi^-\pi^- \to ee)}{2F_\pi^2\, T_{\mathrm{lept}}} = 1 - \frac{m_\pi^2}{(4\pi F_\pi)^2}\left(3\log\frac{\mu^2}{m_\pi^2} + \frac{7}{2} + \frac{\pi^2}{4} + \frac{5}{6} g_\nu^{\pi\pi}(\mu)\right)$$

Lattice calculation yields (statistical error only)

[XF, L. Jin, X. Tuo, S. Xia, PRL122 (2019) 022001]

$$\frac{\mathcal{A}(\pi\pi \to ee)}{2F_\pi^2\, T_{\mathrm{lept}}} = 0.910(3) \quad \Rightarrow \quad g_\nu^{\pi\pi}(m_\rho) = -12.0(3)$$

Chiral perturbation theory for $\pi^- \to \pi^+ ee$

[X. Tuo, XF, L. Jin, PRD100 (2019) 094511]

$$\frac{\mathcal{A}(\pi^- \to \pi^+ ee)}{2F_\pi^2\, T_{\mathrm{lept}}} = 1 + \frac{m_\pi^2}{(4\pi F_\pi)^2}\left(3\log\frac{\mu^2}{m_\pi^2} + 6 + \frac{5}{6} g_\nu^{\pi\pi}(\mu)\right)$$

Lattice calculation yields (statistical + systematical errors)

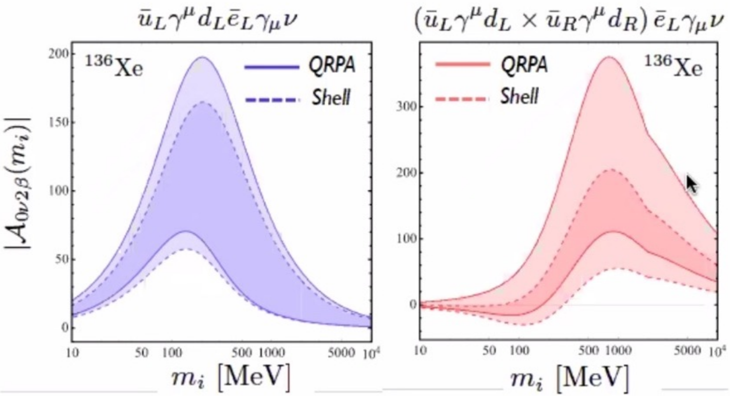
$$\frac{\mathcal{A}(\pi^- \to \pi^+ ee)}{2F_\pi^2\, T_{\mathrm{lept}}} = 1.105(3)(7) \quad \Rightarrow \quad g_\nu^{\pi\pi}(m_\rho) = -10.9(3)(7)$$

Also $g_\nu^{\pi\pi}(m_\rho) = -10.8(1)(5)$ [W. Detmold, D. Murphy, arXiv:2004.07404]

New progress

$0\nu2\beta$ decay amplitude: dependence on the neutrino mass

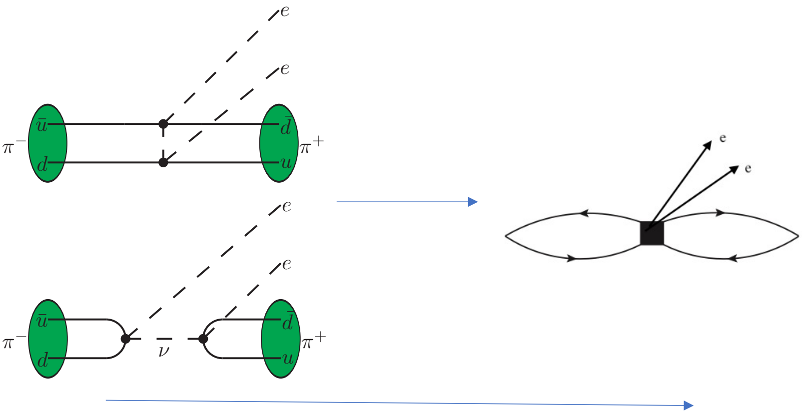
Mass dependence of LECs and NMEs — W. Dekens, J. de Vries, K. Fuyuto, E. Mereghetti, G. Zhou, JHEP06(2020)097



Understanding m_i –behavior is important to give correct prediction in light sterile scenario.

Jordy de Vries - "there was quite some interest in the idea of light sterile neutrinos and the link to 0vbb, in particular for pionic operators that provide a LO contribution. I wanted to see if something could be done by your great machinery."

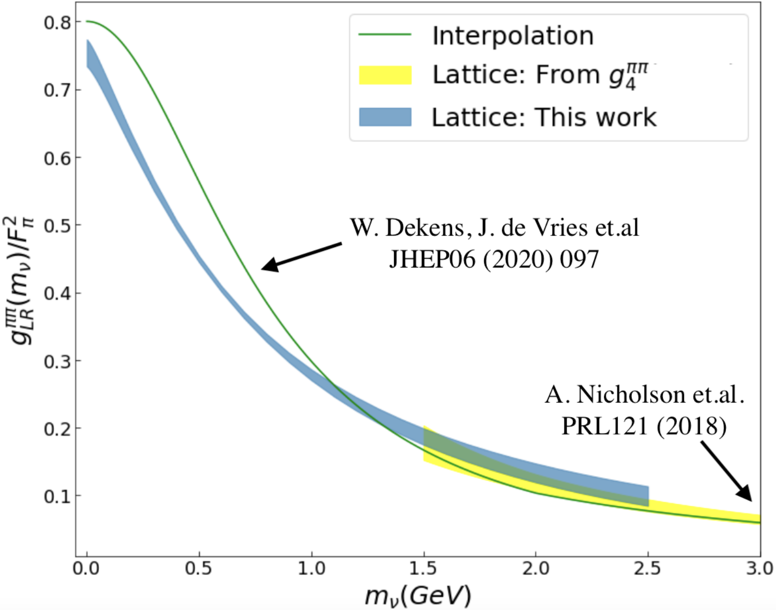
$0\nu2\beta$ decay amplitude: dependence on the neutrino mass



Increase the mass of the neutrino

$0\nu 2\beta$ decay amplitude: dependence on the neutrino mass

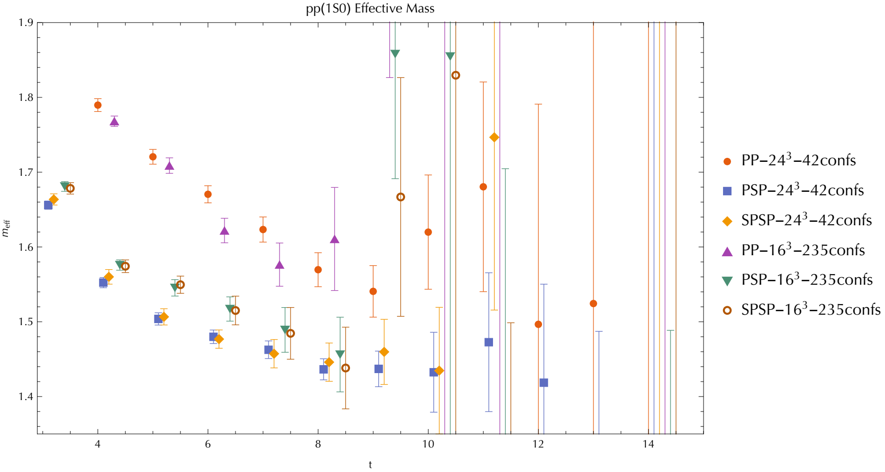
Project lead by Xin-Yu Tuo



$nn \to ppee$ decay amplitude

Project lead by Zi-Yu Wang

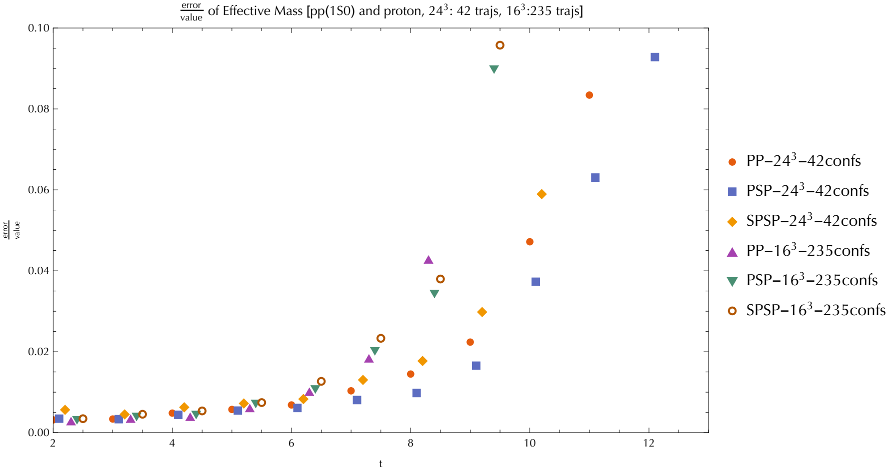
Effective mass for $^1S_0$ nn or pp state



$nn \to ppee$ decay amplitude

Project lead by Zi-Yu Wang

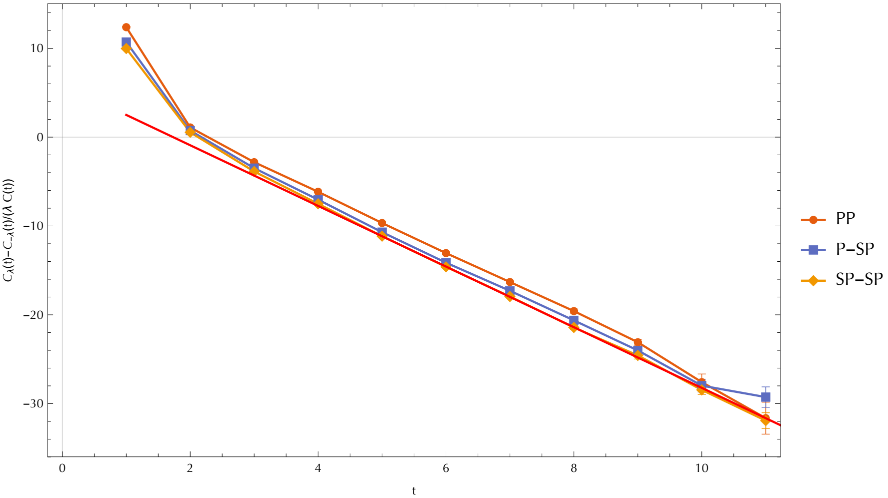
Noise/Signal ratio increases at large time separation $\to$ Sign problem



$nn \to ppee$ decay amplitude

Project lead by Zi-Yu Wang

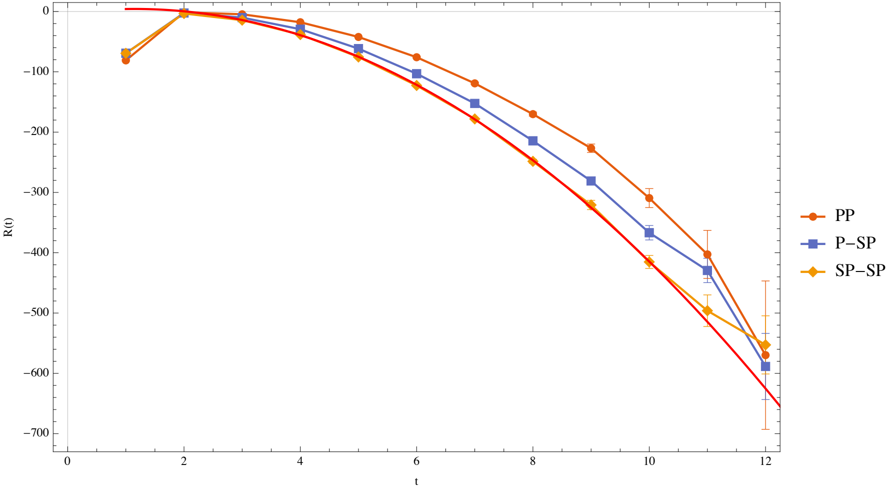
pp fusion: ${}^1S_0$ state $\to$ ${}^3S_1$ state



$nn \to ppee$ decay amplitude

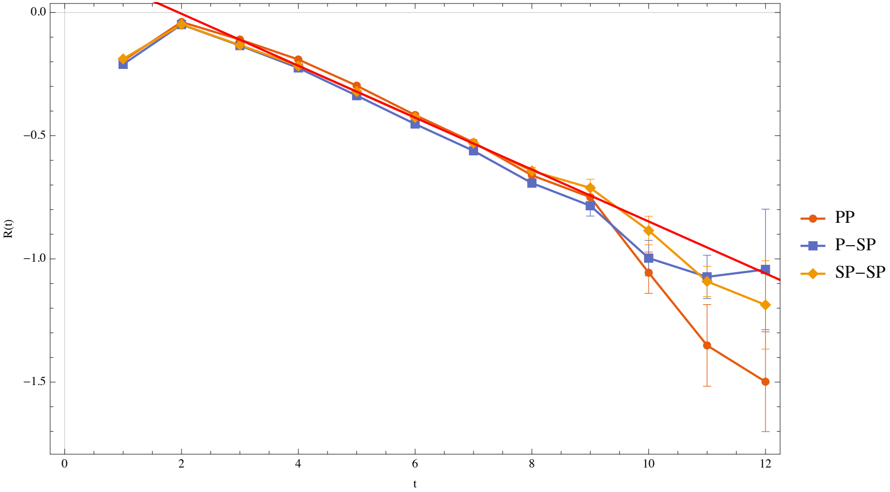
Project lead by Zi-Yu Wang

$2\nu 2\beta$ decay: $nn \to ppee\bar{\nu}\bar{\nu}$



Project lead by Zi-Yu Wang

$0\nu2\beta$ decay: $nn \to ppee$



Outlook

- $0\nu\beta\beta$ is of fundamental interests $\Rightarrow$ Experimental search, worldwide competition

- The interpretation of $0\nu\beta\beta$ experiments relies on the control of theory uncertainty

- Appealing to connect lattice QCD $\Rightarrow$ chiral EFT $\Rightarrow$ many-body nuclear theory