Toward a unified description of nuclei

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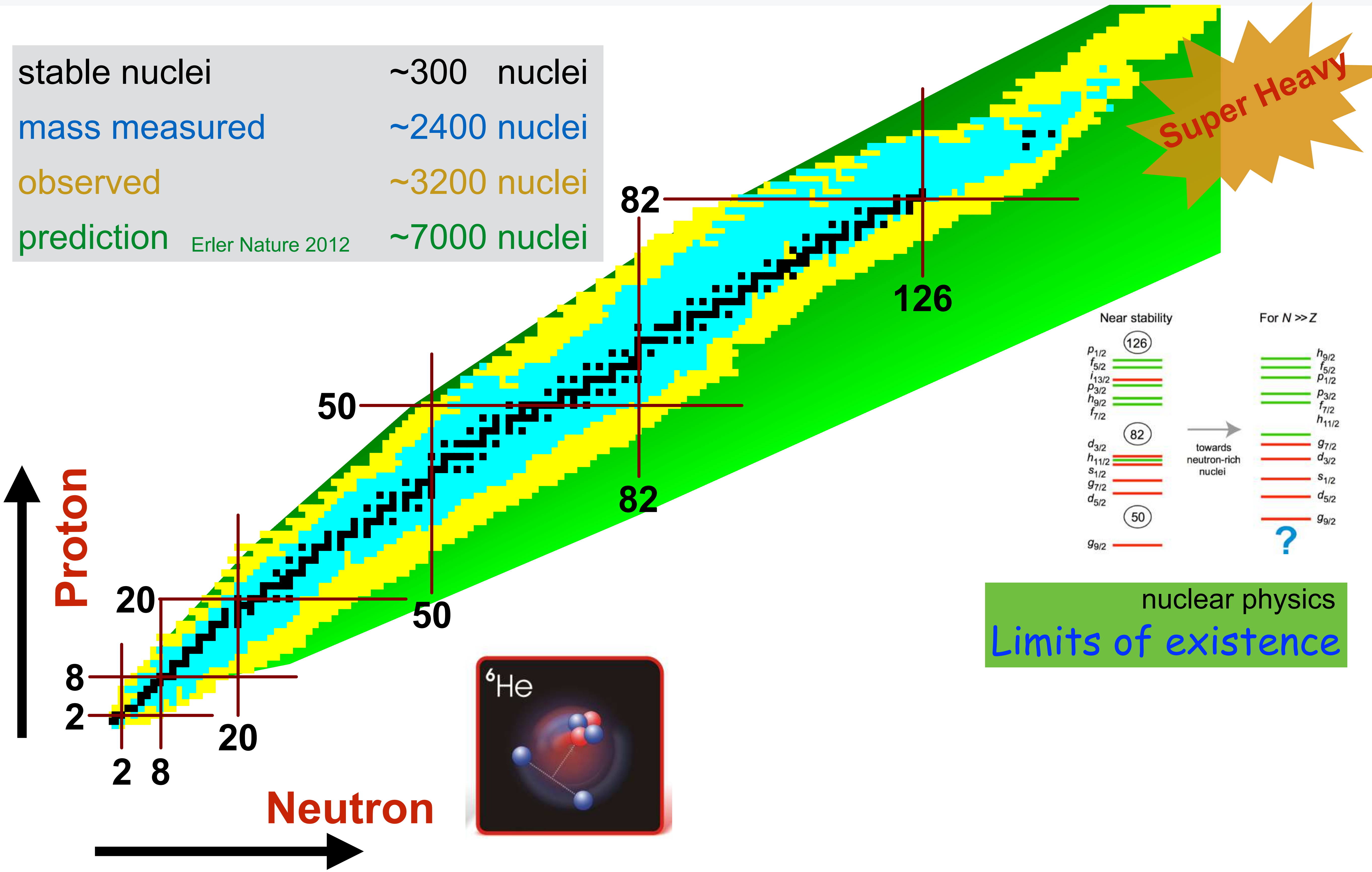
Peking University

Nuclide Chart

stable nuclei	~300 nuclei
mass measured	~2400 nuclei
observed	~3200 nuclei
prediction Erler Nature 2012	~7000 nuclei



nuclear physics

Limits of existence

Nuclide Chart

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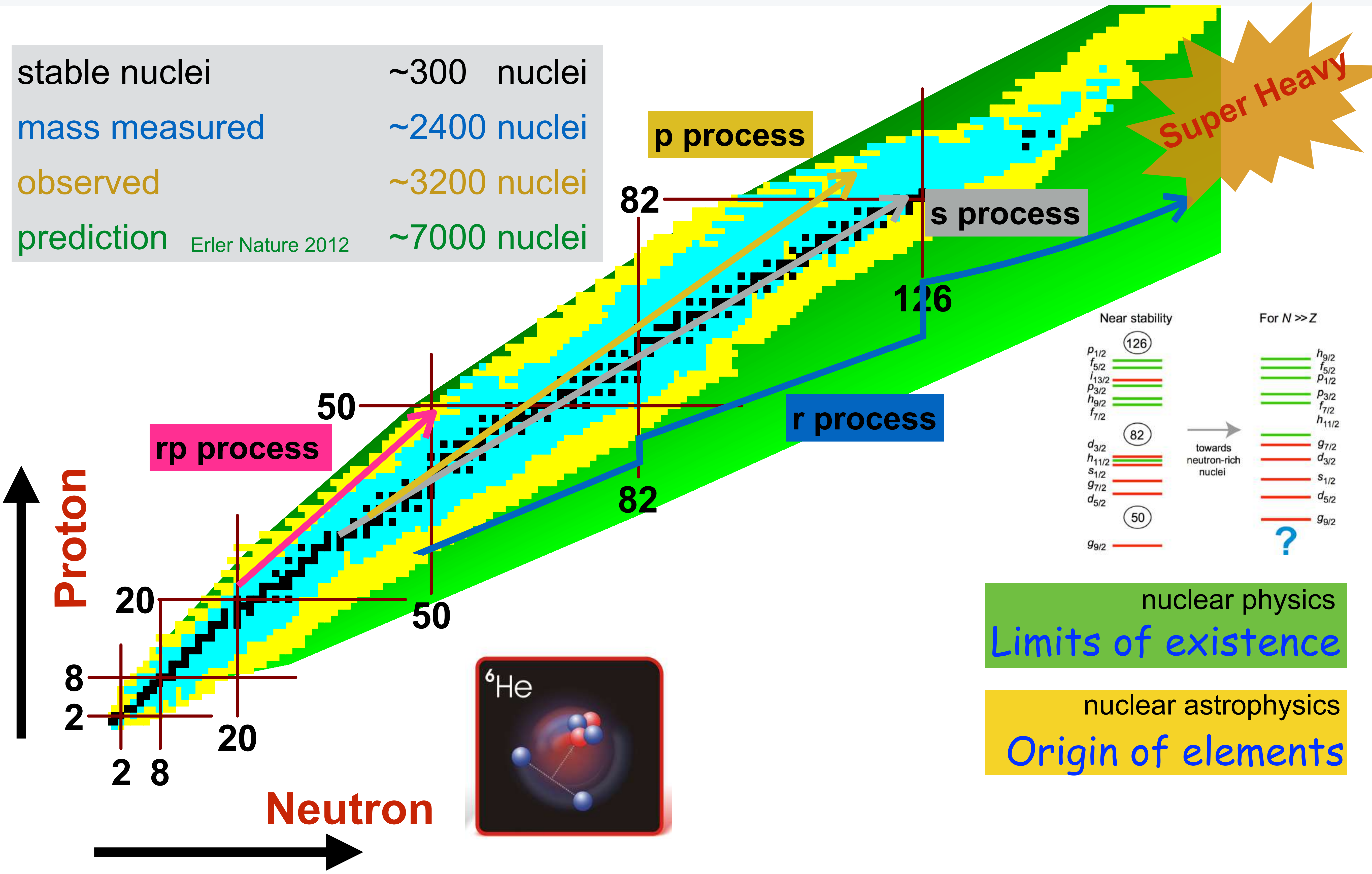

nuclear physics

Limits of existence

nuclear astrophysics

Origin of elements

Nuclide Chart

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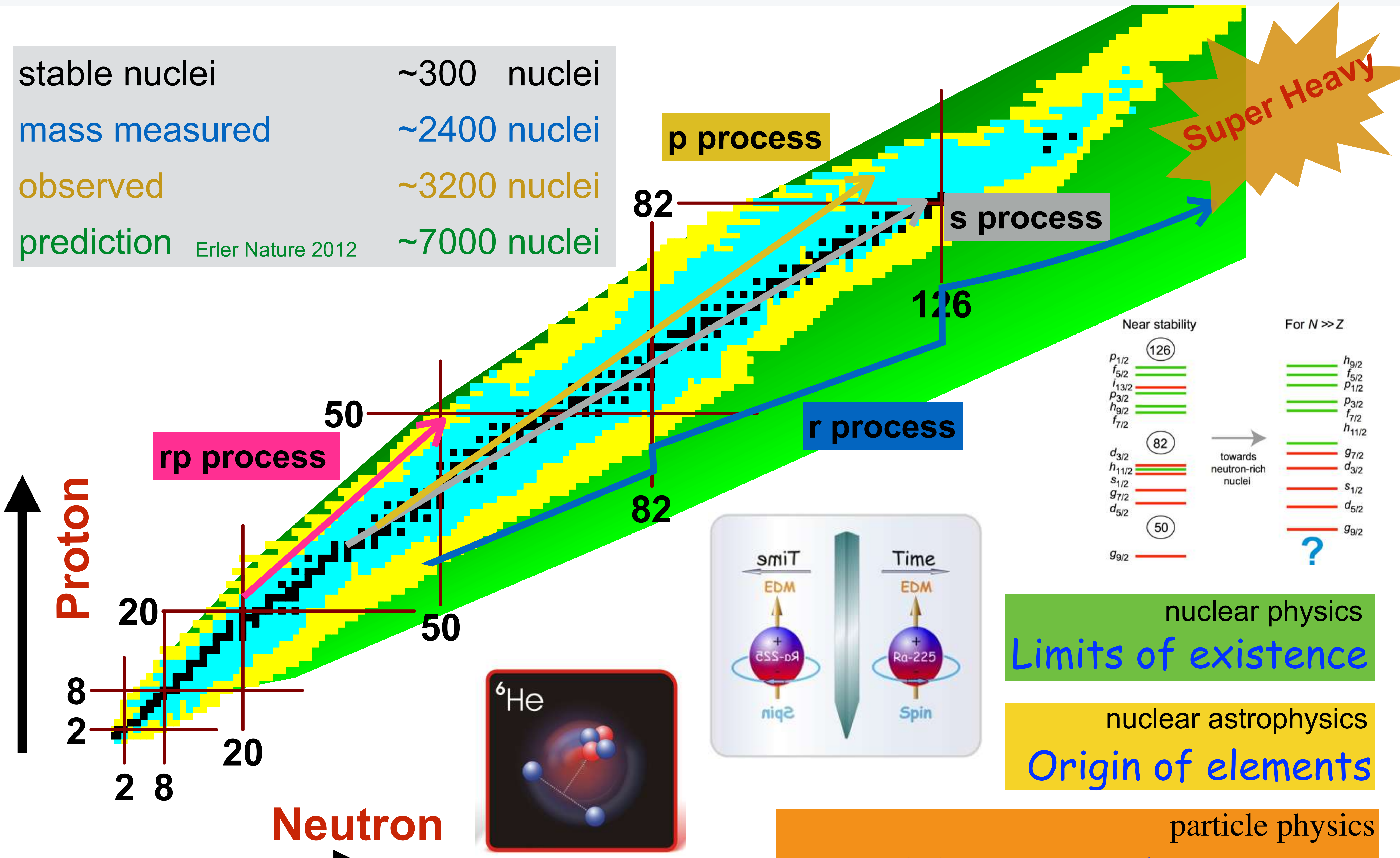


nuclear physics
Limits of existence

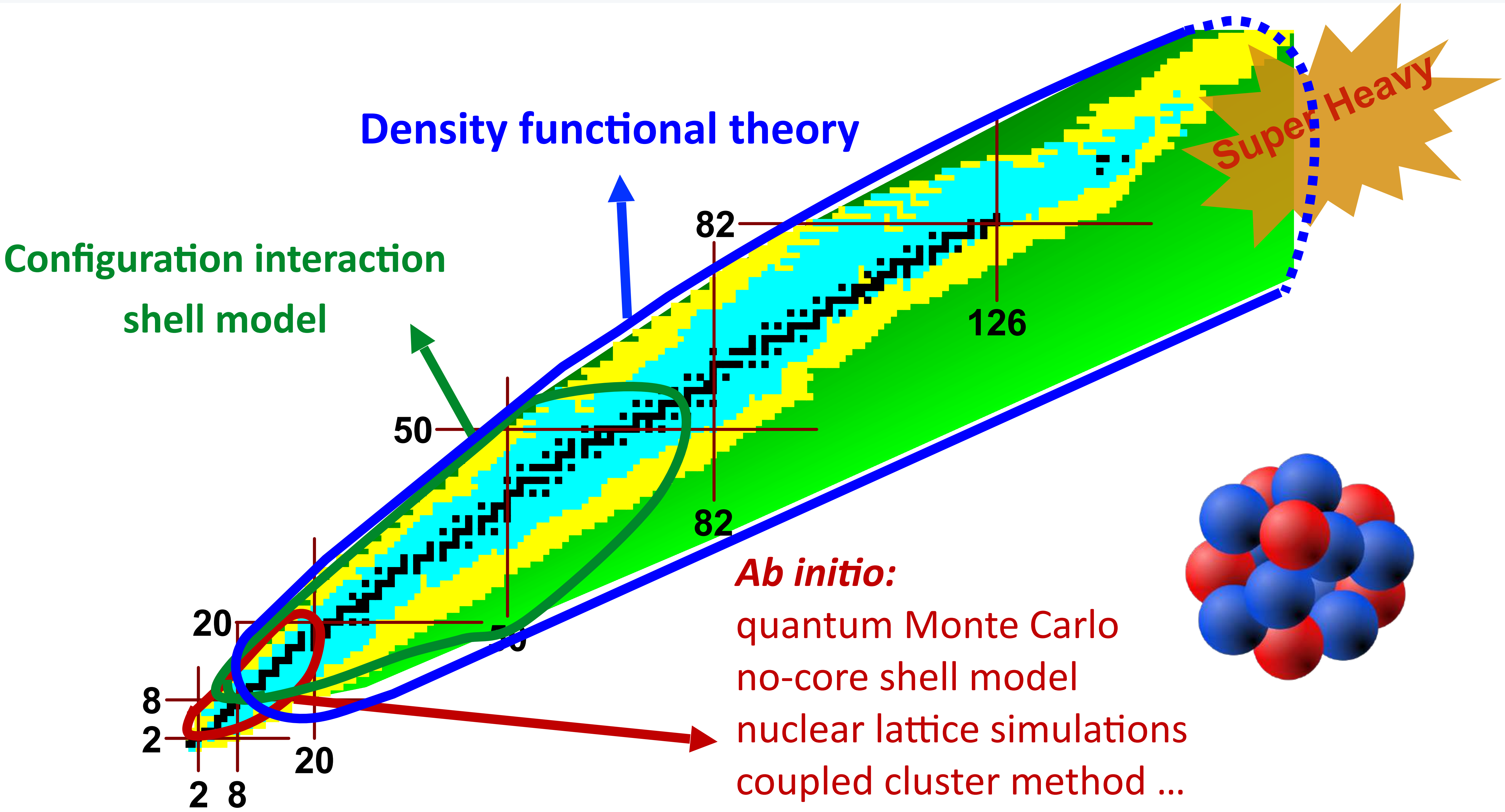
nuclear astrophysics
Origin of elements

particle physics
Test of fundamental symmetries

Toward a unified description of nuclei

- a global approach which is applicable for the whole nuclear chart (light & heavy; neutron-deficient & neutron-rich)
- a consistent treatment for the ground-state and spectroscopic properties (low spin & high spin)
- a unified description of nuclear structure and nuclear reaction
- a microscopic formalism based directly on realistic nuclear interactions
- …

State-of-the-art theories for nuclei



Density functional theory

Configuration interaction shell model

Ab initio:
quantum Monte Carlo
no-core shell model
nuclear lattice simulations
coupled cluster method …

It would be interesting to investigate the intersections between different theories for a unified and comprehensive description of nuclei.

Density functional theory

The many-body problem is mapped onto an one-body problem

Hohenberg-Kohn Theorem

The exact ground-state energy of a quantum mechanical many-body system is a universal functional of the local density.

Kohn-Sham DFT

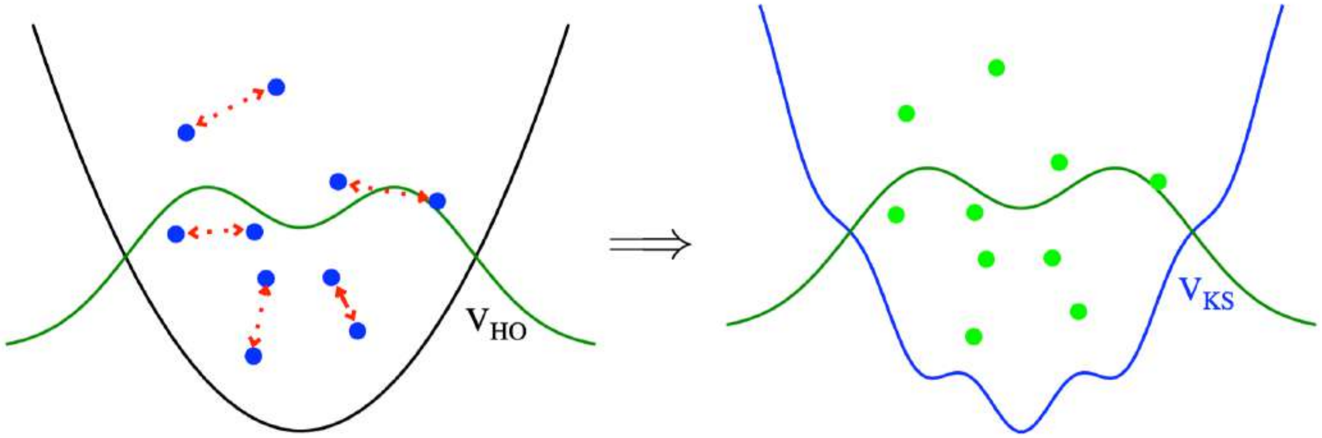
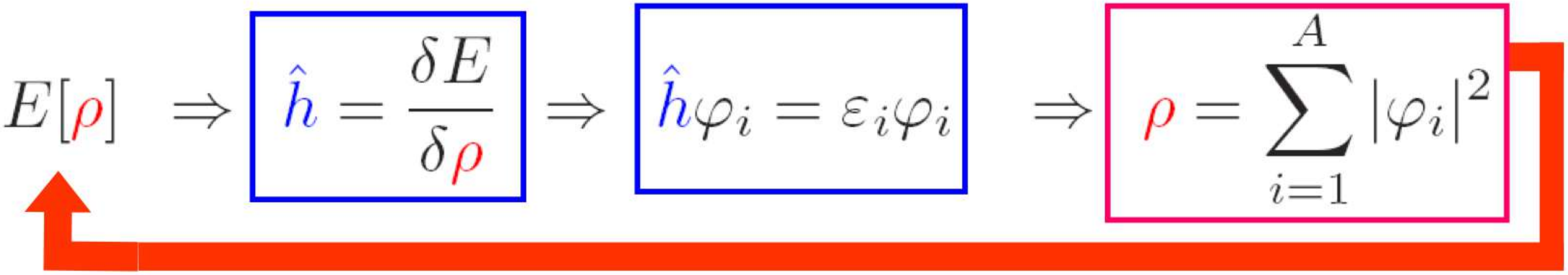


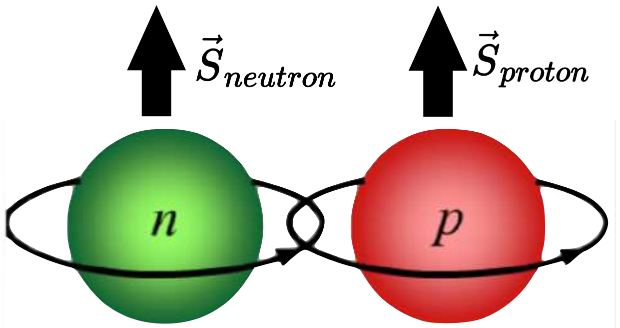
Figure from Drut PPNP 2010



$$E[\rho] \Rightarrow \boxed{\hat{h} = \frac{\delta E}{\delta \rho}} \Rightarrow \boxed{\hat{h}\varphi_i = \varepsilon_i \varphi_i} \Rightarrow \boxed{\rho = \sum_{i=1}^{A} |\varphi_i|^2}$$

The practical usefulness of the Kohn-Sham theory depends entirely on whether an **Accurate Energy Density Functional** can be found!

Density functional theory for nuclei

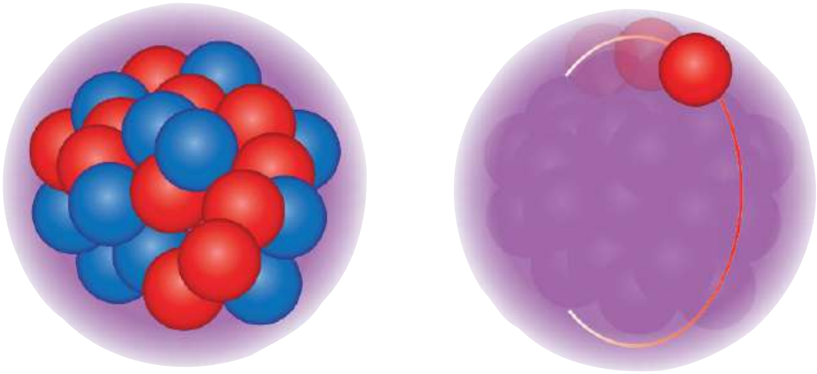


✓ The nuclear force is complicated

✓ More degrees of freedom: spin, isospin, pairing, …

✓ Nuclei are self-bound systems

DFT for the intrinsic density



✓ At present, all successful functionals are phenomenological

not connected to any NN- or NNN-interaction

✓ Adjust to properties of nuclear matter and/or finite nuclei, and

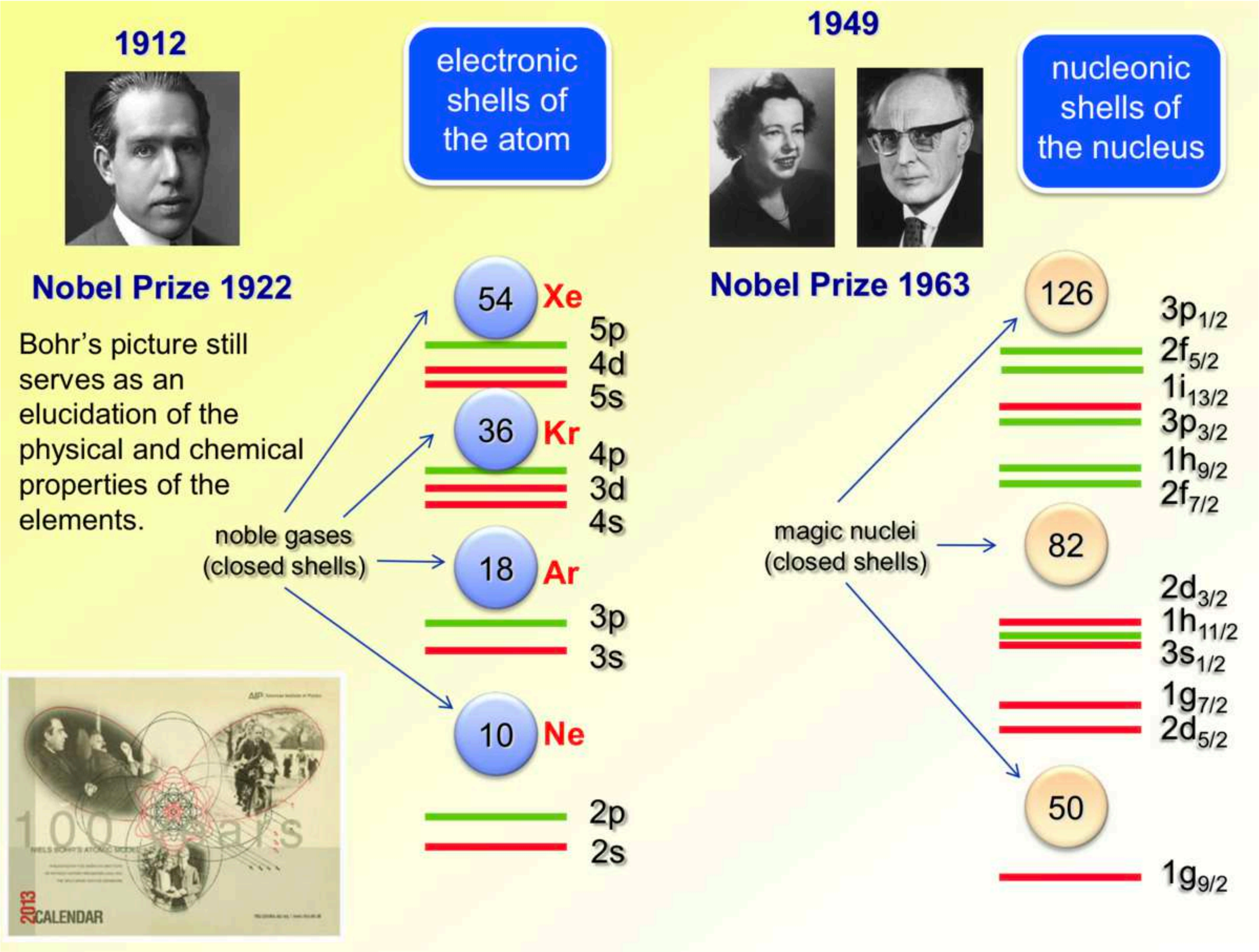
(in future) to ab-initio results

Relativistic vs Nonrelativistic

- ✓ No relativistic kinematics necessary

$$\sqrt{p_F^2 + m_N^2} = m_N \sqrt{1 + 0.075}$$

- ✓ Large spin-orbit splitting

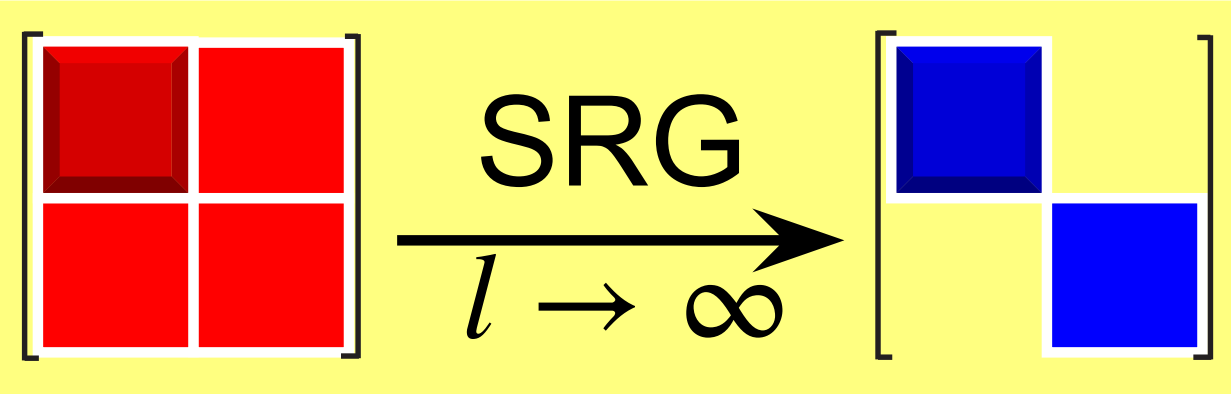


Bohr's picture still serves as an elucidation of the physical and chemical properties of the elements.

Similarity Renormalization Group

Relativistic Kohn-Sham Equation: Dirac Equation

$$\begin{pmatrix} m + V + S & \boldsymbol{\sigma}\cdot\boldsymbol{p} \\ \boldsymbol{\sigma}\cdot\boldsymbol{p} & -m + V - S \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = \varepsilon \begin{pmatrix} f \\ g \end{pmatrix}$$



$$\frac{dH(l)}{dl} = \left[\eta(l), H(l)\right], \quad \eta(l) = \left[\beta, H(l)\right]$$

Bylev, Pirner, Phys. Lett. B 428, 329 (1998)

Flow equation

$$\frac{dH(l)}{dl} = \left[\eta(l), H(l)\right], \quad \eta(l) = \left[\beta, H(l)\right]$$

Expansion with nucleon mass

$$\frac{1}{M}H(l) = \sum_{k=0}^{\infty} \frac{1}{M^k} H_k(l)$$

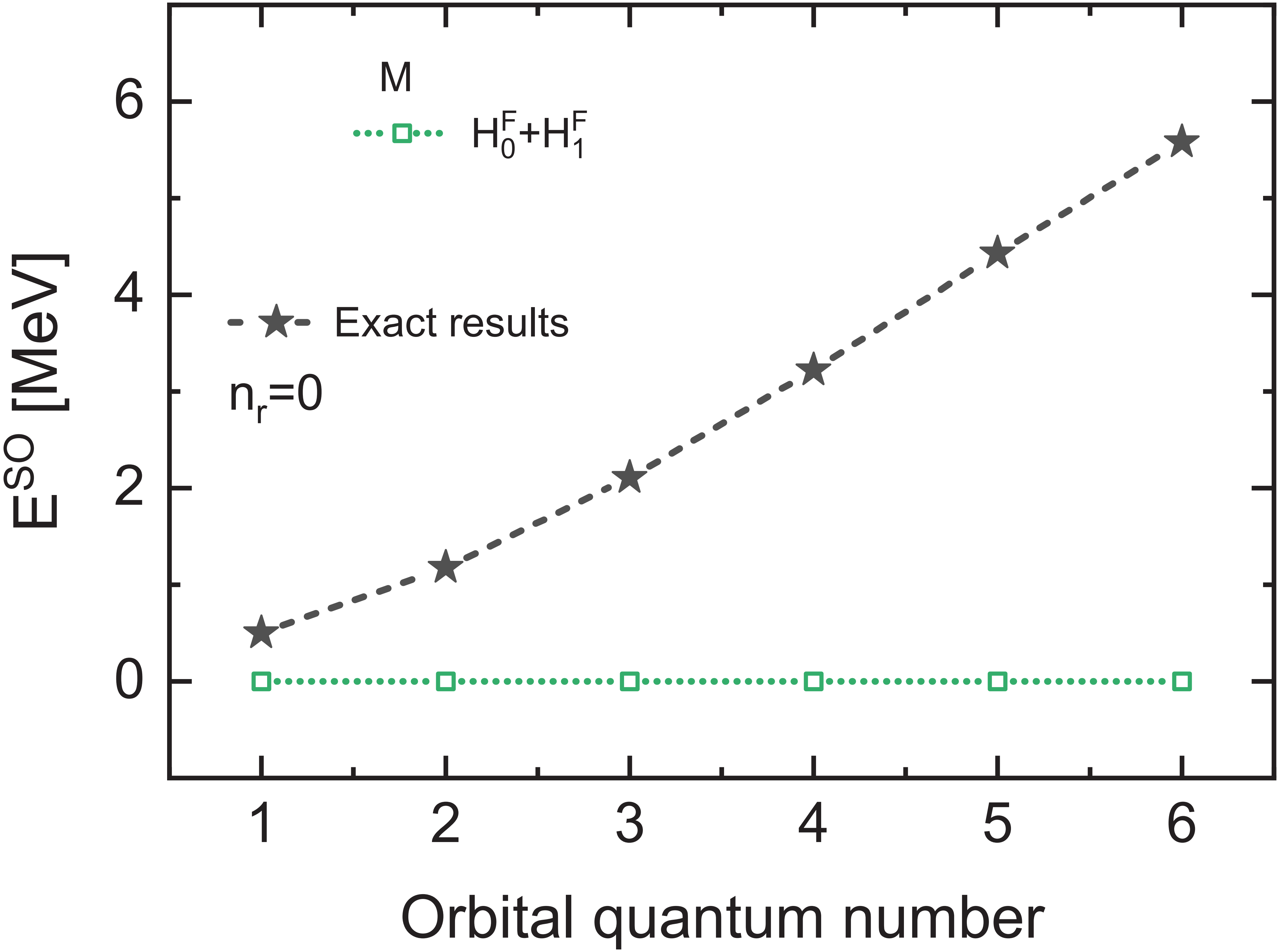
$$\Rightarrow \quad H(\infty) = \begin{pmatrix} H^F + M & 0 \\ 0 & H^D - M \end{pmatrix}$$

Bylev, Pirner, Phys. Lett. B 428, 329 (1998)

Guo, Phys. Rev. C 85, 021302(R) (2012)

Spin-orbit Splitting Energy

Too slow convergence!



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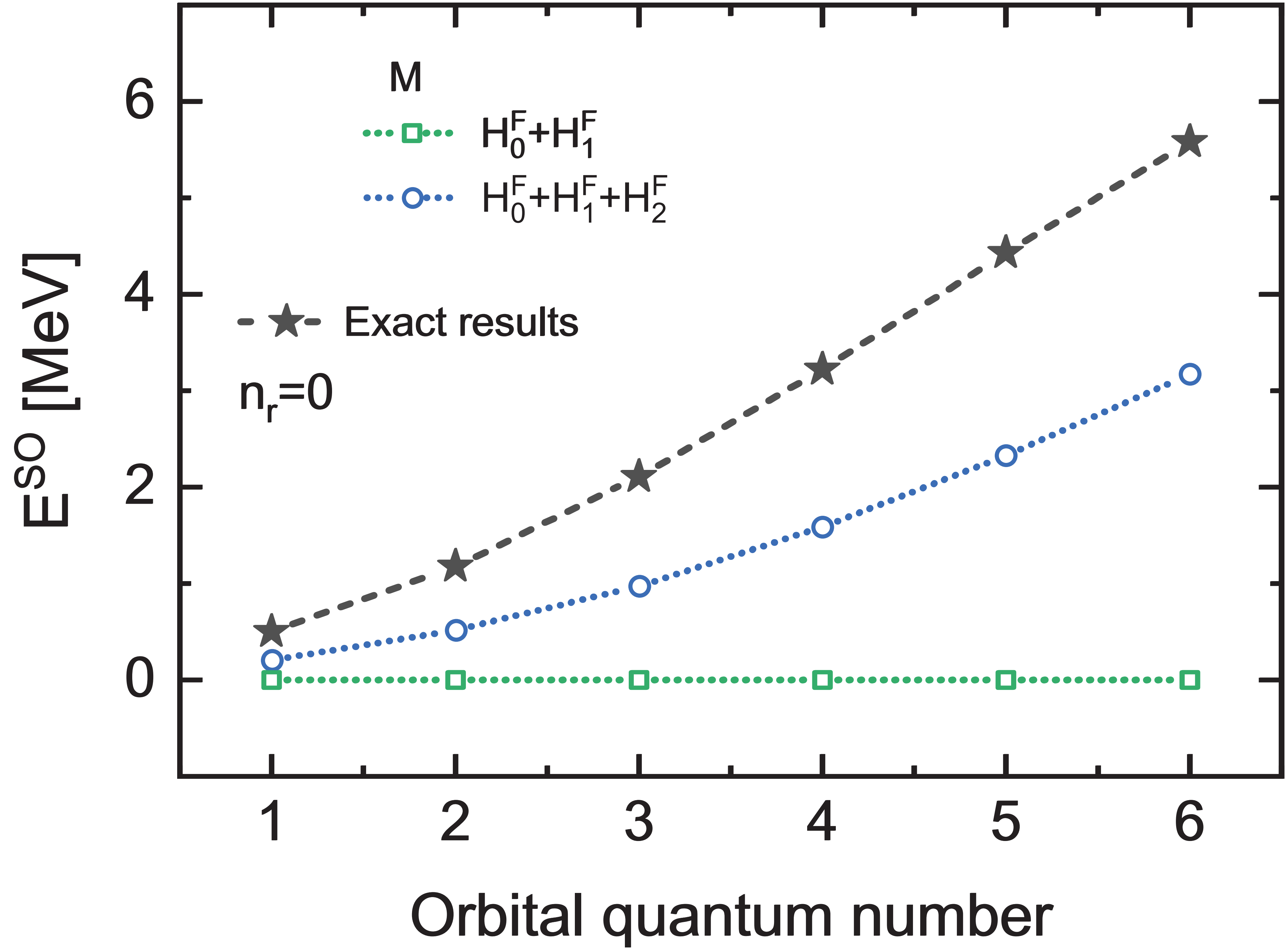

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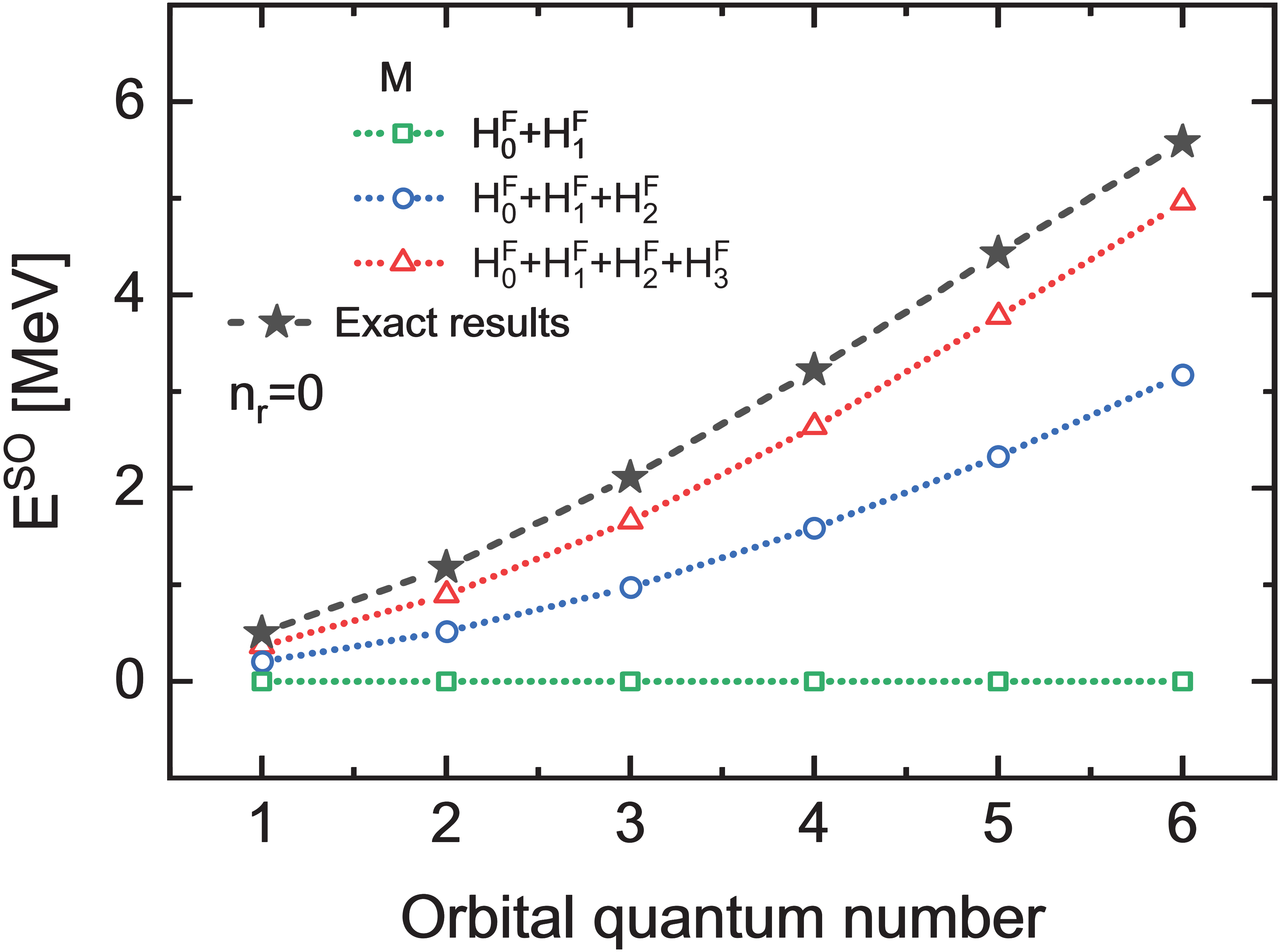


Medium effects

$$\begin{pmatrix} m+V+S & \boldsymbol{\sigma}\cdot\boldsymbol{p} \\ \boldsymbol{\sigma}\cdot\boldsymbol{p} & -m+V-S \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = \varepsilon \begin{pmatrix} f \\ g \end{pmatrix}$$

Expansion with effective nucleon mass

$$\tilde{M}(r) = M + S(r)$$

$$\frac{1}{\tilde{M}} H(l) = \sum_{k=0}^{\infty} \frac{1}{\tilde{M}^k} H_k(l)$$

$$\frac{dH(l)}{dl} = \left[\eta(l), H(l)\right], \quad \eta(l) = \left[\beta, H(l)\right]$$

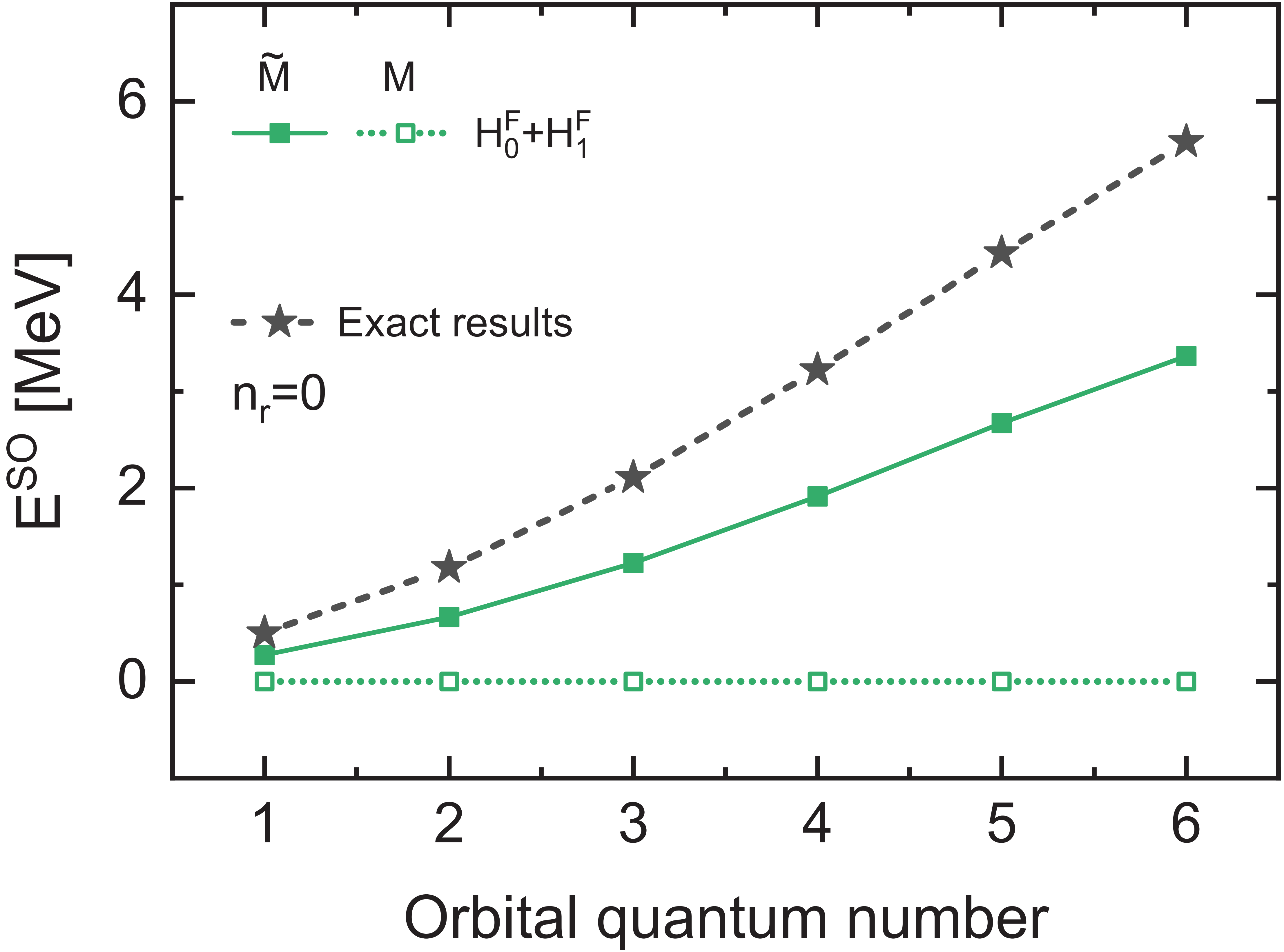
⇒

$$H(\infty) = \begin{pmatrix} H^F + M & 0 \\ 0 & H^D - M \end{pmatrix}$$

Ren, PWZ, Phys. Rev. C, 100, 044322 (2019)

Spin-orbit Splitting Energy

Ren, PWZ, Phys. Rev. C, 100, 044322 (2019)



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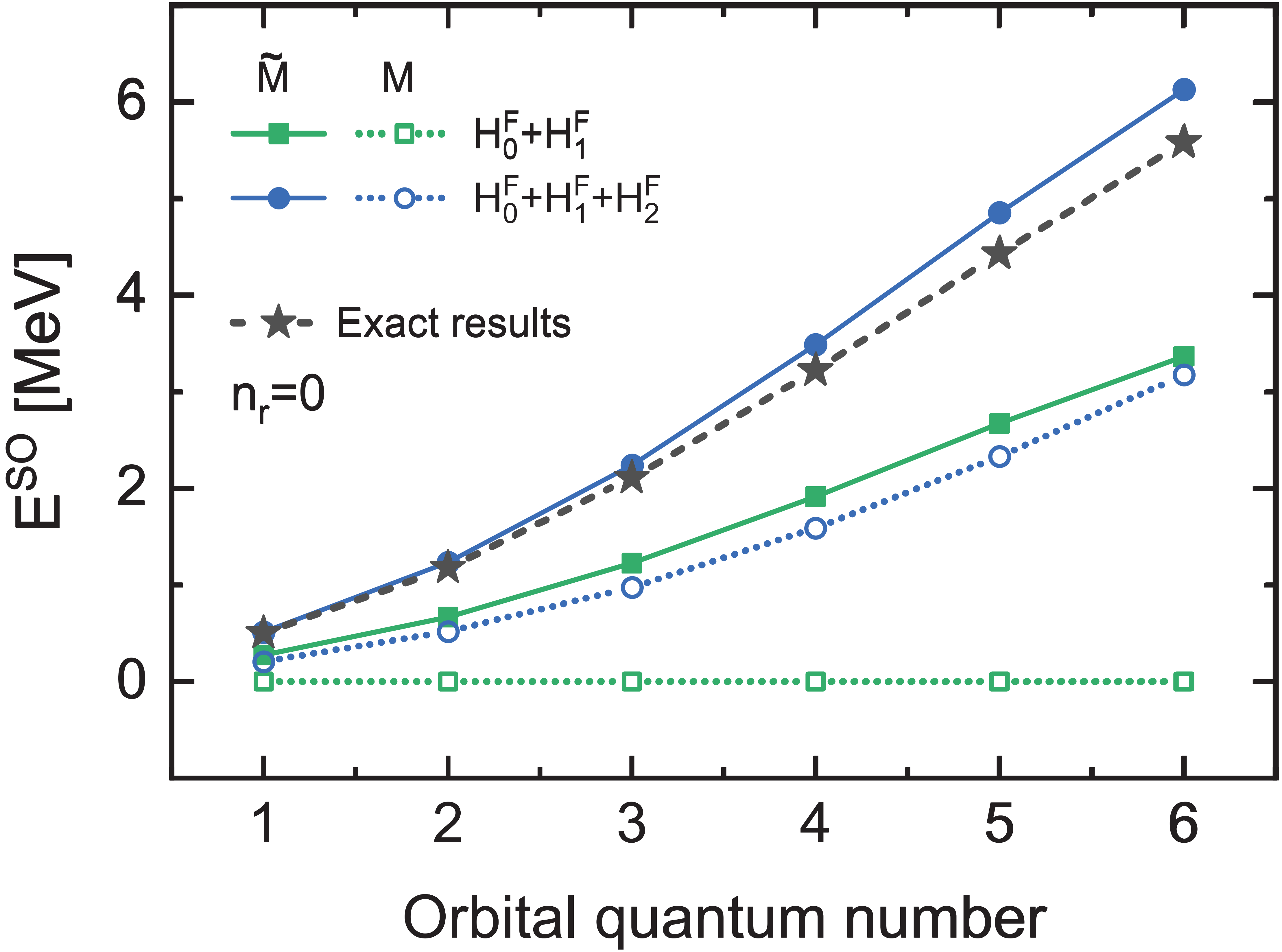
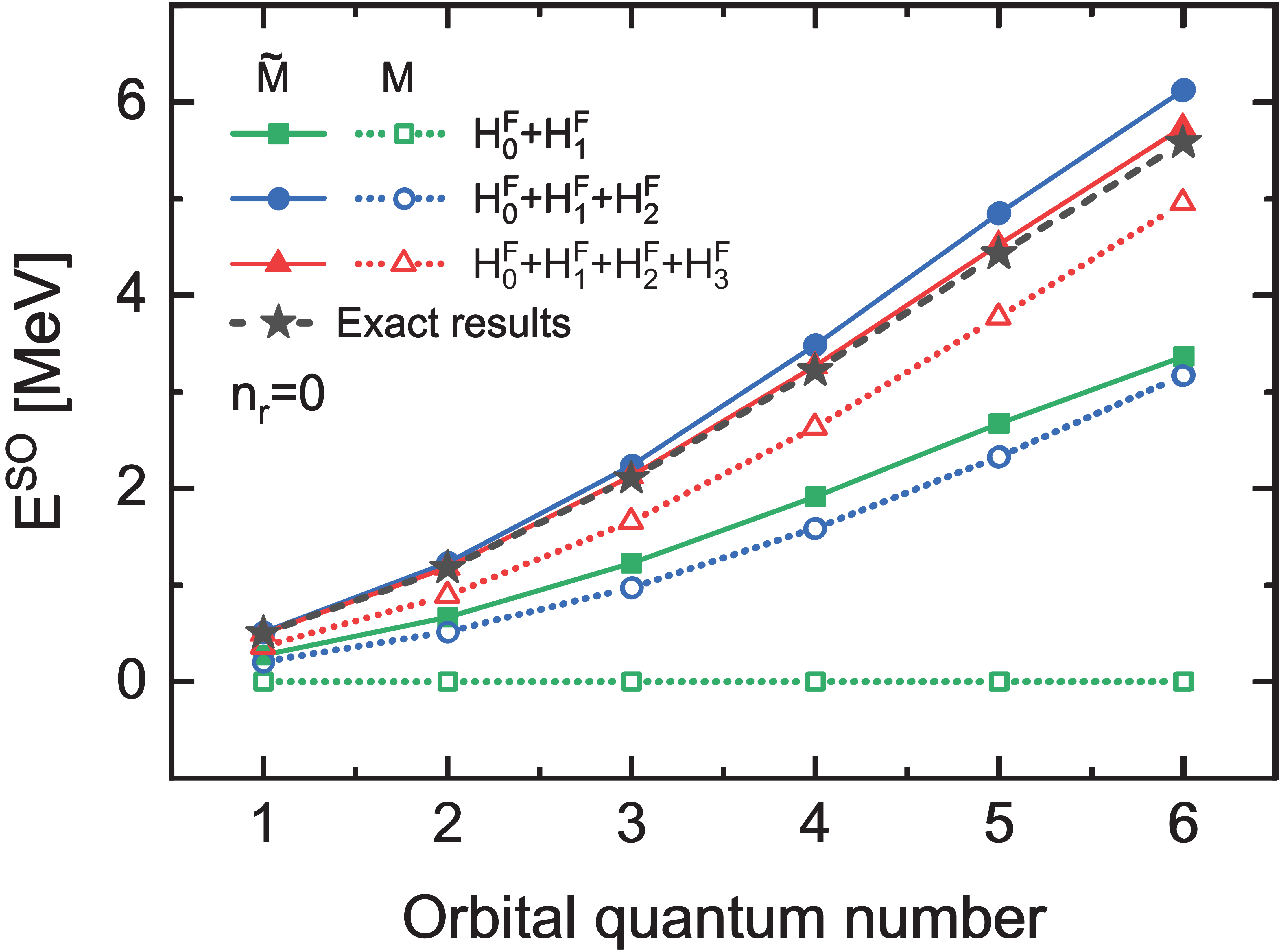
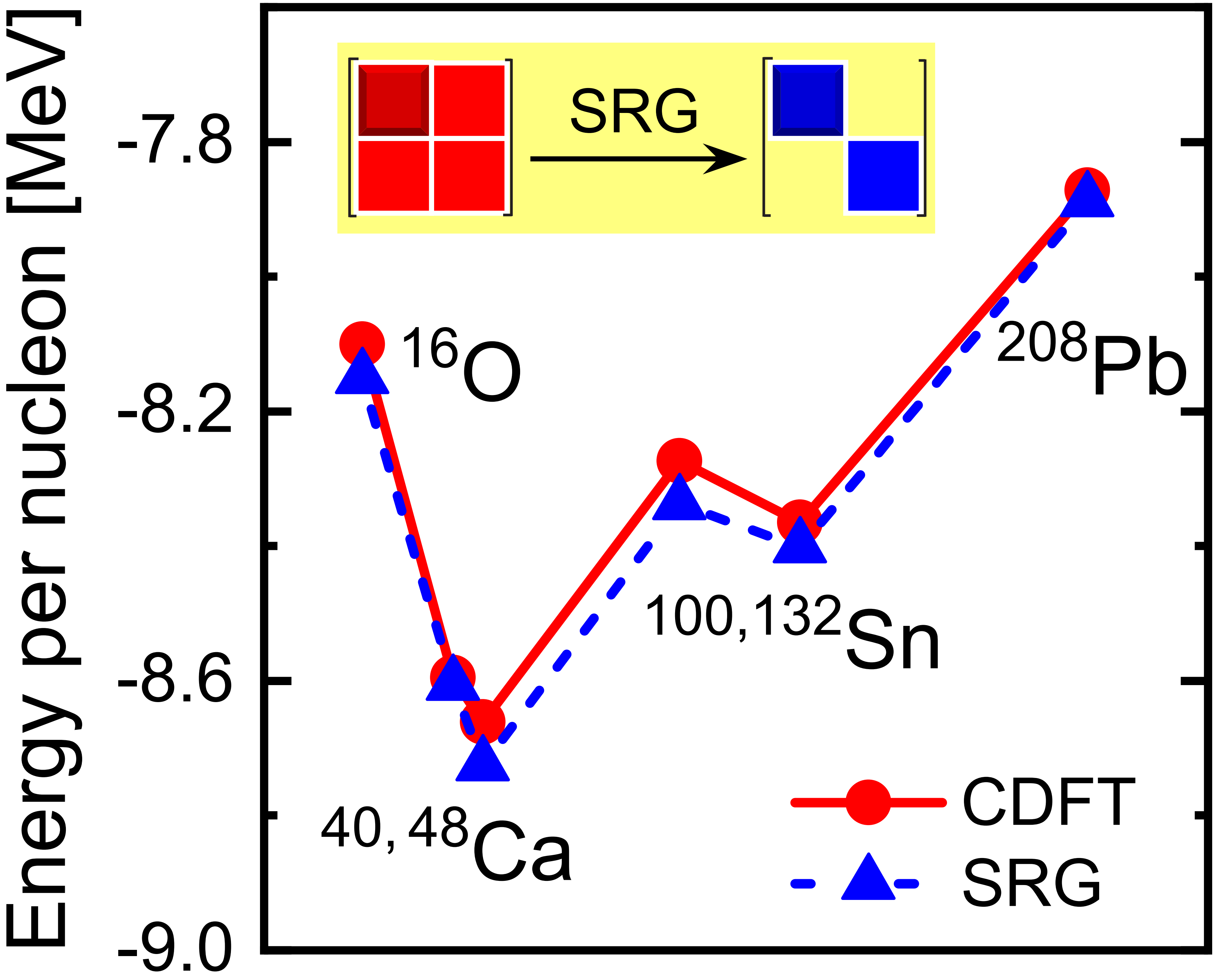


Spin-orbit Splitting Energy

Ren, PWZ, Phys. Rev. C, 100, 044322 (2019)



Toward a bridge between relativistic and nonrelativistic DFT

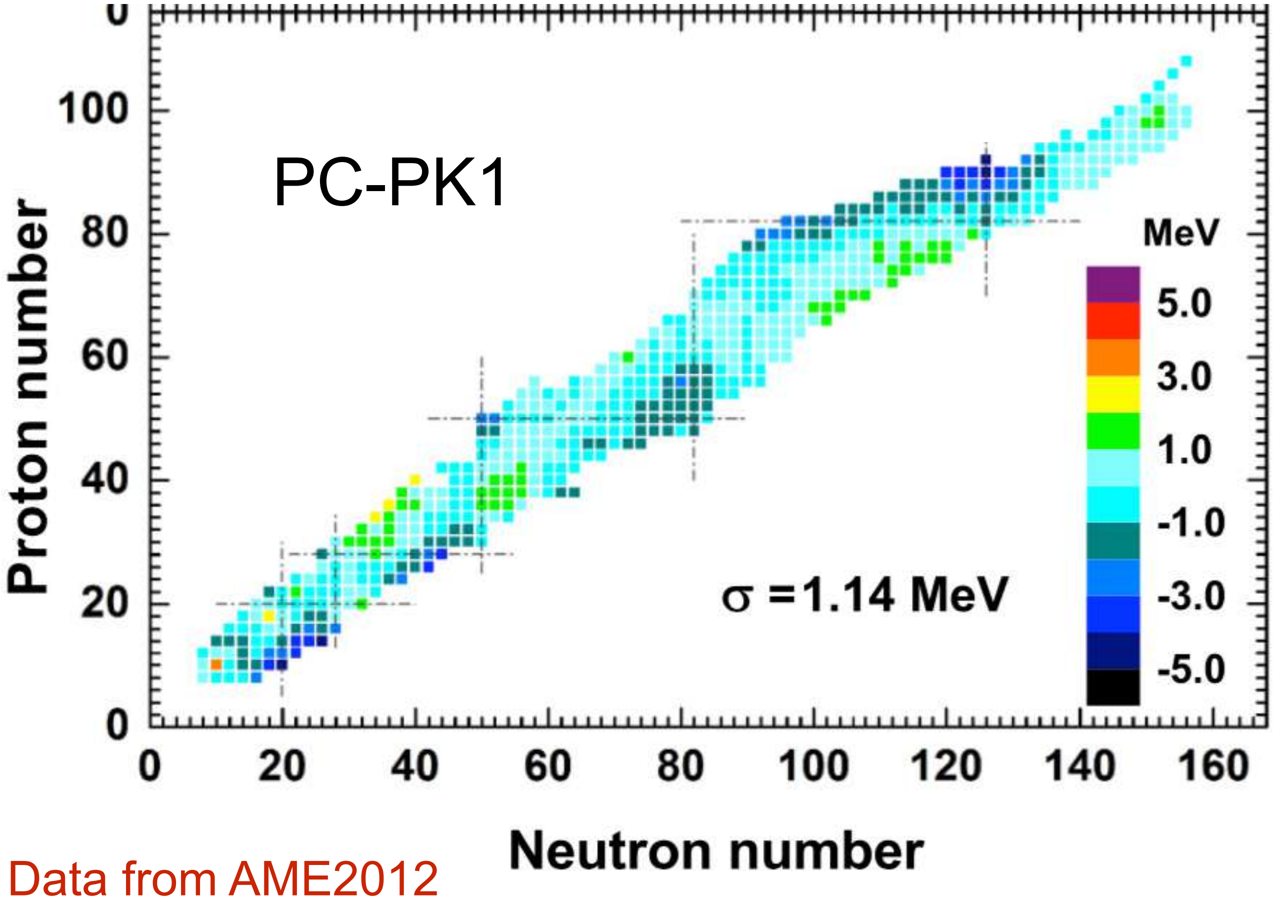


Ren, PWZ, Phys. Rev. C, 102, 021301(R) (2020) **Editors' Suggestion**

Covariant density functional： PC-PK1

~10 parameters fitted to 60 spherical nuclei …

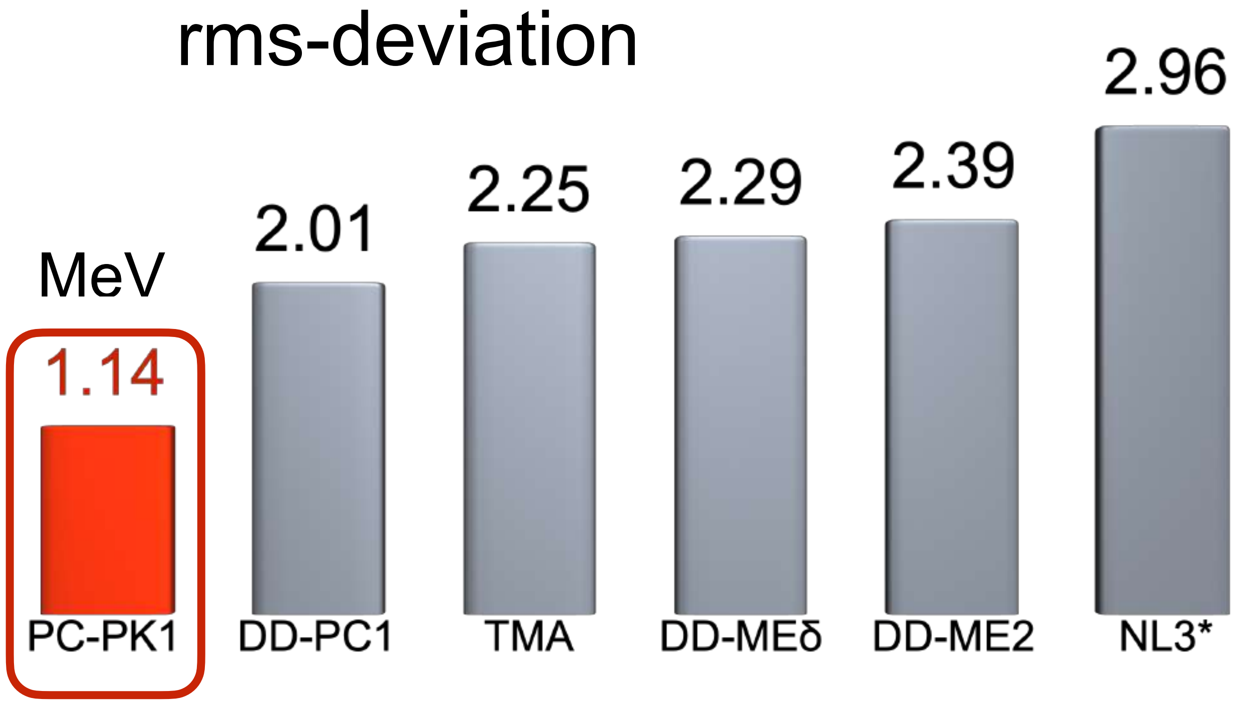
Mass Differences: M_{cal} - M_{exp}



Data from AME2012

PWZ, Li, Yao, Meng, PRC 82, 054319 (2010)

Lu, Li, Li, Yao, Meng PRC 91, 027304 (2015)



Agbemava PRC 2014
Geng PTP 2005

Best density-functional description for nuclear masses so far!

Outline

- Nuclear landscape

  (light and heavy nuclei; neutron-deficient and neutron-rich nuclei)

- Nuclear spectroscopy

  (ground and excited states; low- and high- angular momentum states)

- Nuclear dynamics

  (nuclear structure and reaction)

- Nuclear interactions

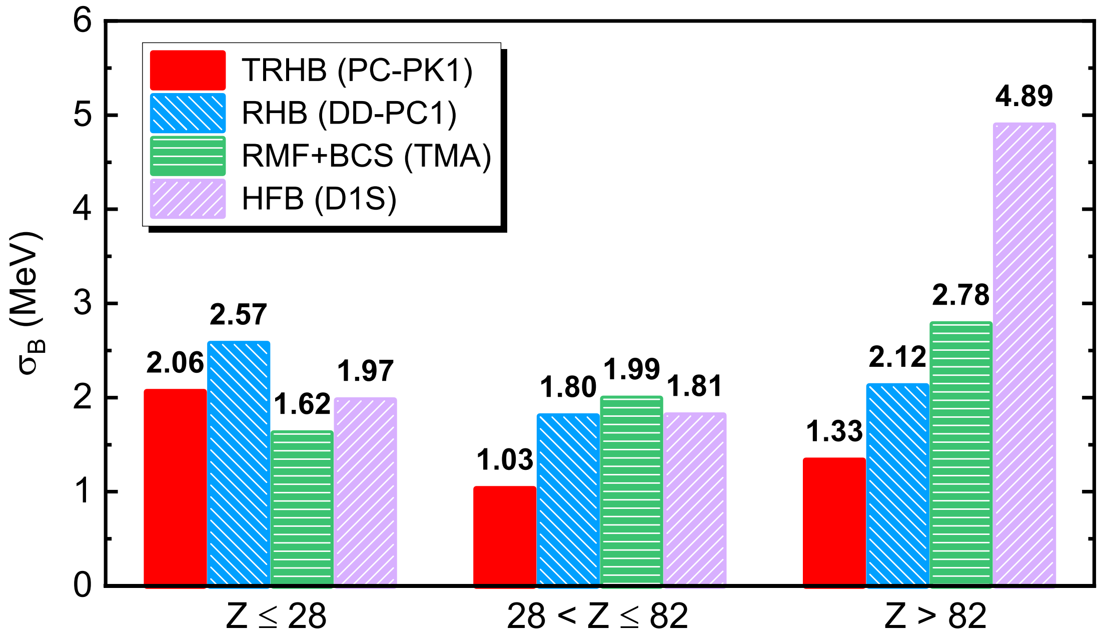
  (towards a microscopic formalism based on bare nuclear interactions)

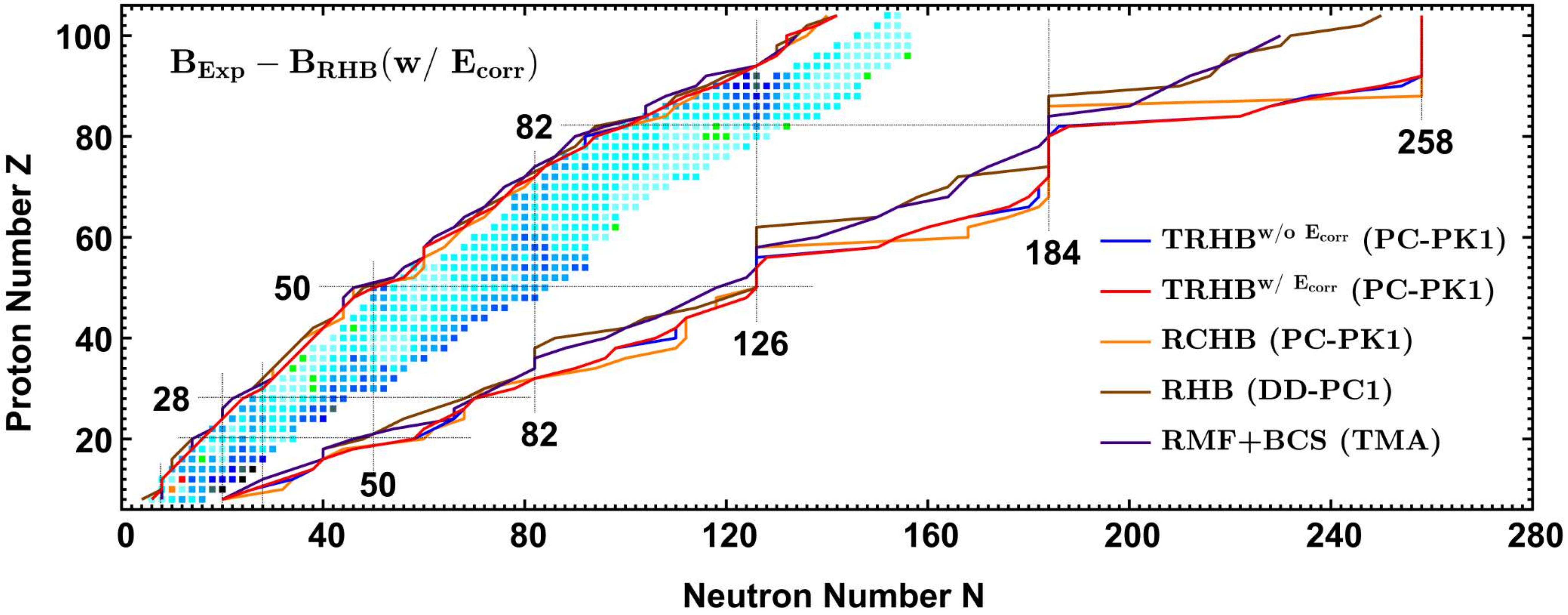
- Summary

Nuclear landscape

Triaxiality + Dynamical correlations

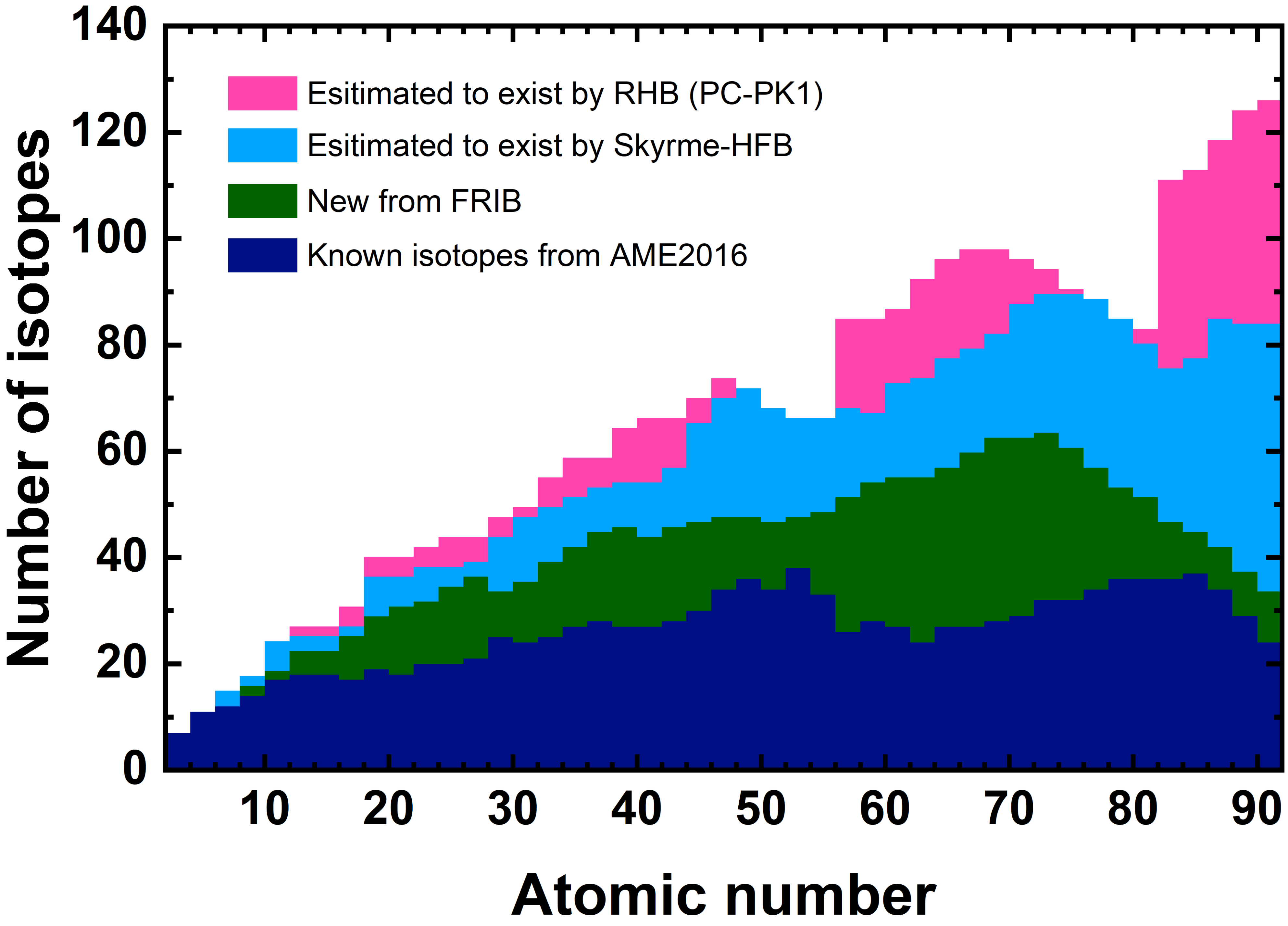
Yang, Wang, PWZ, Li, Submitted





What will the new facilities bring us?

More isotopes …



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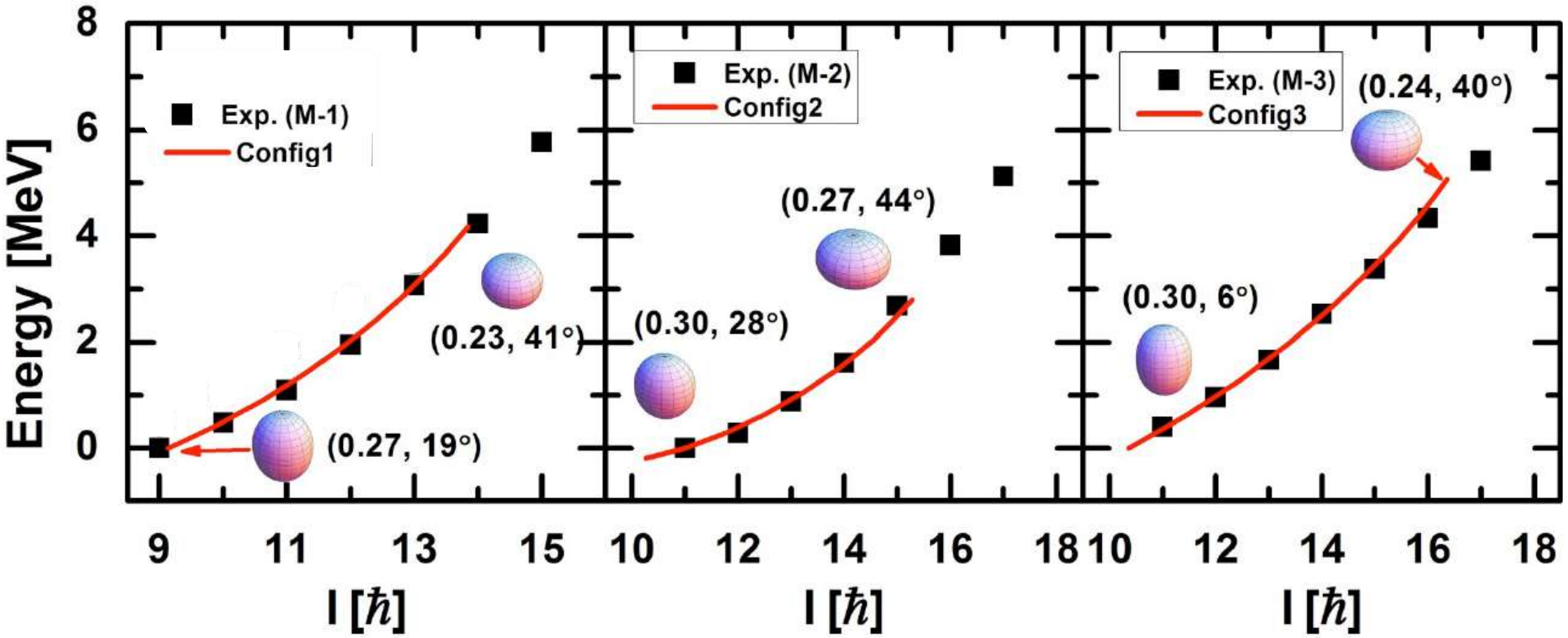
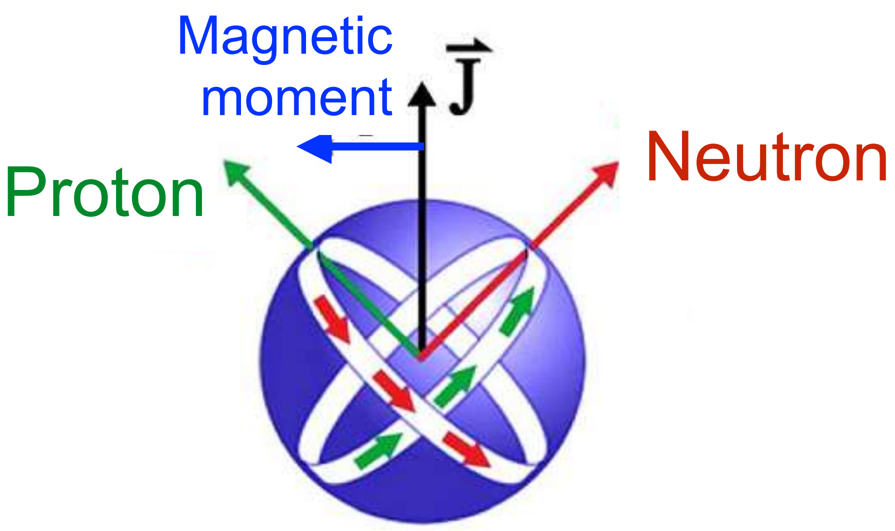
Magnetic and Anti-magnetic Rotation

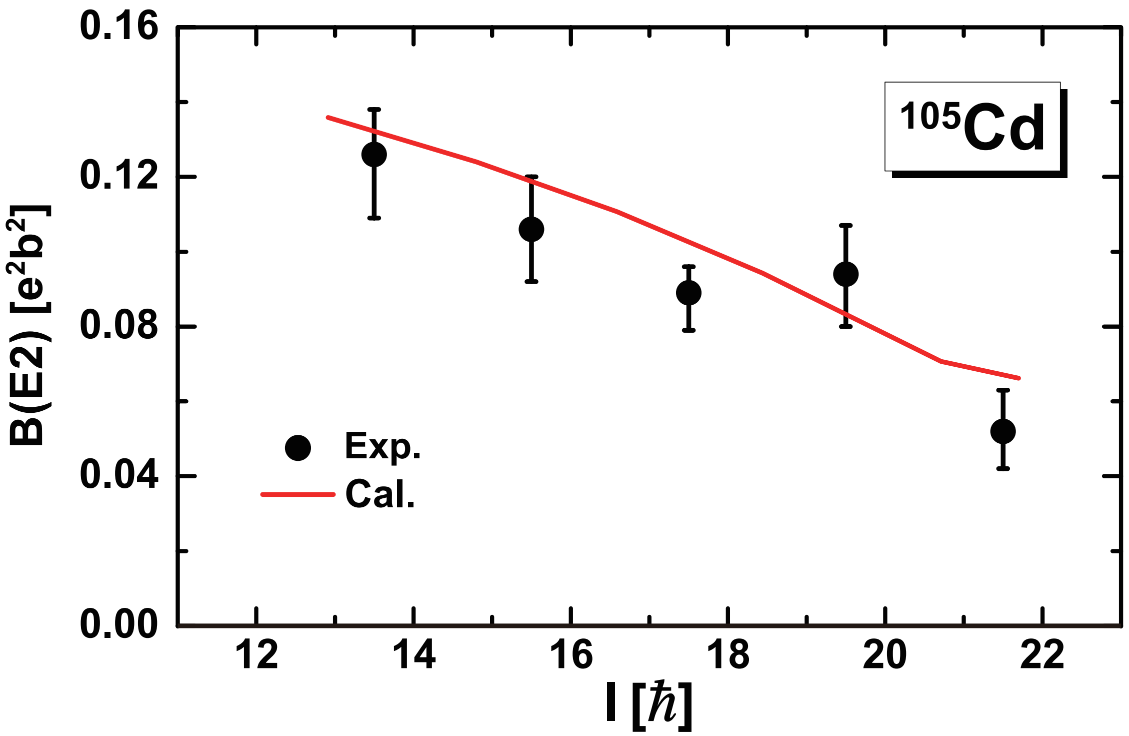
Tilted axis cranking CDFT

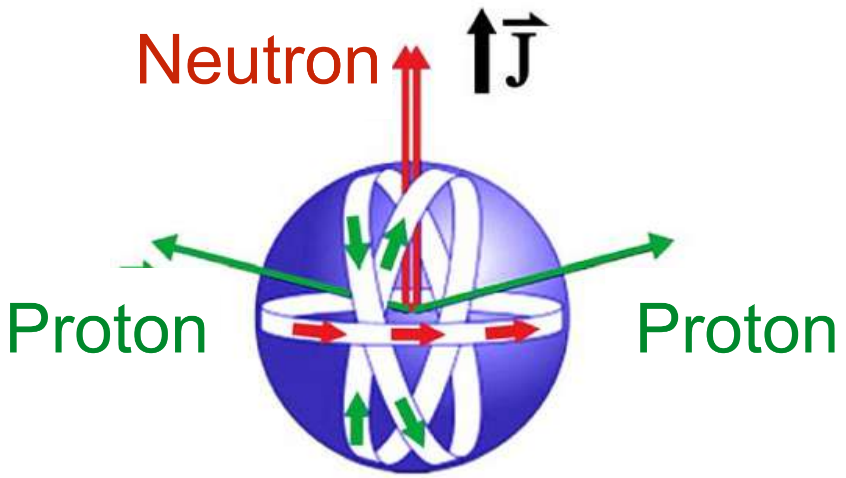
PWZ, Peng, Liang, Ring, Meng, PRL 107, 122501 (2011)

PWZ, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)





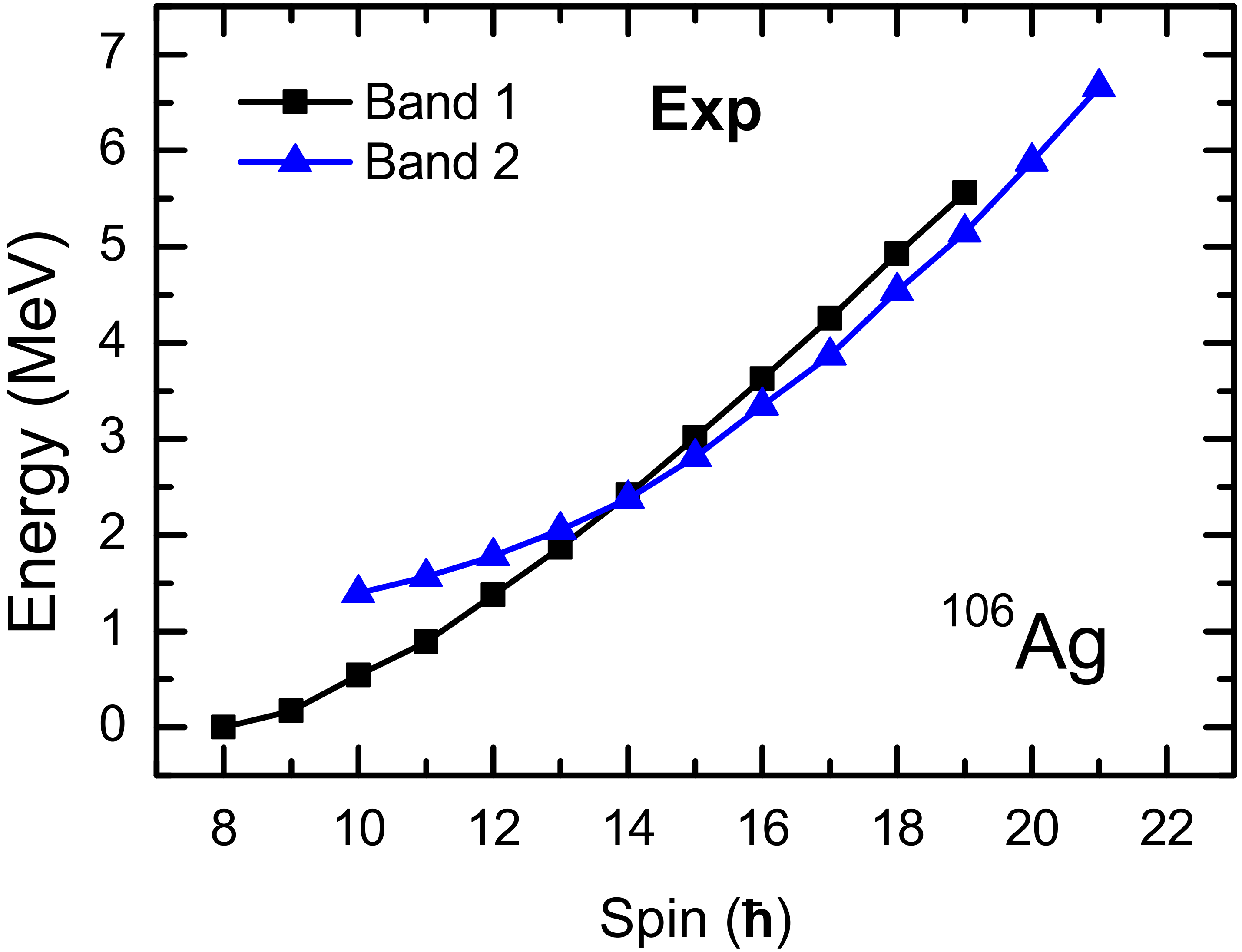


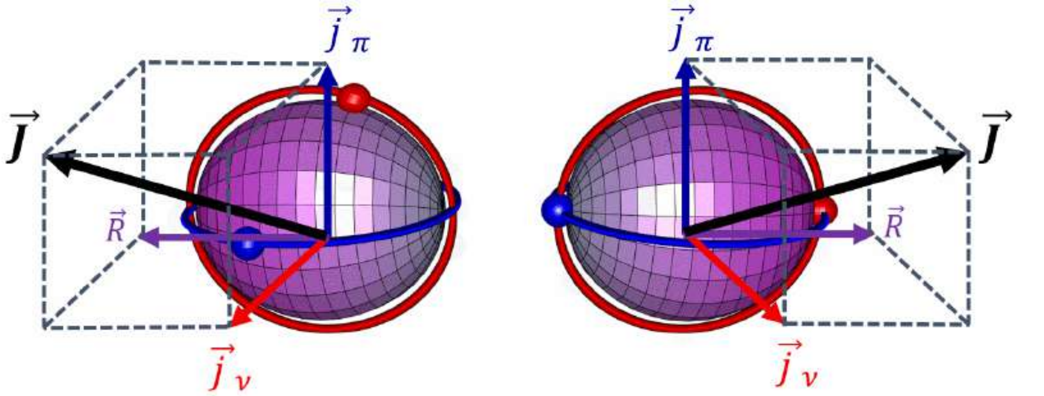


Chiral conundrum in ^{106}Ag

Experimental observations in 2007: Energy Spectrum

Joshi, et al., PRL 98, 102501 (2007)





Frauendorf and Meng, Nucl. Phys. A 617,131 (1997)

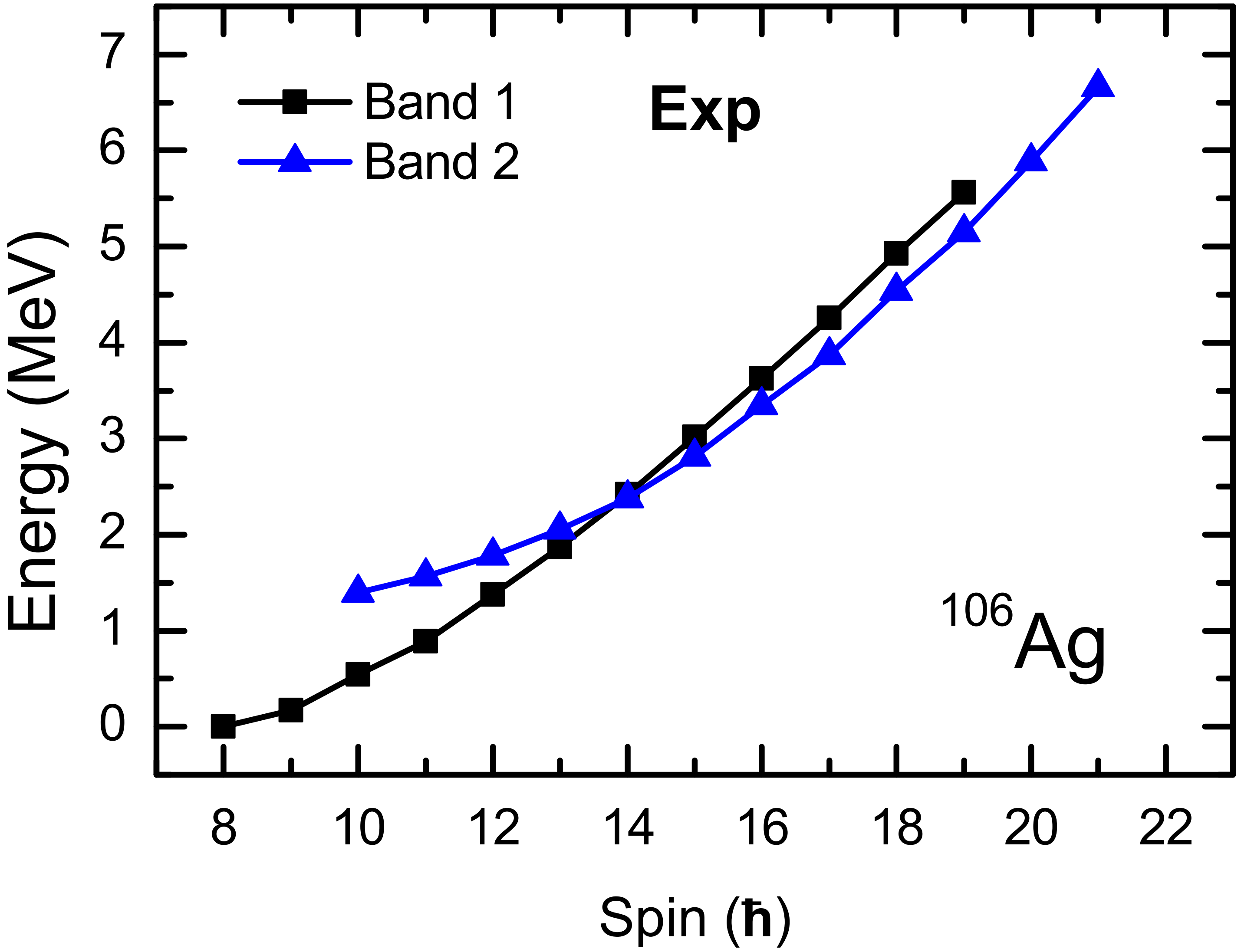
A pair of strongly coupled bands observed

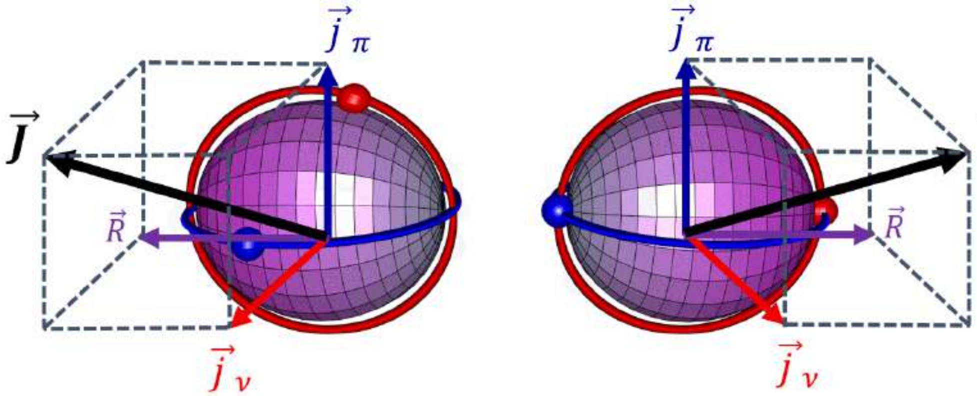
Chiral bands? But why crossing?

Chiral conundrum in ^{106}Ag

Experimental observations in 2007: Energy Spectrum

Joshi, et al., PRL 98, 102501 (2007)





Frauendorf and Meng, Nucl. Phys. A 617,131 (1997)

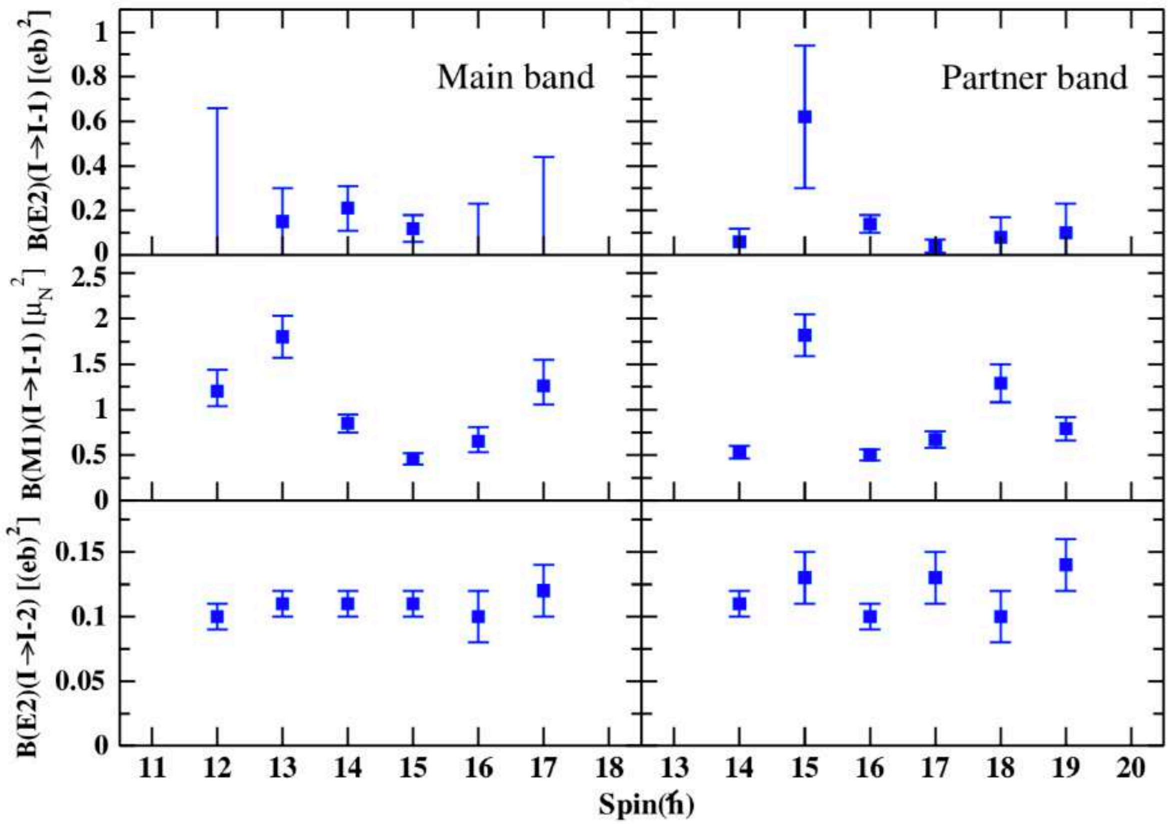
A pair of strongly coupled bands observed

Chiral bands? But why crossing?

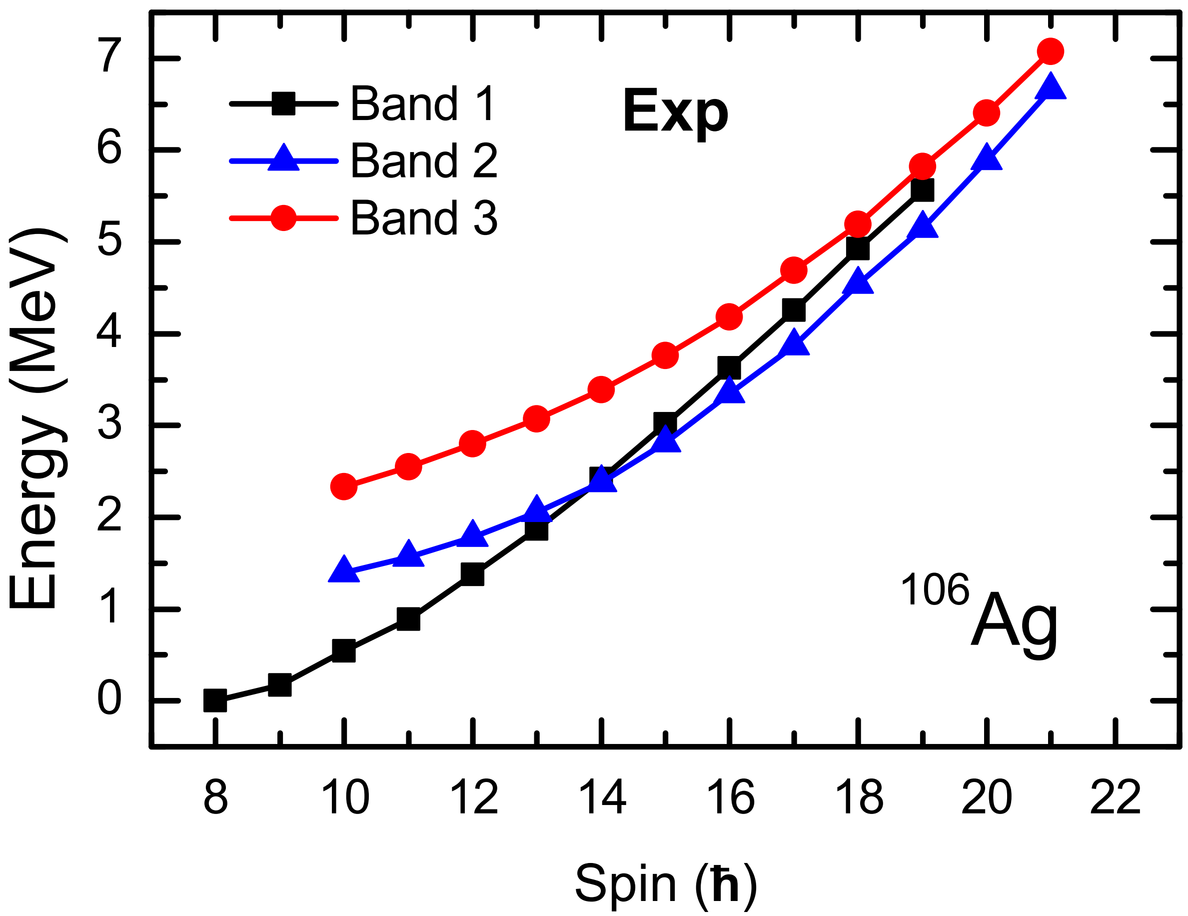
Chiral conundrum in ^{106}Ag

Experimental observations in 2014: Transition strength

Rather, et al., PRL 112, 202503 (2014)



Lieder, et al., PRL 112, 202502 (2014)



B(M1) and B(E2) values were measured in both experiments

A third band is reported in Lieder's experiment

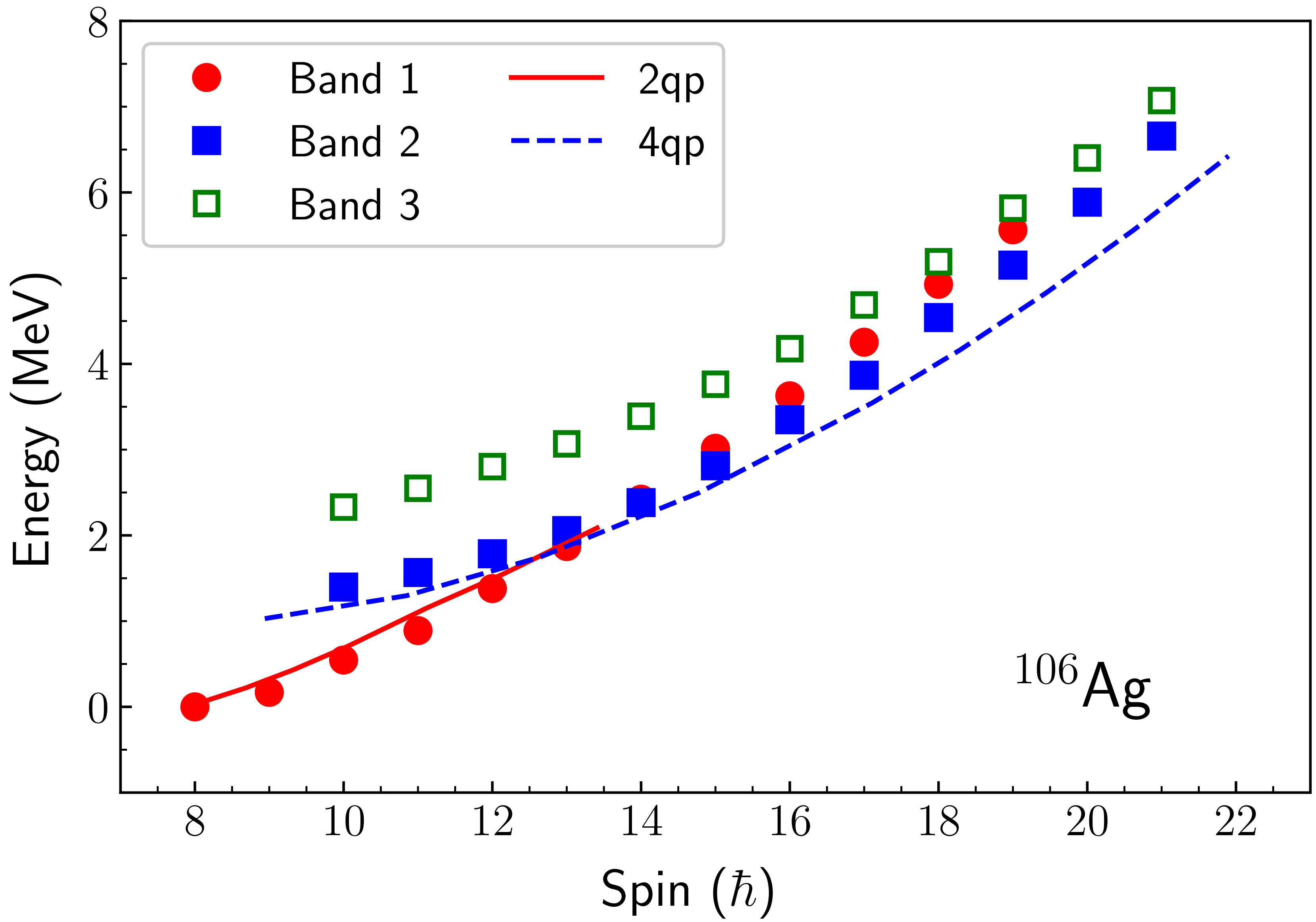
Chiral bands ? But why crossing ? Why three bands ?

Chiral conundrum in ^{106}Ag

PWZ PLB 773,1 (2017)

PWZ, Wang, Chen PRC 99, 054319 (2019)

TAC-CDFT calculations



2qp $\pi g_{9/2} \otimes \nu h_{11/2}$

4qp $\pi g_{9/2} \otimes \nu h_{11/2}(gd)^2$

(C)DFT and Shell Model

(C)DFT

✓ Universal density functionals

Symmetry broken
Single config. fruitful physics
No Configuration mixing

✓ Applicable for almost all nuclei
✗ No spectroscopic properties

Shell Model

✗ Non-universal effective interactions

No symmetry broken
Single config. little physics
Configuration mixing

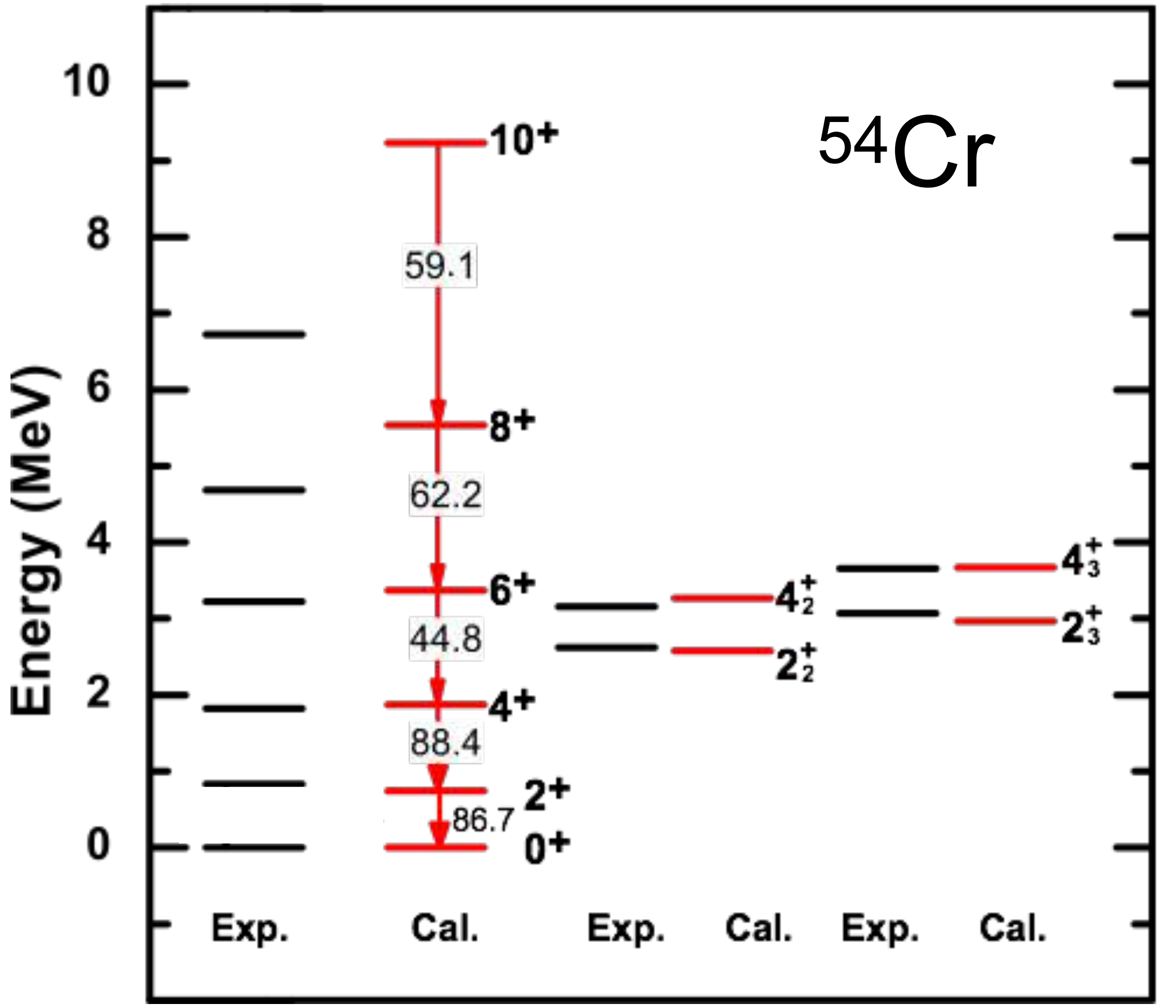
✗ intractable for deformed heavy nuclei
✓ spectroscopy from multi config.

a theory combining the advantages from both approaches

Configuration Interaction Projected DFT (CI-PDFT)

PWZ, Ring, Meng, PRC 94 (2016) 041301(R)

$0\nu\beta\beta$?



Next step: time-odd interaction; beyond 2-qp configurations;

Towards neutron-rich nuclei ?

Towards triaxial nuclei ?

Outline

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  (towards a microscopic formalism based on bare nuclear interactions)

- Summary

Time-dependent density functional theory

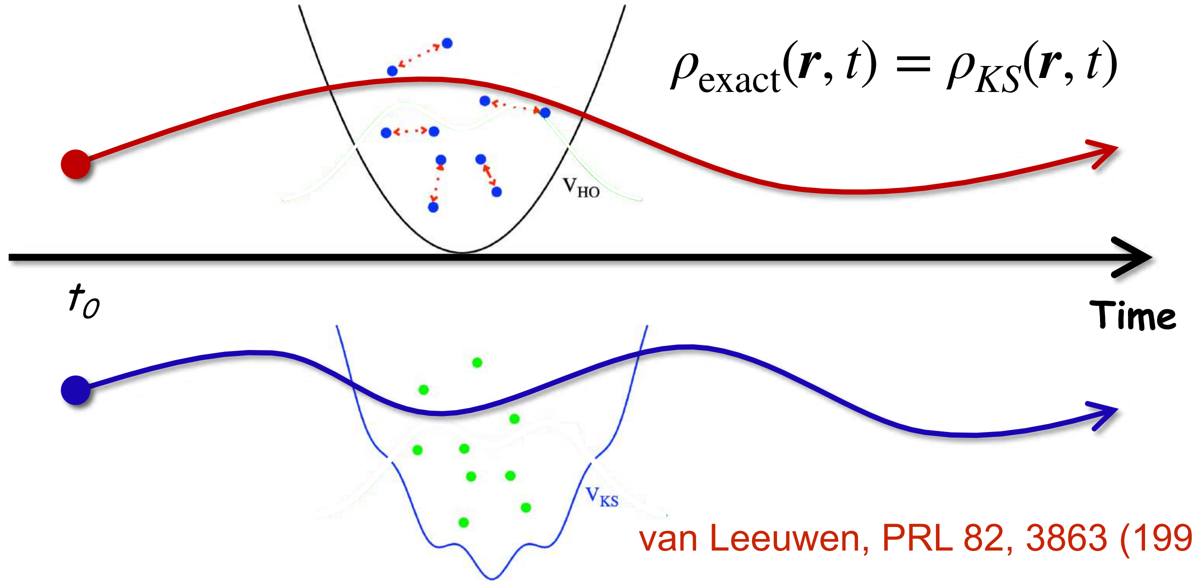
The many-body problem is mapped onto a one-body problem

Runge-Gross Theorem

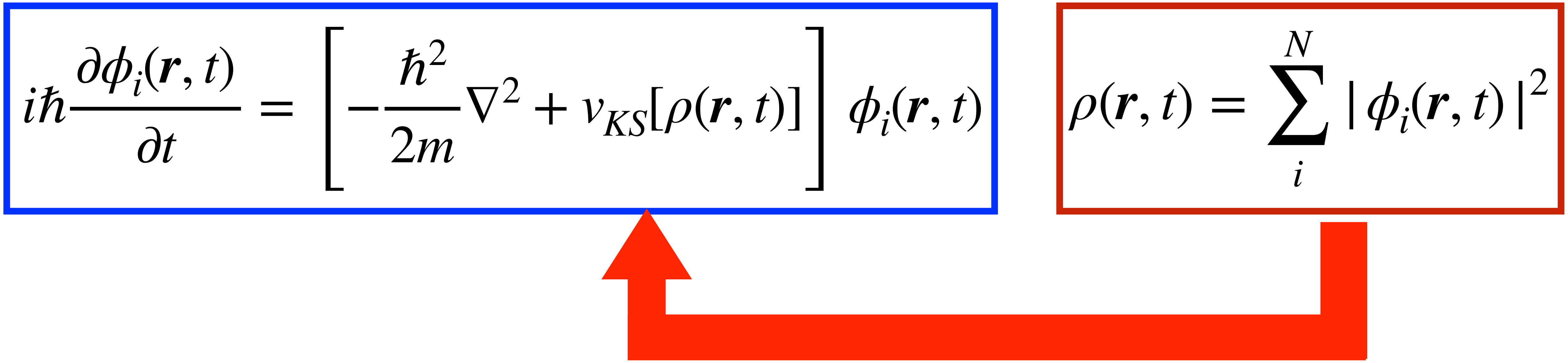
There is a unique mapping between the time dependent external potential and the density, for many body systems evolving from a given initial state.

Runge and Gross, PRL 52, 997 (1984)

Time-dependent Kohn-Sham DFT



van Leeuwen, PRL 82, 3863 (1999)

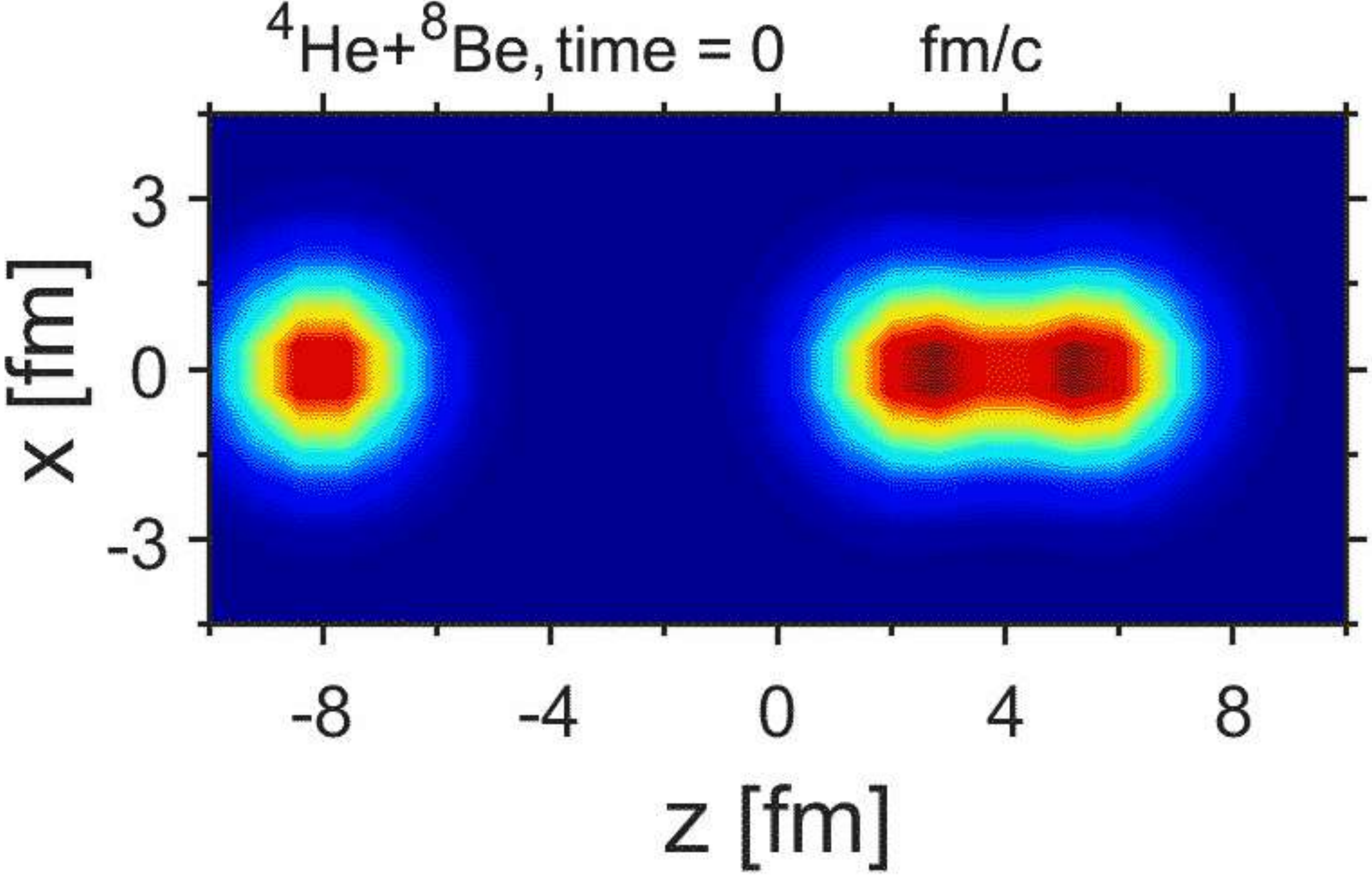


$$i\hbar \frac{\partial \phi_i(\boldsymbol{r}, t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + v_{KS}[\rho(\boldsymbol{r}, t)] \right] \phi_i(\boldsymbol{r}, t)$$

$$\rho(\boldsymbol{r}, t) = \sum_i^N |\phi_i(\boldsymbol{r}, t)|^2$$

Resonant scattering of ^{4}He + ^{8}Be

Ren, PWZ, Meng, PLB 801, 135194 (2020)



The linear-chain states are generated, and then evolve to a triangular configuration, and finally to a more compact shape.

Resonant scattering of ^{4}He + ^{8}Be

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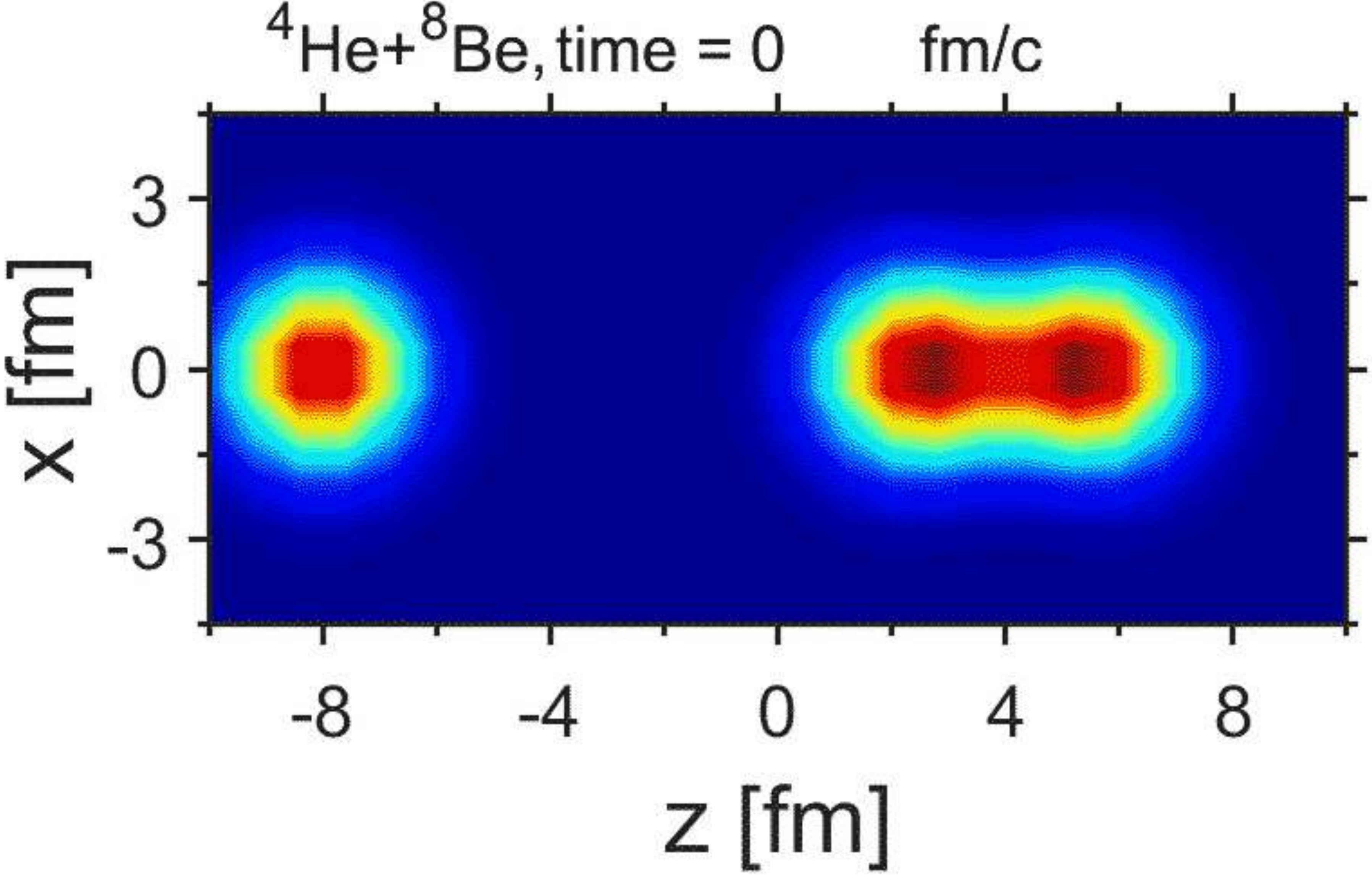

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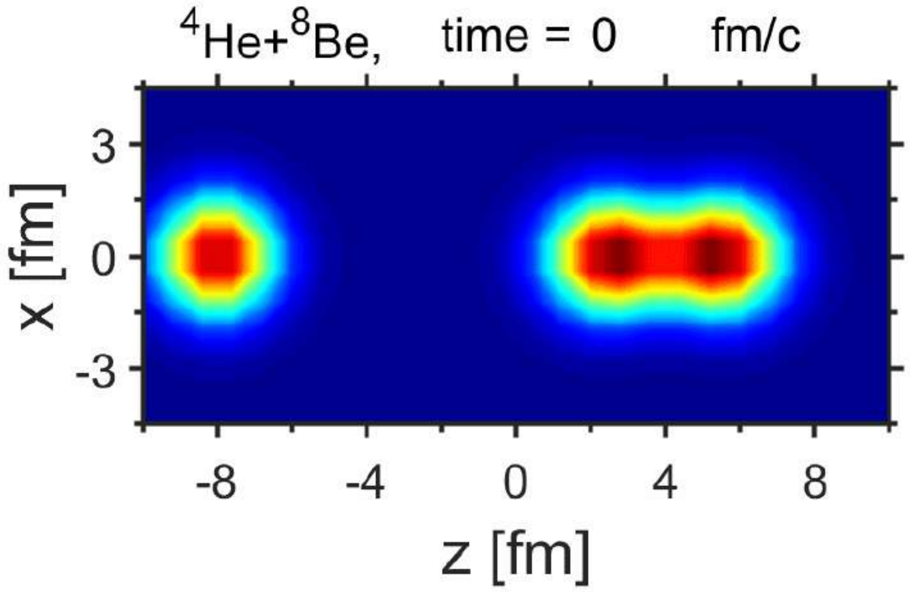
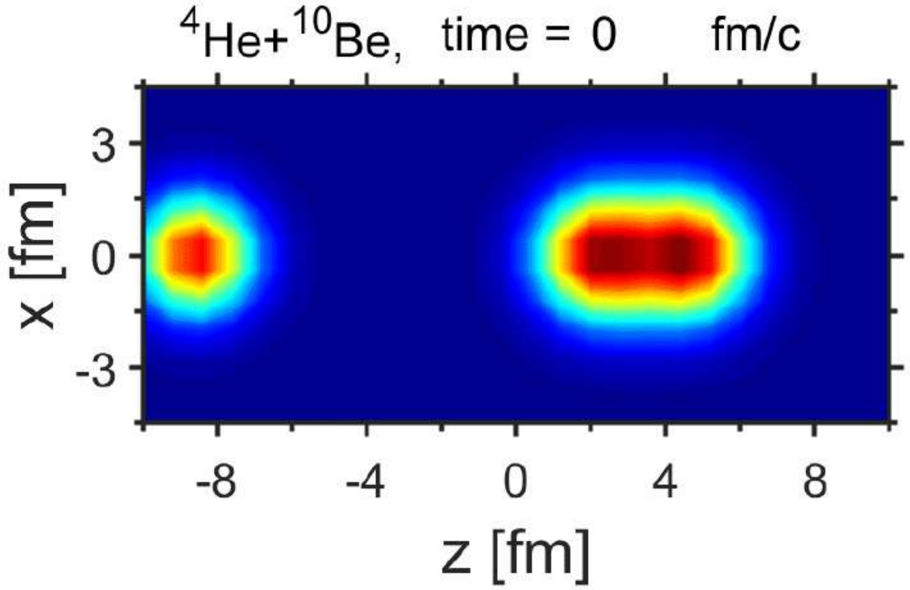
Adding more neutrons: ^{4}He +^{10}Be

Ren, PWZ, Meng, PLB 801, 135194 (2020)





- ✓ The metastable linear chains can be formed in ^{4}He+^{8}Be and ^{4}He+^{10}Be collisions.
- ✓ During the time evolution of the linear-chain configuration, moving clusters can be found.

Adding more neutrons: ^{4}He +^{10}Be

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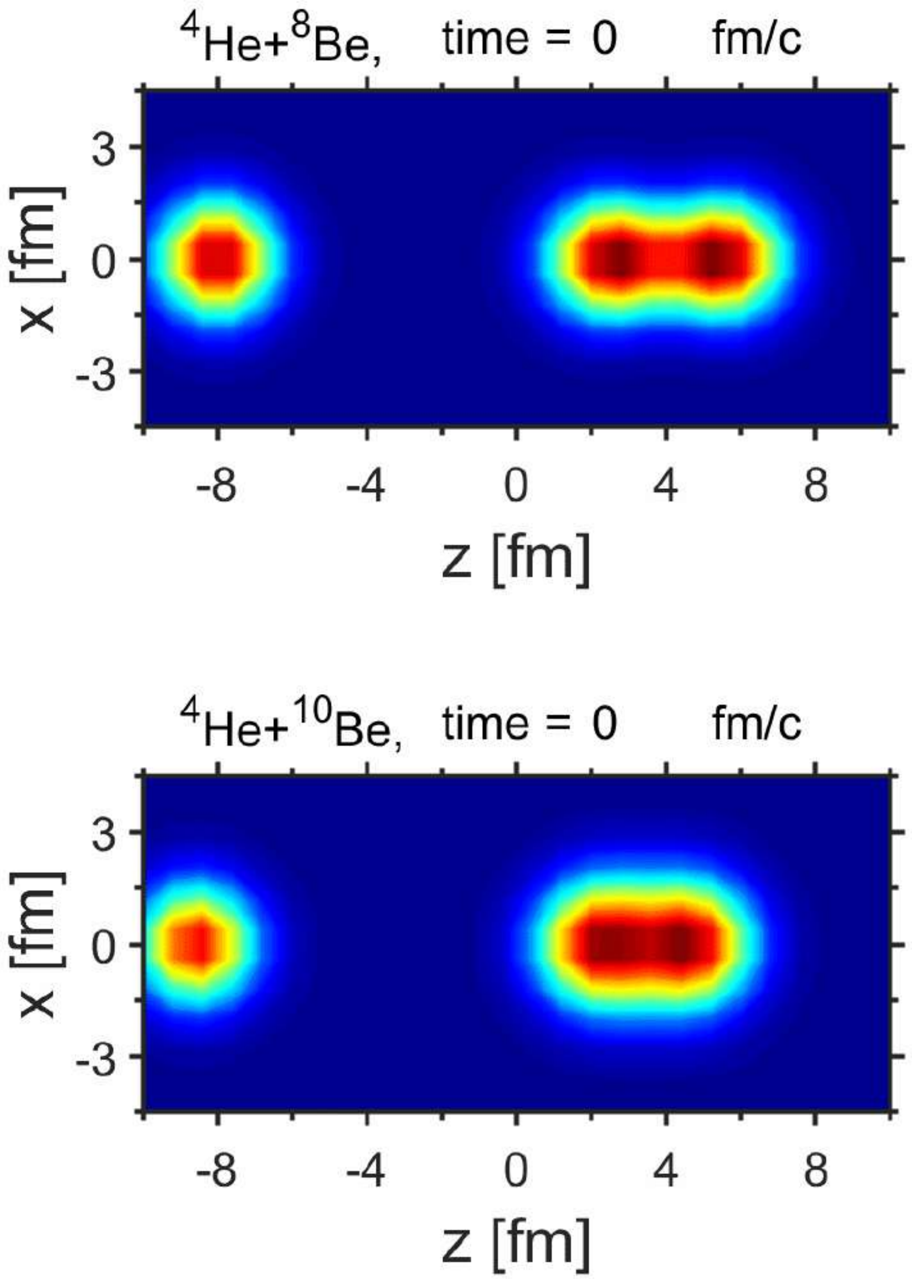

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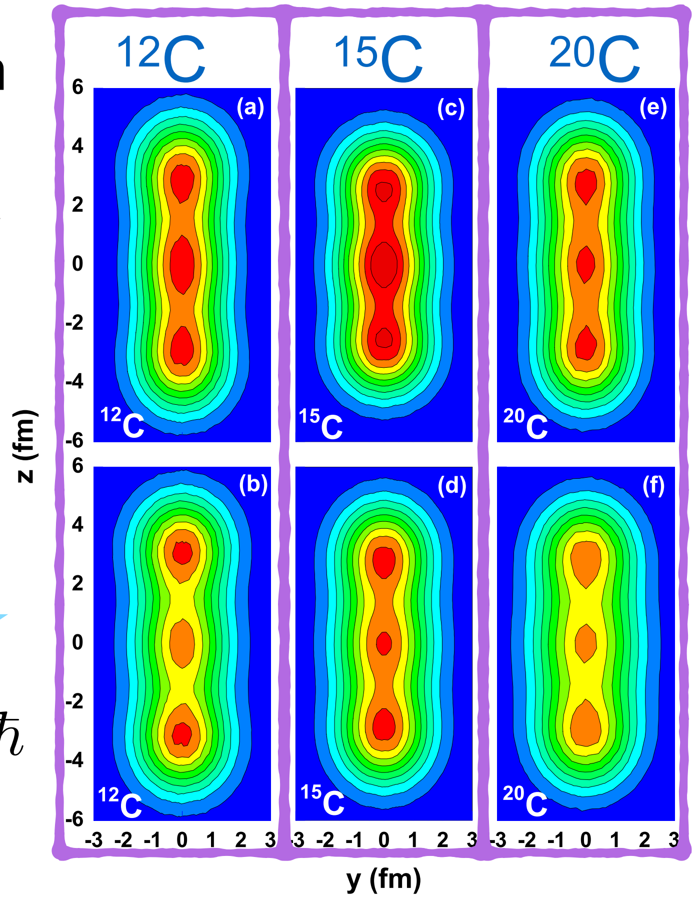
Statical calculations

spin

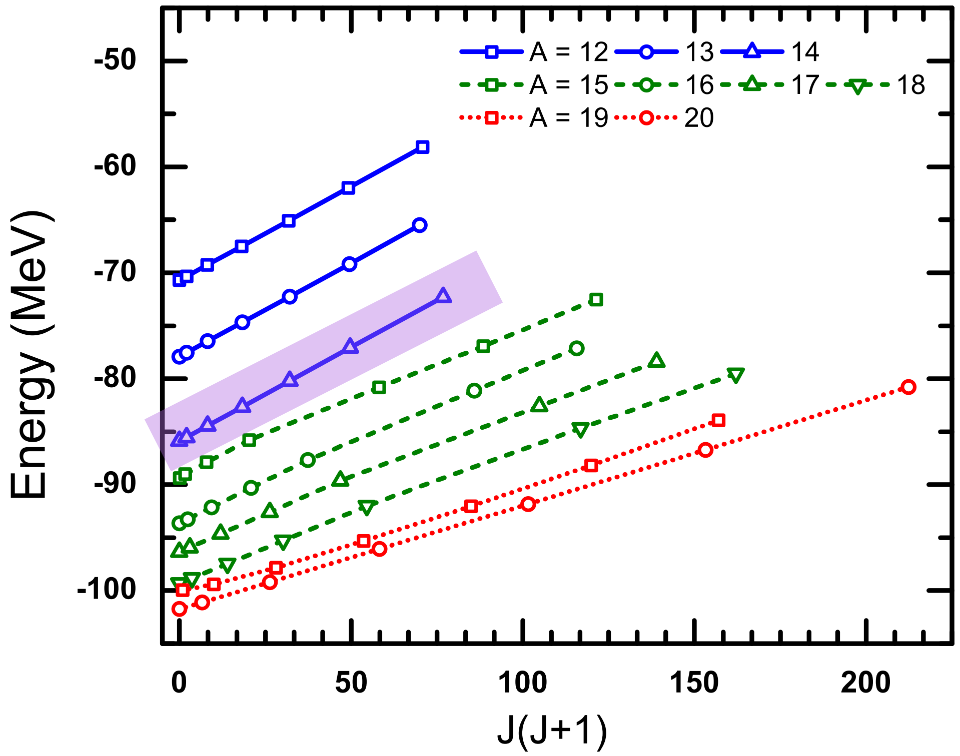
~0 $\hbar$

~10 $\hbar$

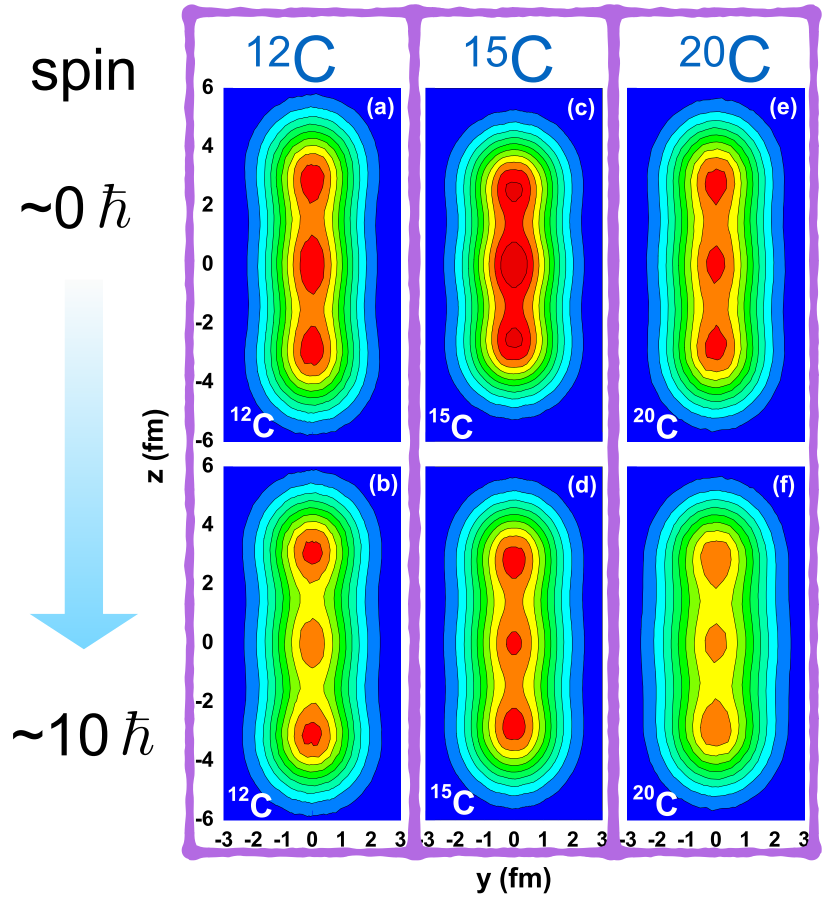


Rod-like shapes could be realized in nuclei with large total spin and isospin.

PWZ, Itagaki, Meng, PRL 115, 022501 (2015)



Statical calculations



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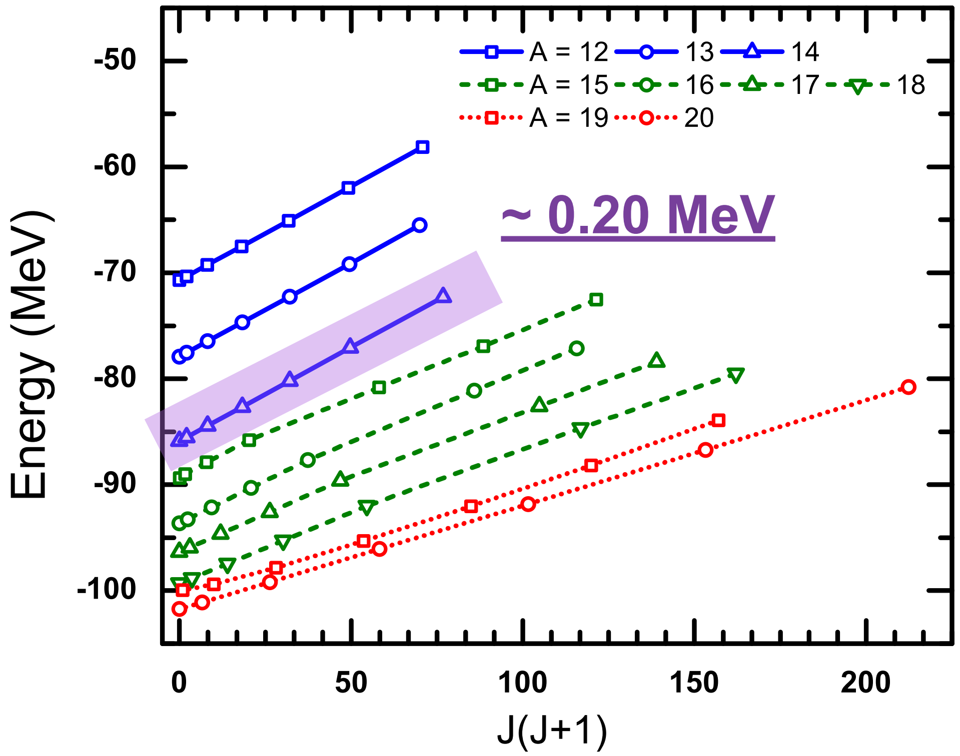
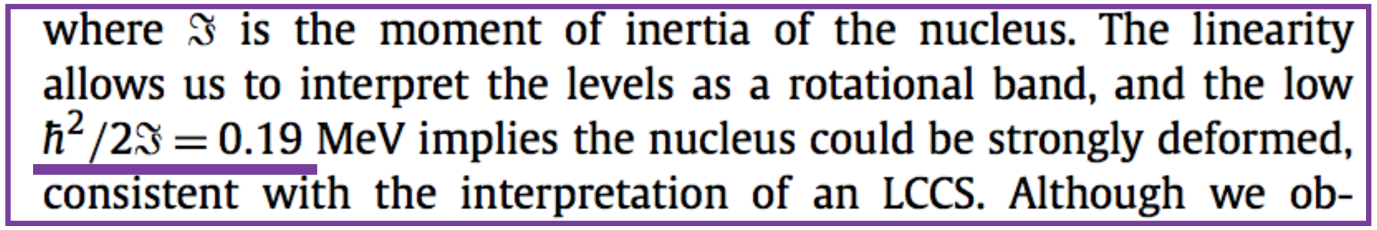


Exp @RIKEN



where $\Im$ is the moment of inertia of the nucleus. The linearity allows us to interpret the levels as a rotational band, and the low $\hbar^2/2\Im = 0.19$ MeV implies the nucleus could be strongly deformed, consistent with the interpretation of an LCCS. Although we ob-

Yamaguchi et al., PLB 766 (2017) 11–16

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  (ground and excited states; low- and high- angular momentum states)

- Nuclear dynamics

  (nuclear structure and reaction)

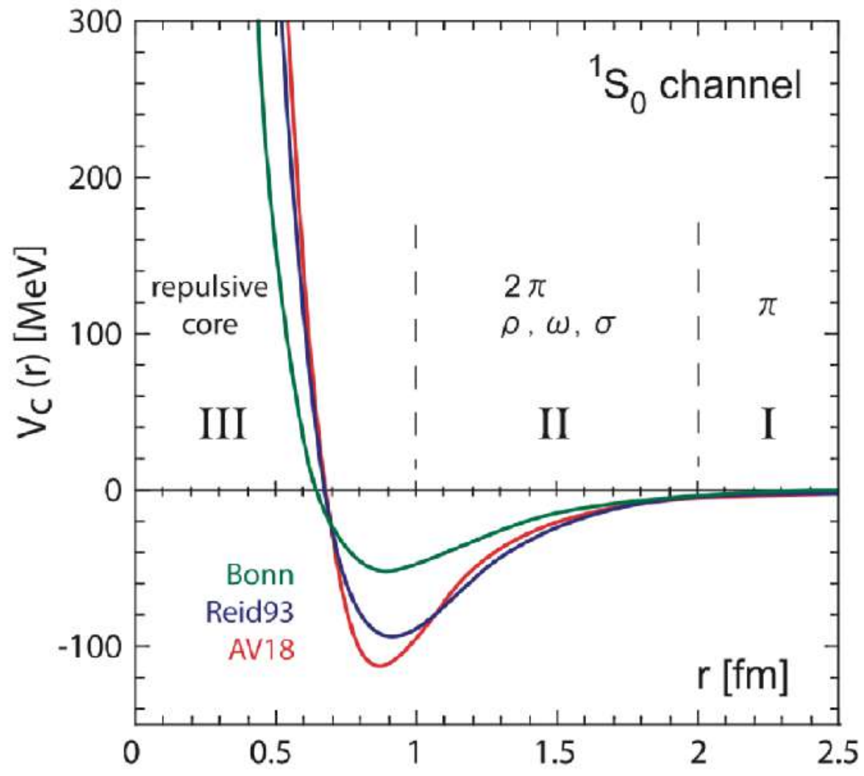
- Nuclear interactions

  (towards a microscopic formalism based on bare nuclear interactions)

- Summary

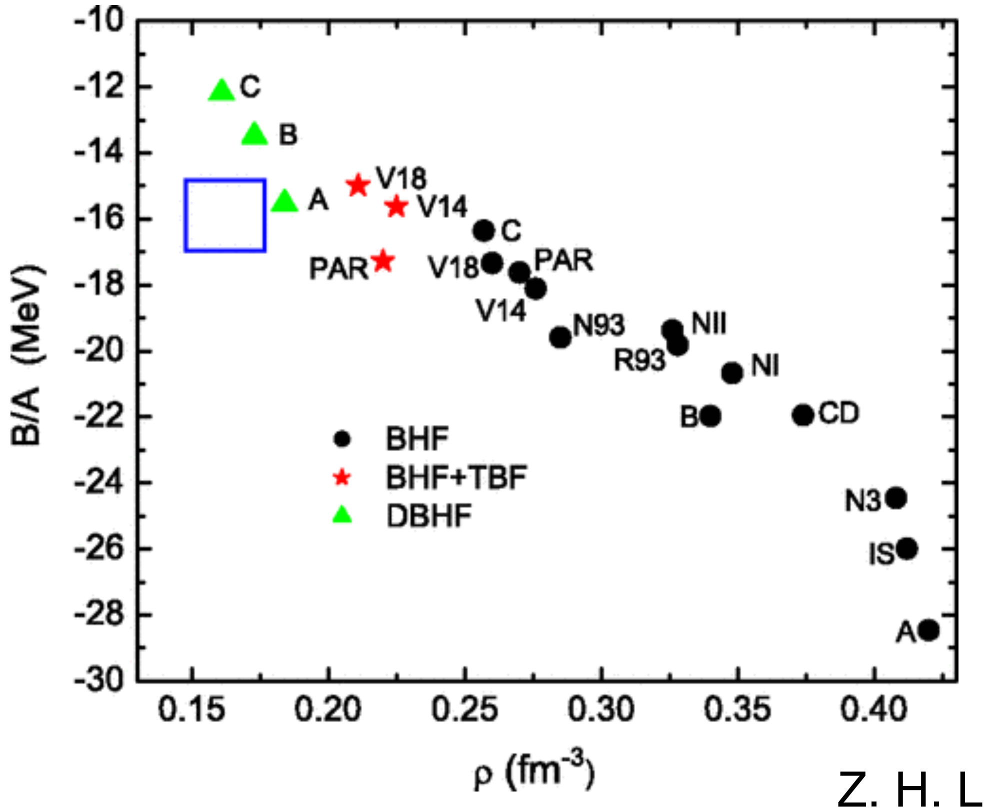
Complicated nuclear force

NN force



S. Aoki 2008

Nuclear matter saturation



Z. H. Li 2006

Short-range repulsive; Three-body force; Covariance

NN forces from EFTs of QCD

北京航空航天大学 BEIHANG UNIVERSITY 1952

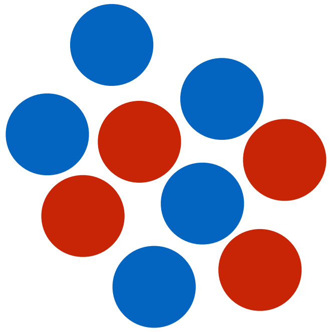
UNIVERSITÄT BONN

PEKING UNIVERSITY 1898

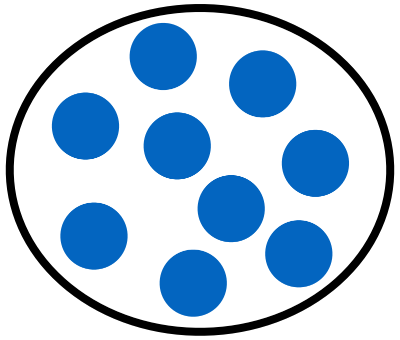
JÜLICH Forschungszentrum

Neutron Drops

Why study neutron drops? (A pure simple toy model?)



Self-bound



Confined

Pudliner, et al., PRL 76, 2416 (1996)
Gandolfi, et al., PRL 106, 012501 (2011)
Maris, et al., PRC 87, 054318 (2013)
Potter, et al., PLB 739, 445 (2014)
PWZ and Gandolfi, PRC 94 (2016) 041302(R)
Shen, et al., PLB 778, 344 (2018)
……

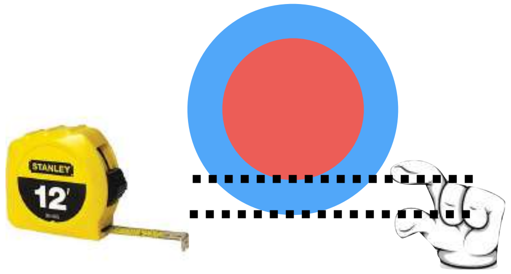
1. benchmark various nuclear many-body methods
2. calibrate nuclear interactions / energy density functionals
3. model neutron-rich nuclei and neutron stars
4. predict few neutron resonances
5. connections with the skin thickness and symmetry energy

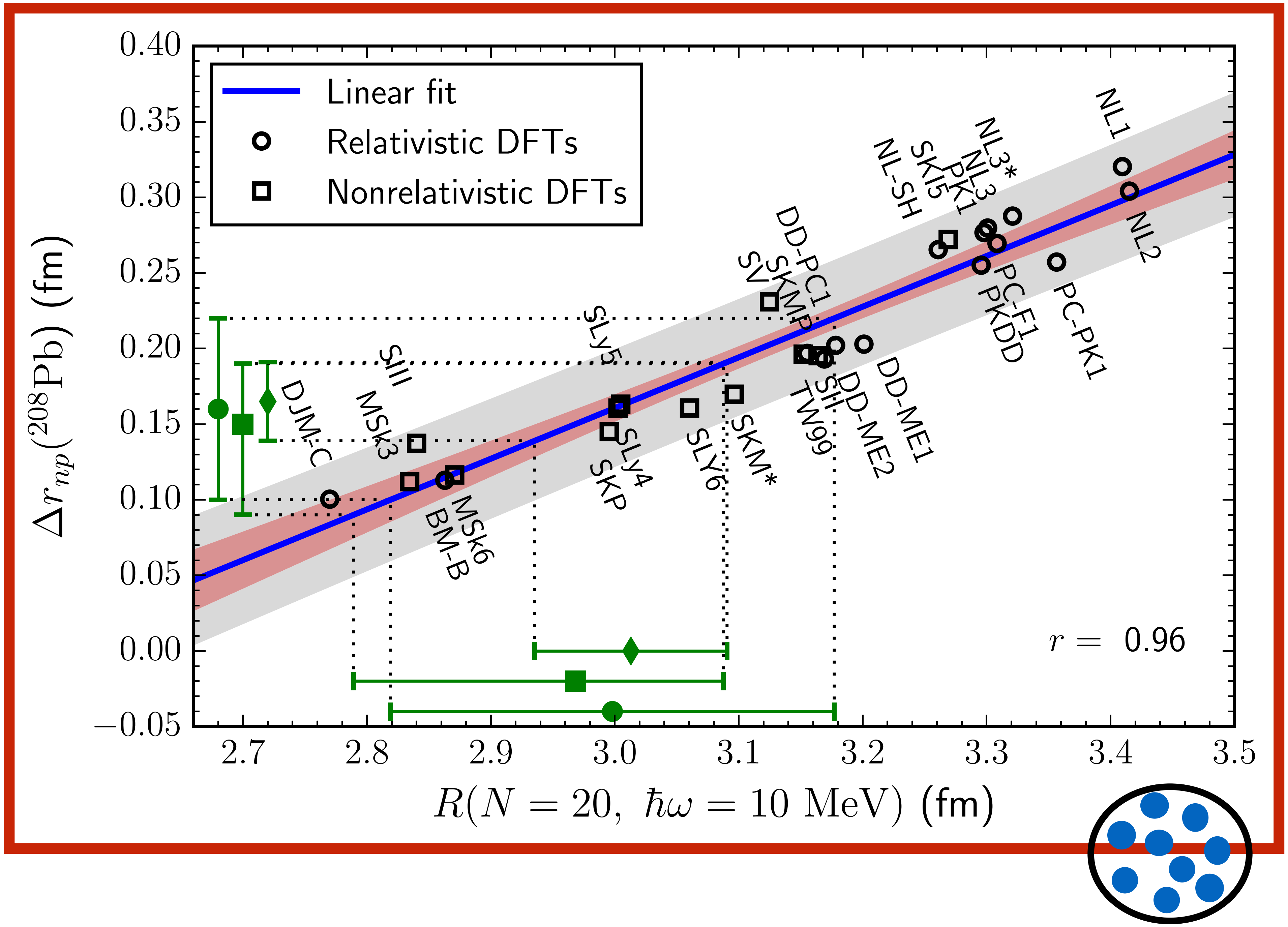
Neutron drop as a “toy” neutron-rich nucleus

Neutron skin thickness

PWZ and Gandolfi PRC 94 (2016) 041302(R)

$$\Delta r_{np} = r_n - r_p$$

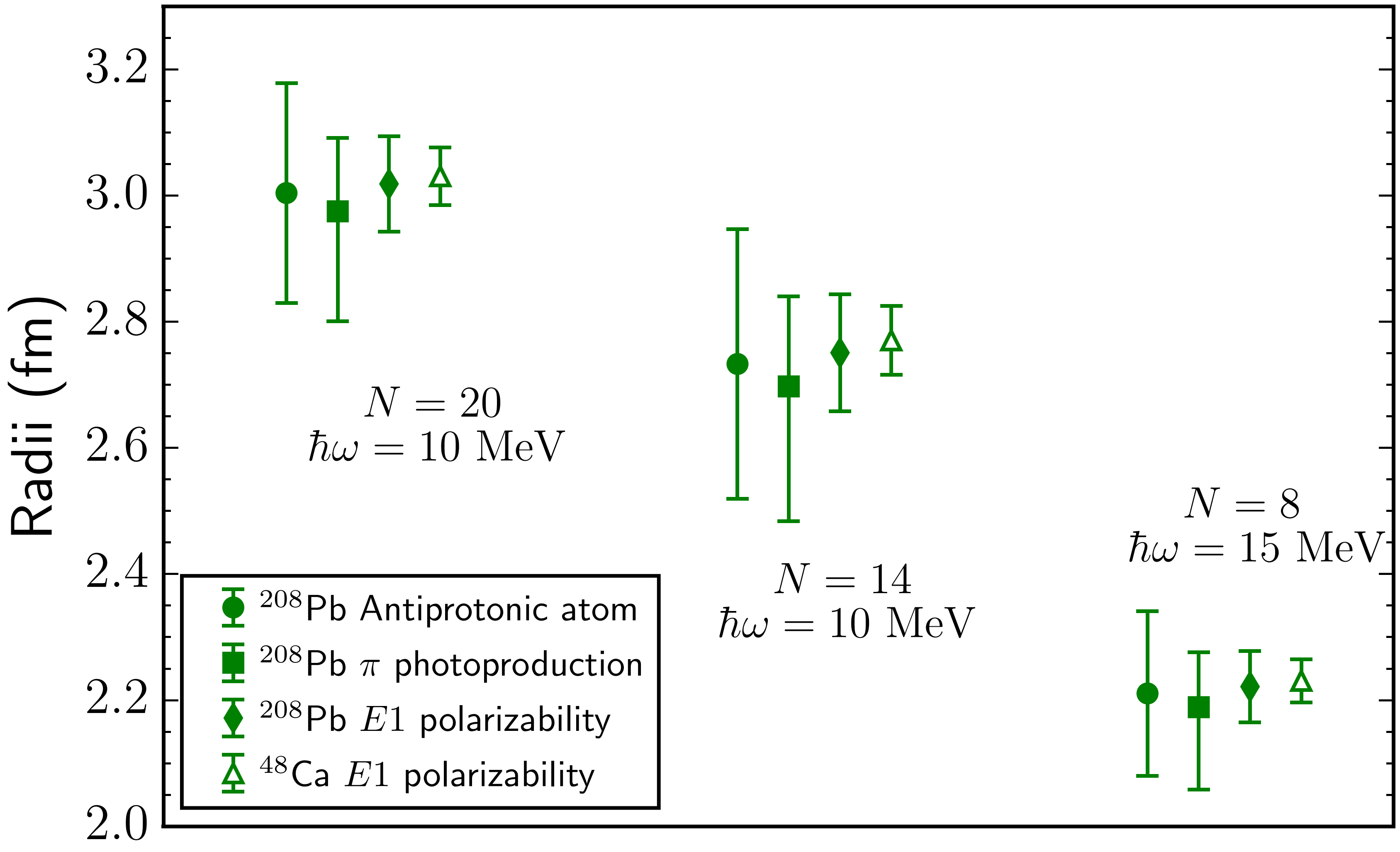




Constraining three-body forces

Radii: QMC calculations with various Hamiltonians, compared to what is extracted from "experiments".

PWZ and Gandolfi PRC 94 (2016) 041302(R)

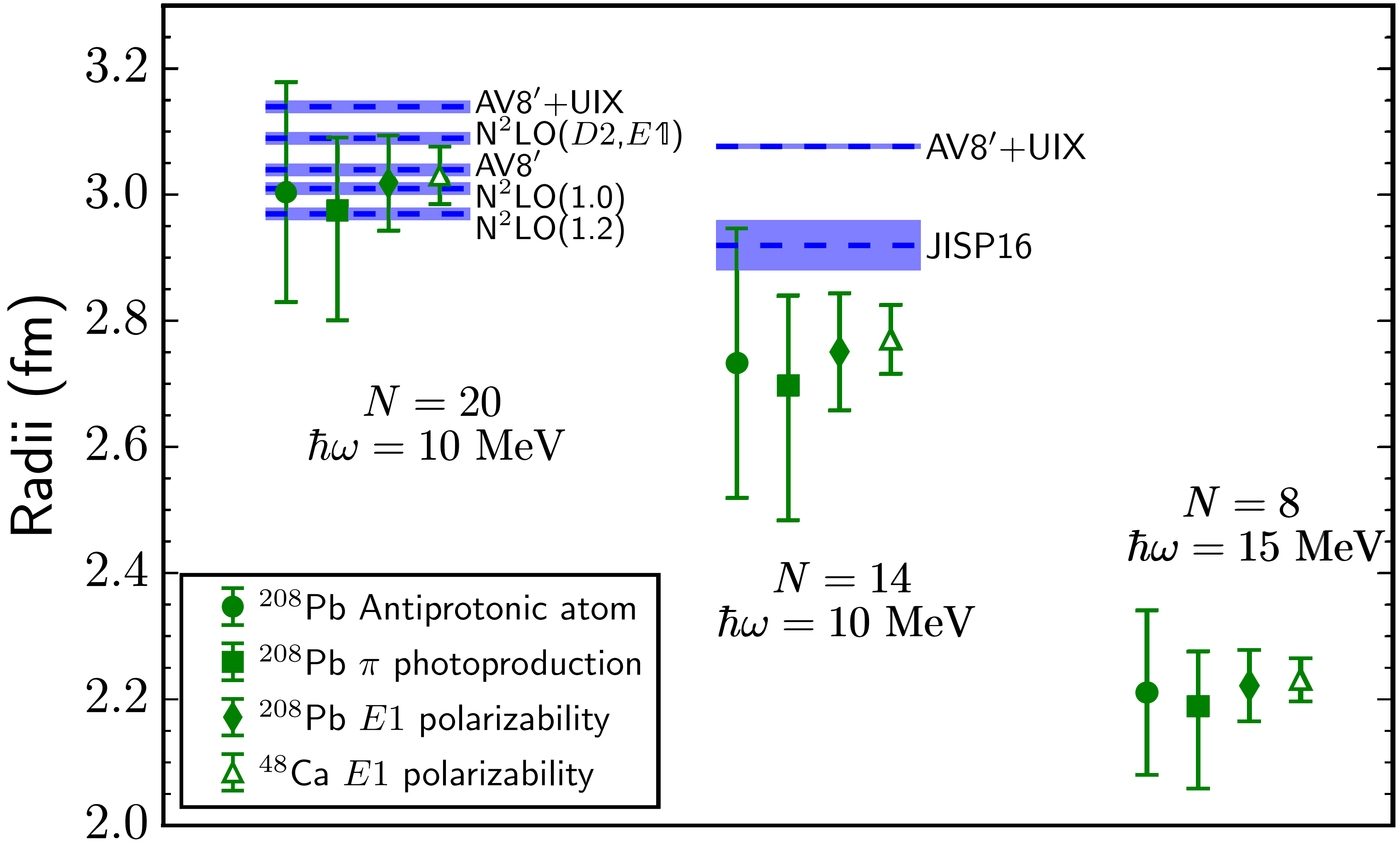


With the development of the high-accuracy measurements of neutron skin thickness, the radii obtained for neutron drops will provide a useful constraint for realistic three neutron forces.

Constraining three-body forces

Radii: QMC calculations with various Hamiltonians, compared to what is extracted from "experiments".

PWZ and Gandolfi PRC 94 (2016) 041302(R)

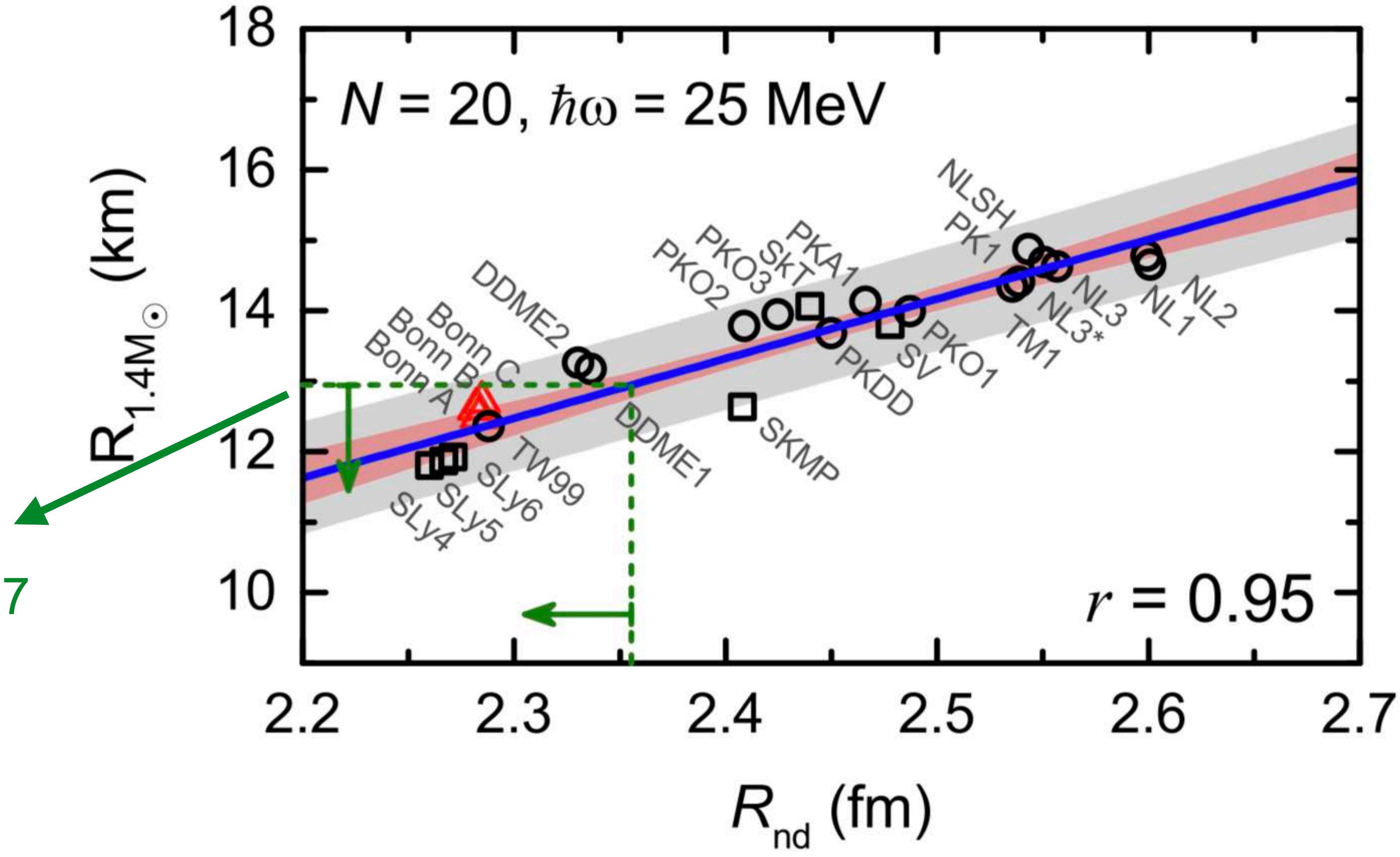


With the development of the high-accuracy measurements of neutron skin thickness, the radii obtained for neutron drops will provide a useful constraint for realistic three neutron forces.

Neutron drop as a “toy” neutron star

Going to high densities …

Tong, PWZ, Meng, PRC 101, 035802 (2020)



From GW170817

The strong correlation can be used to constrain the radius of the neutron drop

Outline

- Nuclear landscape

  (light and heavy nuclei; neutron-deficient and neutron-rich nuclei)

- Nuclear spectroscopy

  (ground and excited states; low- and high- angular momentum states)

- Nuclear dynamics

  (nuclear structure and reaction)

- Nuclear interactions

  (towards a microscopic formalism based on bare nuclear interactions)

- Summary

Summary

Covariant density functional theory is being improved and extended for a unified description of nuclei.

➢ Nuclear landscape

a systematic study with PC-PK1

triaxiality + dynamical correlations!

➢ Nuclear spectroscopy

a new tool for global studies of nuclear spectroscopy: CI-PDFT

merits of (C)DFT and Shell Model preserved

➢ Nuclear dynamics

time-dependent CDFT has been developed

pave the way for a unified description of nuclear structure and reaction

➢ Nuclear interaction

neutron drop, neutron star, and neutron skin are found to be strongly correlated

likely to have an enduring impact on the understanding of multi-neutron interactions

Collaborations

Beijing
Jie Meng
Jing Peng
Zhengxue Ren
Yakun Wang
Shuangquan Zhang

Munich
Qibo Chen
Peter Ring

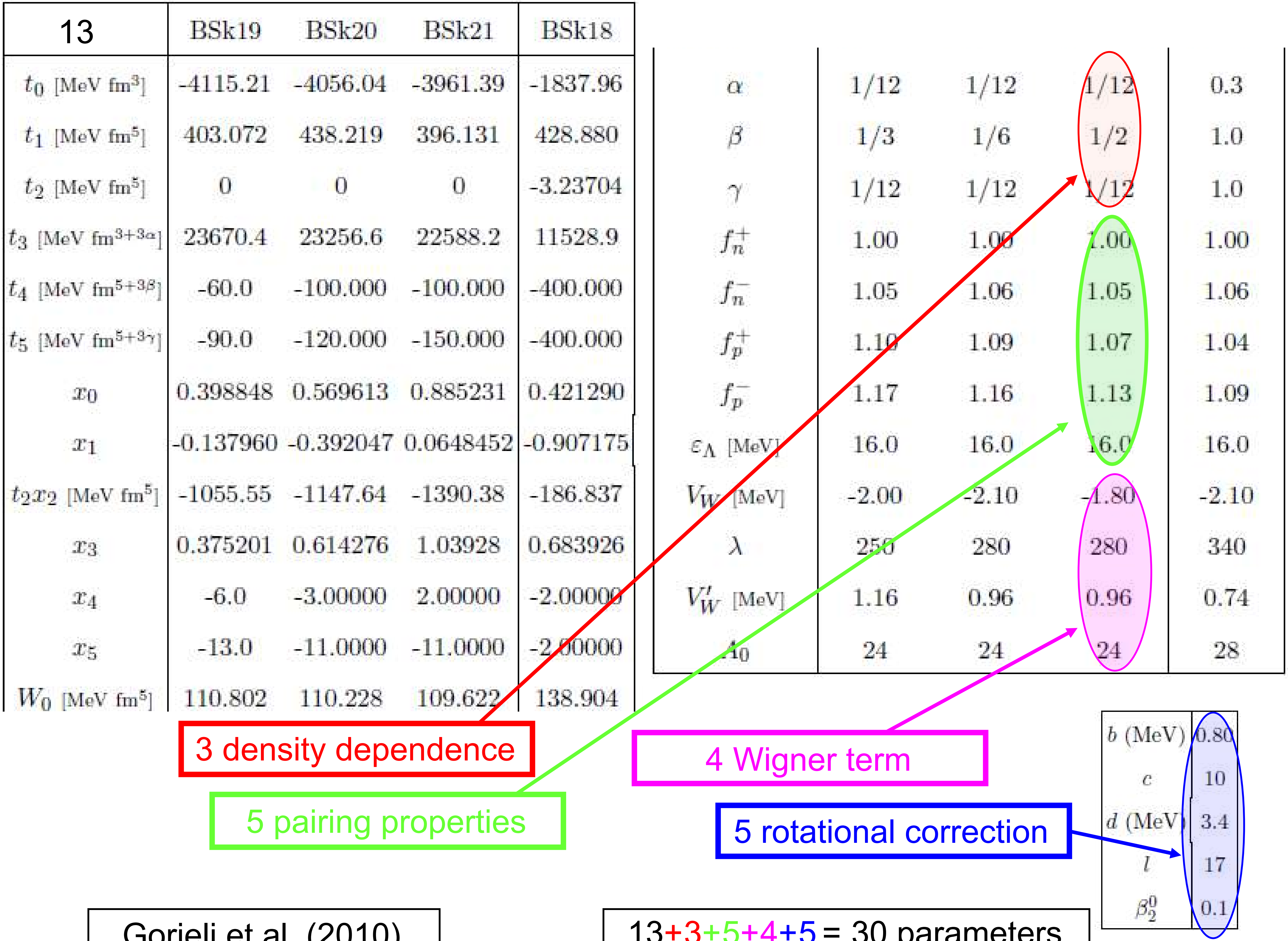
Chongqing
Zhipan Li

Kyoto
Naoyuki Itagaki

Los Alamos
Stefano Gandolfi

…

Thank you for your attention!



13	BSk19	BSk20	BSk21	BSk18
t_0 [MeV fm^3]	-4115.21	-4056.04	-3961.39	-1837.96
t_1 [MeV fm^5]	403.072	438.219	396.131	428.880
t_2 [MeV fm^5]	0	0	0	-3.23704
t_3 [MeV fm$^{3+3\alpha}$]	23670.4	23256.6	22588.2	11528.9
t_4 [MeV fm$^{5+3\beta}$]	-60.0	-100.000	-100.000	-400.000
t_5 [MeV fm$^{5+3\gamma}$]	-90.0	-120.000	-150.000	-400.000
x_0	0.398848	0.569613	0.885231	0.421290
x_1	-0.137960	-0.392047	0.0648452	-0.907175
t_2x_2 [MeV fm^5]	-1055.55	-1147.64	-1390.38	-186.837
x_3	0.375201	0.614276	1.03928	0.683926
x_4	-6.0	-3.00000	2.00000	-2.00000
x_5	-13.0	-11.0000	-11.0000	-2.00000
W_0 [MeV fm^5]	110.802	110.228	109.622	138.904
α	1/12	1/12	1/12	0.3
β	1/3	1/6	1/2	1.0
γ	1/12	1/12	1/12	1.0
f_n^+	1.00	1.00	1.00	1.00
f_n^-	1.05	1.06	1.05	1.06
f_p^+	1.10	1.09	1.07	1.04
f_p^-	1.17	1.16	1.13	1.09
ε_Λ [MeV]	16.0	16.0	16.0	16.0
V_W [MeV]	-2.00	-2.10	-1.80	-2.10
λ	250	280	280	340
V'_W [MeV]	1.16	0.96	0.96	0.74
A_0	24	24	24	28

b (MeV)	0.80
c	10
d (MeV)	3.4
l	17
β_2^0	0.1

3 density dependence

5 pairing properties

4 Wigner term

5 rotational correction

Gorieli et al, (2010)

13+3+5+4+5 = 30 parameters