Effective field theory approach to lepton number violating processes

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Introduction

Why we explore lepton number violation (LNV)?

- Neutrino oscillation ⇒ non-vanishing neutrino mass ⇐ Majorana neutrino mass
- The nature of dark matter
- The asymmetry of matter and anti-matter
- ...
- LNV processes are definite signal for **New physics (NP)**
- lepton number violation (LNV) → BAU via leptogenesis

Introduction

The ways to lepton number violation (LNV)

- **Experimental**
  - High-energy frontier: The production of the same sign charged leptons
  - **High-intensity frontier:** Search for the LNV signals in low energy experiments
- **Theoretical**
  - Top-down approach: Study signals in explicit NP models
  - **Bottom-up approach:** Work with Effective field theories (EFTs)

Both approaches are necessary and complementary!

Introduction

Low energy LNV processes

- **Nuclear processes**
  - Neutrinoless double beta decay $X \to X' e^{\mp} e^{\mp}$
    - KamLAND-Zen, Gerda, EXO-200, SNO+, Majorana…
    - $T_{1/2}^{0\nu}(^{136}Xe) > 1.06 \times 10^{26}$ yr from KamLAND-Zen
  - Muon to positron or anti-muon $\mu^- X \to e^+(\mu^+) X'$ in the upcoming Mu2e experiment

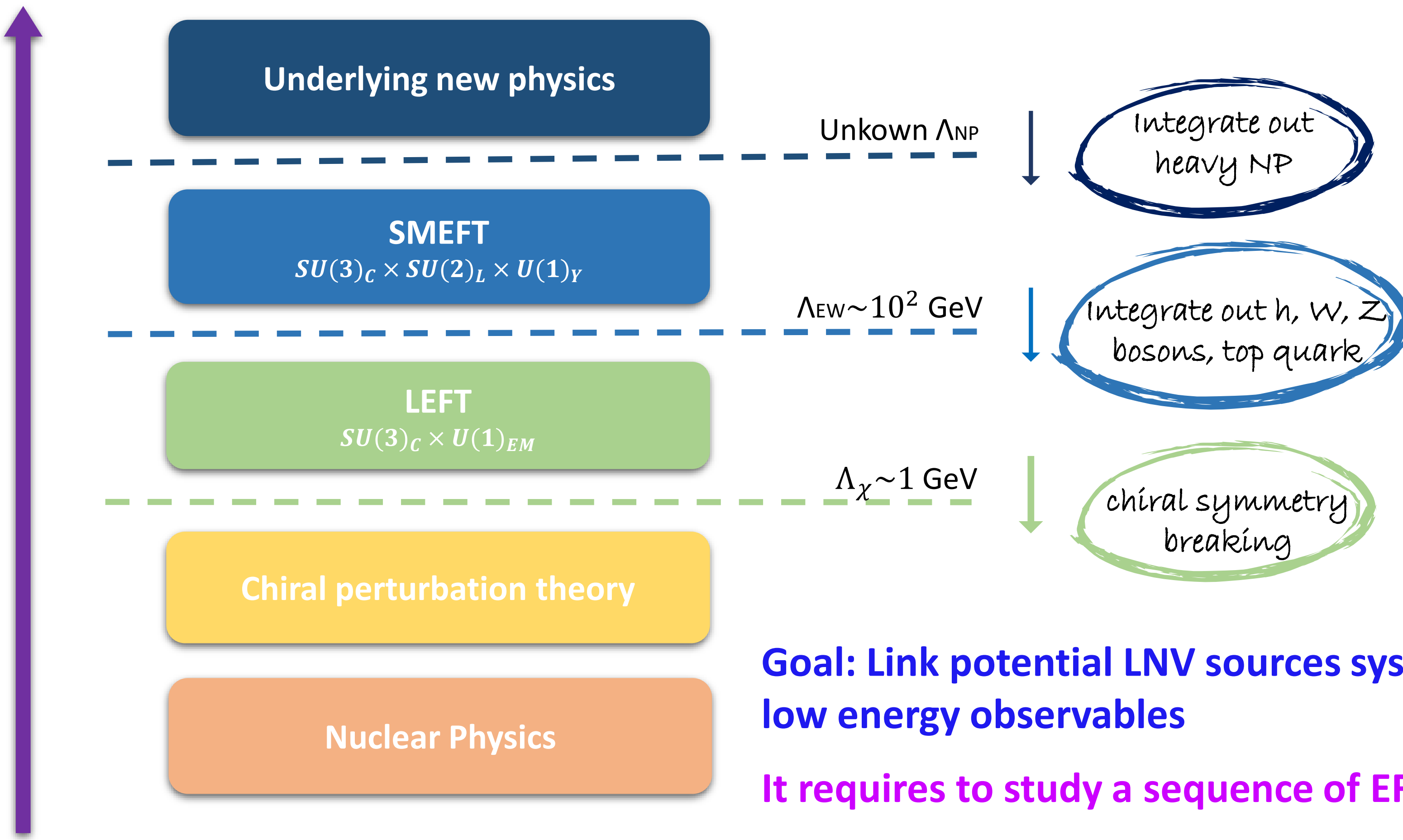
- **Rare mesons & lepton decays**
  - LNV K, B, D decays
    - Babar, Belle, LHCb
  - LNV Tau decays
    - Belle

Modes for $l_\alpha l_\beta$=	ee	$e\mu$	$\mu\mu$
$K^- \to \pi^+ l_\alpha^- l_\beta^-$	2.2×10^{-10}	5.0×10^{-10}	4.2×10^{-11}
$D^- \to \pi^+ l_\alpha^- l_\beta^-$	1.1×10^{-6}	2.0×10^{-6}	2.2×10^{-8}
$D^- \to K^+ l_\alpha^- l_\beta^-$	9×10^{-7}	1.9×10^{-6}	1.0×10^{-5}
$B^- \to \pi^+ l_\alpha^- l_\beta^-$	2.3×10^{-8}	1.5×10^{-7}	4.0×10^{-9}
$B^- \to K^+ l_\alpha^- l_\beta^-$	3.0×10^{-8}	1.6×10^{-7}	4.1×10^{-8}
$\cdots$	$\cdots$	$\cdots$	$\cdots$

Modes for l=	e	μ
$\tau^- \to \pi^- \pi^- l^+$	2.0×10^{-8}	3.9×10^{-8}
$\tau^- \to \pi^- K^- l^+$	3.2×10^{-8}	4.8×10^{-8}
$\tau^- \to K^- K^- l^+$	3.3×10^{-8}	4.7×10^{-8}

Introduction

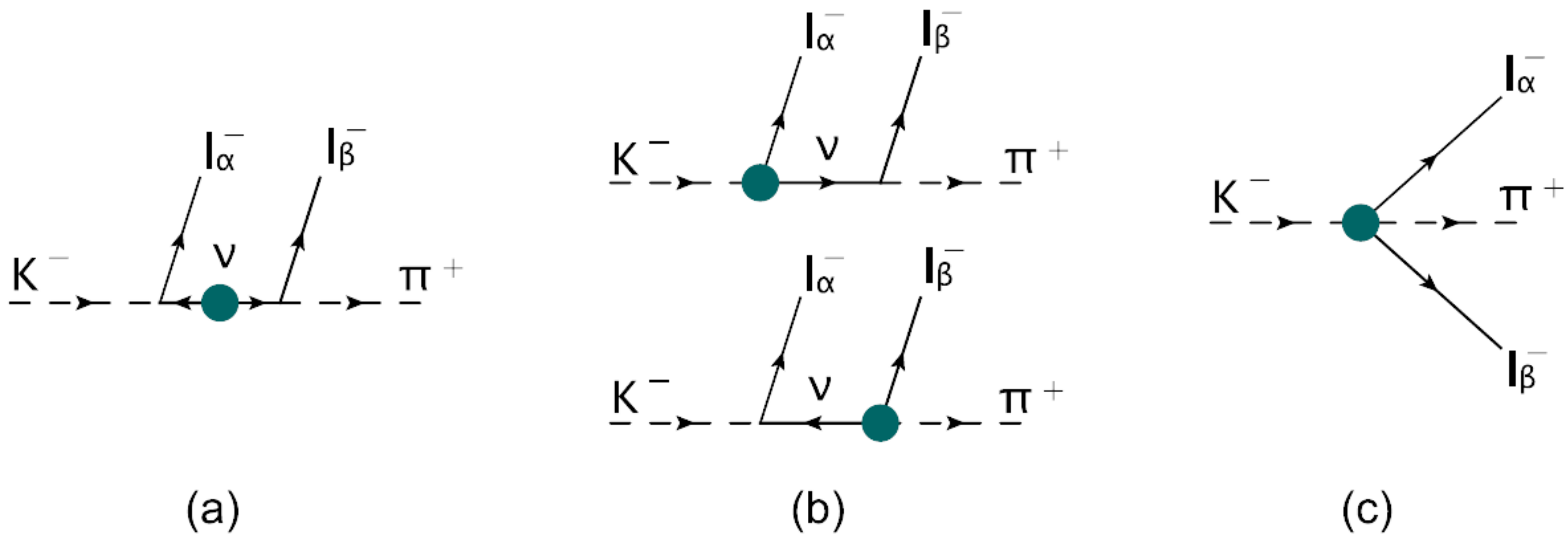
A general picture of EFT approach for LNV processes



Goal: Link potential LNV sources systematically to low energy observables

It requires to study a sequence of EFTs!

Take $K^- \to \pi^+ l_\alpha^- l_\beta^-$ as an example...



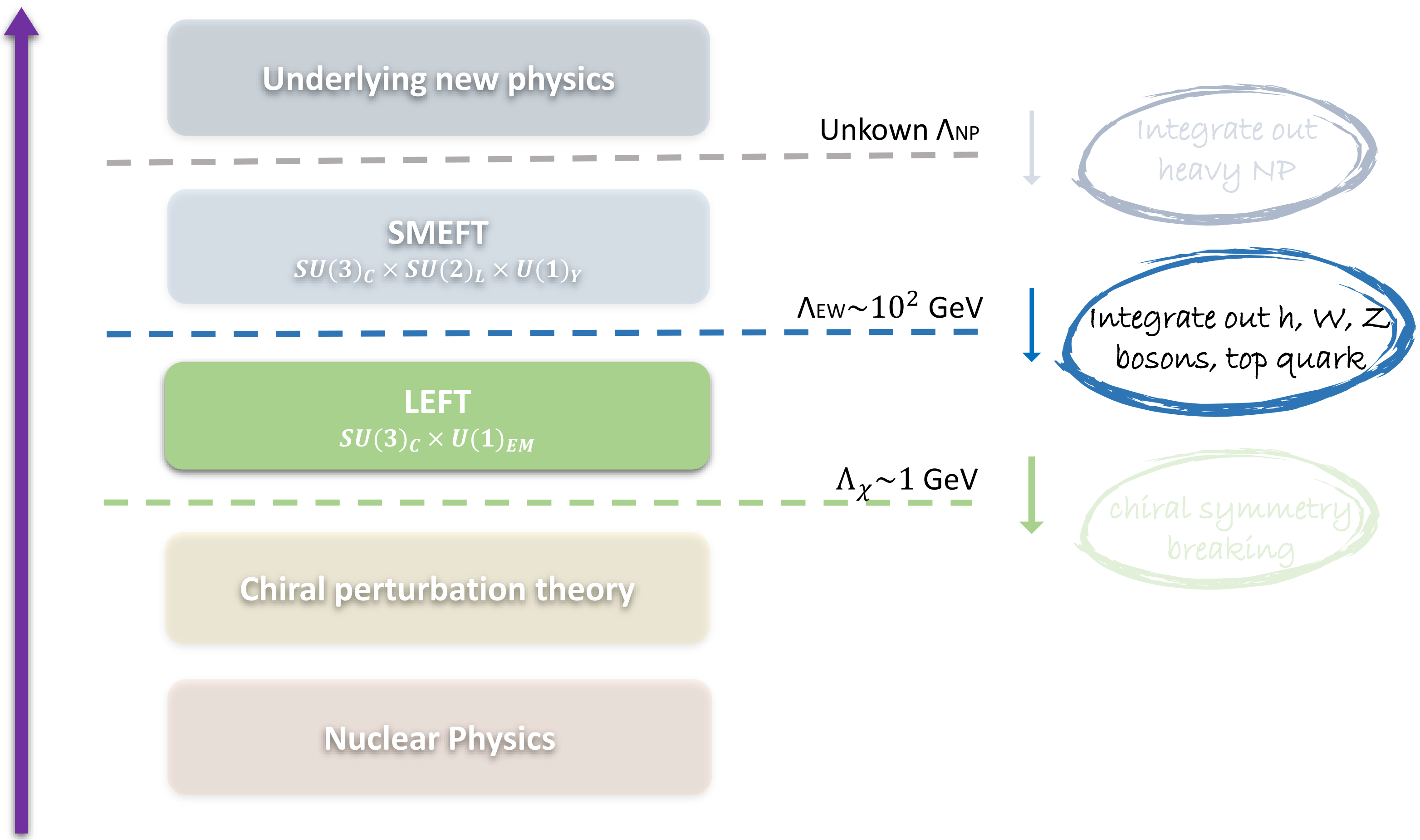
(a) Mass mechanism

(b) Long-distance interaction

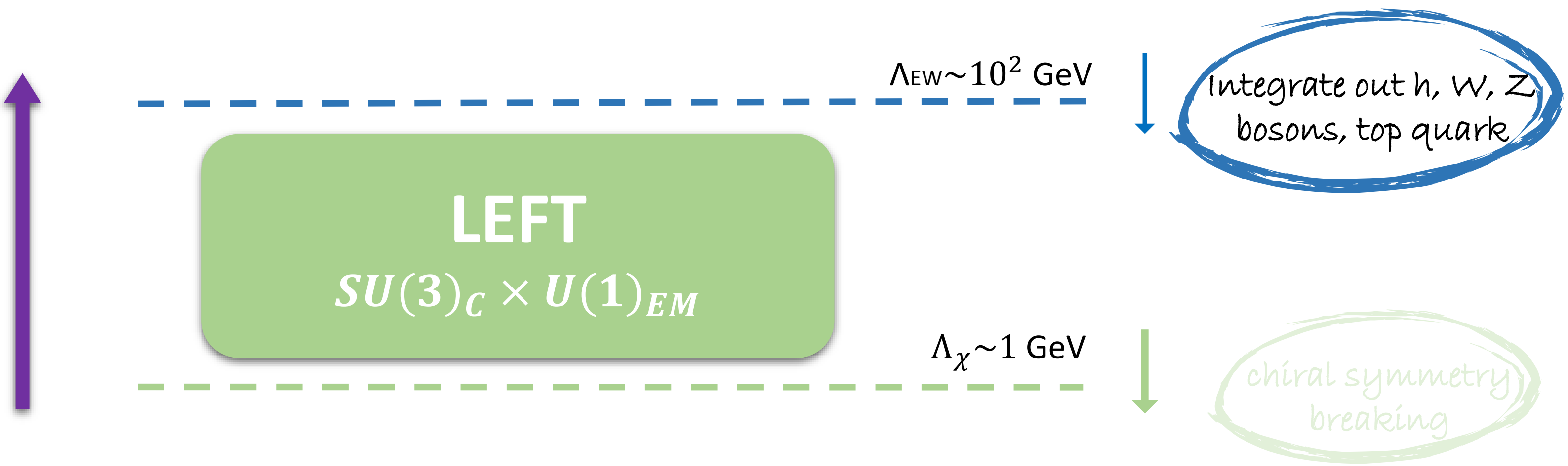
(c) Short-distance interaction

Start from LEFT

A general picture of EFT approach for LNV processes



Start from LEFT



- **LEFT** = all possible local, $SU(3)_C \times U(1)_{EM}$ invariant operators constructed from the relevant fields ordered by the inverse power of Λ_{EW}

Jenkins, Manohar, Stoffer, 2018 Li, Ren, Xiao, Yu, Zheng, 2020

$$\mathscr{L}_{\rm LEFT} = \mathscr{L}_{\dim\leq 4} + \sum_{\dim 5,i} \frac{\hat{C}_{5,i}}{\Lambda} \mathscr{Q}^i_{\dim-5} + \sum_{\dim 6,i} \frac{\hat{C}_{6,i}}{\Lambda^2} \mathscr{Q}^i_{\dim-6} + \sum_{\dim 7,i} \frac{\hat{C}_{7,i}}{\Lambda^3} \mathscr{Q}^i_{\dim-7} + \sum_{\dim 8,i} \frac{\hat{C}_{8,i}}{\Lambda^4} \mathscr{Q}^i_{\dim-8} + \sum_{\dim 9,i} \frac{\hat{C}_{9,i}}{\Lambda^5} \mathscr{Q}^i_{\dim-9} + \cdots$$

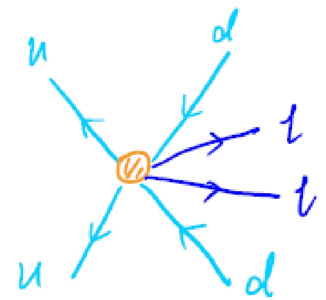
Liao, XDMA, Wang, 2020 Murphy, 2020 Liao, XDMA, Wang, 2019

Operators in LEFT

Relevant dim-9 operator basis

Operator	Specific form	Operator	Specific form
$\mathcal{O}^{LLLL,\,S/P}_{prst}$	$(\overline{u^p_L}\gamma^\mu d^r_L)[\overline{u^s_L}\gamma_\mu d^t_L](j^{\alpha\beta}/j_5^{\alpha\beta})$	$\mathcal{O}^{RRRR,\,S/P}_{prst}$	$(\overline{u^p_R}\gamma^\mu d^r_R)[\overline{u^s_R}\gamma_\mu d^t_R](j^{\alpha\beta}/j_5^{\alpha\beta})$
$\mathcal{O}^{LLLL,\,T}_{prst}$	$(\overline{u^p_L}\gamma^\mu d^r_L)[\overline{u^s_L}\gamma^\nu d^t_L](j^{\alpha\beta}_{\mu\nu})$	$\mathcal{O}^{RRRR,\,T}_{prst}$	$(\overline{u^p_R}\gamma^\mu d^r_R)[\overline{u^s_R}\gamma^\nu d^t_R](j^{\alpha\beta}_{\mu\nu})$
$\tilde{\mathcal{O}}^{LLLL,\,T}_{prst}$	$(\overline{u^p_L}\gamma^\mu d^r_L][\overline{u^s_L}\gamma^\nu d^t_L)(j^{\alpha\beta}_{\mu\nu})$	$\tilde{\mathcal{O}}^{RRRR,\,T}_{prst}$	$(\overline{u^p_R}\gamma^\mu d^r_R][\overline{u^s_R}\gamma^\nu d^t_R)(j^{\alpha\beta}_{\mu\nu})$
$\mathcal{O}^{LRLR,\,S/P}_{prst}$	$(\overline{u^p_L} d^r_R)[\overline{u^s_L} d^t_R](j^{\alpha\beta}/j_5^{\alpha\beta})$	$\mathcal{O}^{RLRL,\,S/P}_{prst}$	$(\overline{u^p_R} d^r_L)[\overline{u^s_R} d^t_L](j^{\alpha\beta}/j_5^{\alpha\beta})$
$\tilde{\mathcal{O}}^{LRLR,\,S/P}_{prst}$	$(\overline{u^p_L} d^r_R][\overline{u^s_L} d^t_R)(j^{\alpha\beta}/j_5^{\alpha\beta})$	$\tilde{\mathcal{O}}^{RLRL,\,S/P}_{prst}$	$(\overline{u^p_R} d^r_L][\overline{u^s_R} d^t_L)(j^{\alpha\beta}/j_5^{\alpha\beta})$
$\mathcal{O}^{LRLR,\,T}_{prst}$	$(\overline{u^p_L} i\sigma^{\mu\nu} d^r_R)[\overline{u^s_L} d^t_R](j^{\alpha\beta}_{\mu\nu})$	$\mathcal{O}^{RLRL,\,T}_{prst}$	$(\overline{u^p_R} i\sigma^{\mu\nu} d^r_L)[\overline{u^s_R} d^t_L](j^{\alpha\beta}_{\mu\nu})$
$\tilde{\mathcal{O}}^{LRLR,\,T}_{prst}$	$(\overline{u^p_L}\sigma^{\mu\rho} d^r_R)[\overline{u^s_L}\sigma^{\nu}{}_{\rho} d^t_R](j^{\alpha\beta}_{\mu\nu})$	$\tilde{\mathcal{O}}^{RLRL,\,T}_{prst}$	$(\overline{u^p_R}\sigma^{\mu\rho} d^r_L)[\overline{u^s_R}\sigma^{\nu}{}_{\rho} d^t_L](j^{\alpha\beta}_{\mu\nu})$
$\mathcal{O}^{LRLL,\,V/A}_{prst}$	$(\overline{u^p_L} d^r_R)[\overline{u^s_L}\gamma^\mu d^t_L](j^{\alpha\beta}_\mu/j^{\alpha\beta}_{5,\,\mu})$	$\mathcal{O}^{RLRR,\,V/A}_{prst}$	$(\overline{u^p_R} d^r_L)[\overline{u^s_R}\gamma^\mu d^t_R](j^{\alpha\beta}_\mu/j^{\alpha\beta}_{5,\,\mu})$
$\tilde{\mathcal{O}}^{LRLL,\,V/A}_{prst}$	$(\overline{u^p_L} d^r_R][\overline{u^s_L}\gamma^\mu d^t_L)(j^{\alpha\beta}_\mu/j^{\alpha\beta}_{5,\,\mu})$	$\tilde{\mathcal{O}}^{RLRR,\,V/A}_{prst}$	$(\overline{u^p_R} d^r_L][\overline{u^s_R}\gamma^\mu d^t_R)(j^{\alpha\beta}_\mu/j^{\alpha\beta}_{5,\,\mu})$
$\mathcal{O}^{LRRR,\,V/A}_{prst}$	$(\overline{u^p_L} d^r_R)[\overline{u^s_R}\gamma^\mu d^t_R](j^{\alpha\beta}_\mu/j^{\alpha\beta}_{5,\,\mu})$	$\mathcal{O}^{RLLL,\,V/A}_{prst}$	$(\overline{u^p_R} d^r_L)[\overline{u^s_L}\gamma^\mu d^t_L](j^{\alpha\beta}_\mu/j^{\alpha\beta}_{5,\,\mu})$
$\tilde{\mathcal{O}}^{LRRR,\,V/A}_{prst}$	$(\overline{u^p_L} d^r_R][\overline{u^s_R}\gamma^\mu d^t_R)(j_\mu/j^{\alpha\beta}_{5,\,\mu})$	$\tilde{\mathcal{O}}^{RLLL,\,V/A}_{prst}$	$(\overline{u^p_R} d^r_L][\overline{u^s_L}\gamma^\mu d^t_L)(j_\mu/j^{\alpha\beta}_{5,\,\mu})$
$\mathcal{O}^{LRRL,\,T}_{prst}$	$(\overline{u^p_L} i\sigma^{\mu\nu} d^r_R)[\overline{u^s_R} d^t_L](j^{\alpha\beta}_{\mu\nu})$	$\mathcal{O}^{RLLR,\,T}_{prst}$	$(\overline{u^p_R} i\sigma^{\mu\nu} d^r_L)[\overline{u^s_L} d^t_R](j^{\alpha\beta}_{\mu\nu})$
$\tilde{\mathcal{O}}^{LRRL,\,T}_{prst}$	$(\overline{u^p_L} i\sigma^{\mu\nu} d^r_R][\overline{u^s_R} d^t_L)(j^{\alpha\beta}_{\mu\nu})$	$\tilde{\mathcal{O}}^{RLLR,\,T}_{prst}$	$(\overline{u^p_R} i\sigma^{\mu\nu} d^r_L][\overline{u^s_L} d^t_R)(j^{\alpha\beta}_{\mu\nu})$
$\mathcal{O}^{LRRL,\,S/P}_{prst}$	$(\overline{u^p_L} d^r_R)[\overline{u^s_R} d^t_L](j^{\alpha\beta}/j_5^{\alpha\beta})$	$\tilde{\mathcal{O}}^{LRRL,\,S/P}_{prst}$	$(\overline{u^p_L} d^r_R][\overline{u^s_R} d^t_L)(j^{\alpha\beta}/j_5^{\alpha\beta})$

- $j^{\alpha\beta} = (\overline{l_\alpha} l^C_\beta)$, $j_5^{\alpha\beta} = (\overline{l_\alpha}\gamma_5 l^C_\beta)$, $j^{\alpha\beta}_{5,\,\mu} = (\overline{l_\alpha}\gamma_\mu\gamma_5 l^C_\beta)$ (symmetric)
- $j^{\alpha\beta}_\mu = (\overline{l_\alpha}\gamma_\mu l^C_\beta)$, $j^{\alpha\beta}_{\mu\nu} = (\overline{l_\alpha}\sigma_{\mu\nu} l^C_\beta)$p (anti-symmetric)



Operators in LEFT

Relevant dim-6 and -7 operator basis

- The leading order contributions to LD are from dim-6 operators in LEFT [Manohar et al 17, 18]

$$\mathcal{O}^{RL,S}_{pr\alpha\beta} = (\overline{u^p_R} d^r_L)(\overline{l_{L\alpha}} \nu^C_\beta), \qquad \mathcal{O}^{LR,S}_{pr\alpha\beta} = (\overline{u^p_L} d^r_R)(\overline{l_{L\alpha}} \nu^C_\beta),$$

$$\mathcal{O}^{LL,V}_{pr\alpha\beta} = (\overline{u^p_L} \gamma_\mu d^r_L)(\overline{l_{R\alpha}} \gamma^\mu \nu^C_\beta), \qquad \mathcal{O}^{RR,V}_{pr\alpha\beta} = (\overline{u^p_R} \gamma_\mu d^r_R)(\overline{l_{R\alpha}} \gamma^\mu \nu^C_\beta),$$

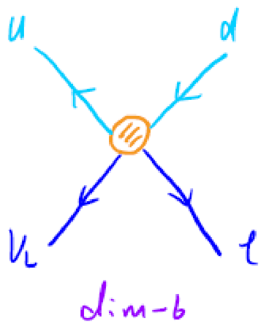
$$\mathcal{O}^{LR,T}_{pr\alpha\beta} = (\overline{u^p_L} \sigma_{\mu\nu} d^r_R)(\overline{l_{L\alpha}} \sigma^{\mu\nu} \nu^C_\beta).$$

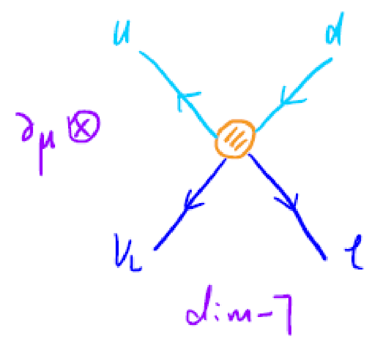
- Including also the contributions of dim-7 operators

$$\mathcal{O}^{LL,VD}_{pr\alpha\beta} = (\overline{u^p_L} \gamma_\mu d^r_L)(\overline{l_{L\alpha}} i \overleftrightarrow{D}^\mu \nu^C_\beta), \qquad \mathcal{O}^{RR,VD}_{pr\alpha\beta} = (\overline{u^p_R} \gamma_\mu d^r_R)(\overline{l_{L\alpha}} i \overleftrightarrow{D}^\mu \nu^C_\beta),$$

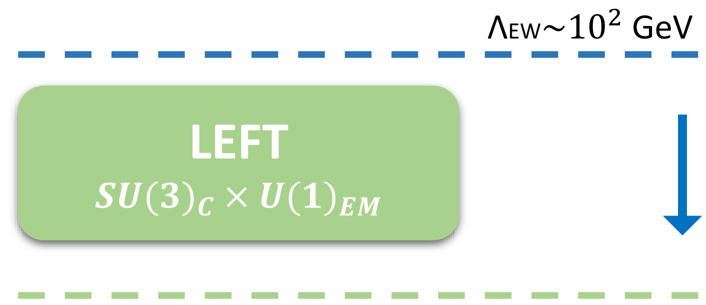
$$\mathcal{O}^{LR,TD}_{pr\alpha\beta} = (\overline{u^p_L} \sigma_{\mu\nu} d^r_R)(\overline{l_{R\alpha}} \gamma^{[\mu} \overleftrightarrow{D}^{\nu]} \nu^C_\beta), \qquad \mathcal{O}^{RL,TD}_{pr\alpha\beta} = (\overline{u^p_R} \sigma_{\mu\nu} d^r_L)(\overline{l_{R\alpha}} \gamma^{[\mu} \overleftrightarrow{D}^{\nu]} \nu^C_\beta),$$

where $\bar{A}\overleftrightarrow{D}^\mu B = \bar{A}(D^\mu B) - \bar{A}(\overleftarrow{D^\mu} B)$ and $\gamma^{[\mu} D^{\nu]} = \gamma^\mu D^\nu - \gamma^\nu D^\mu$





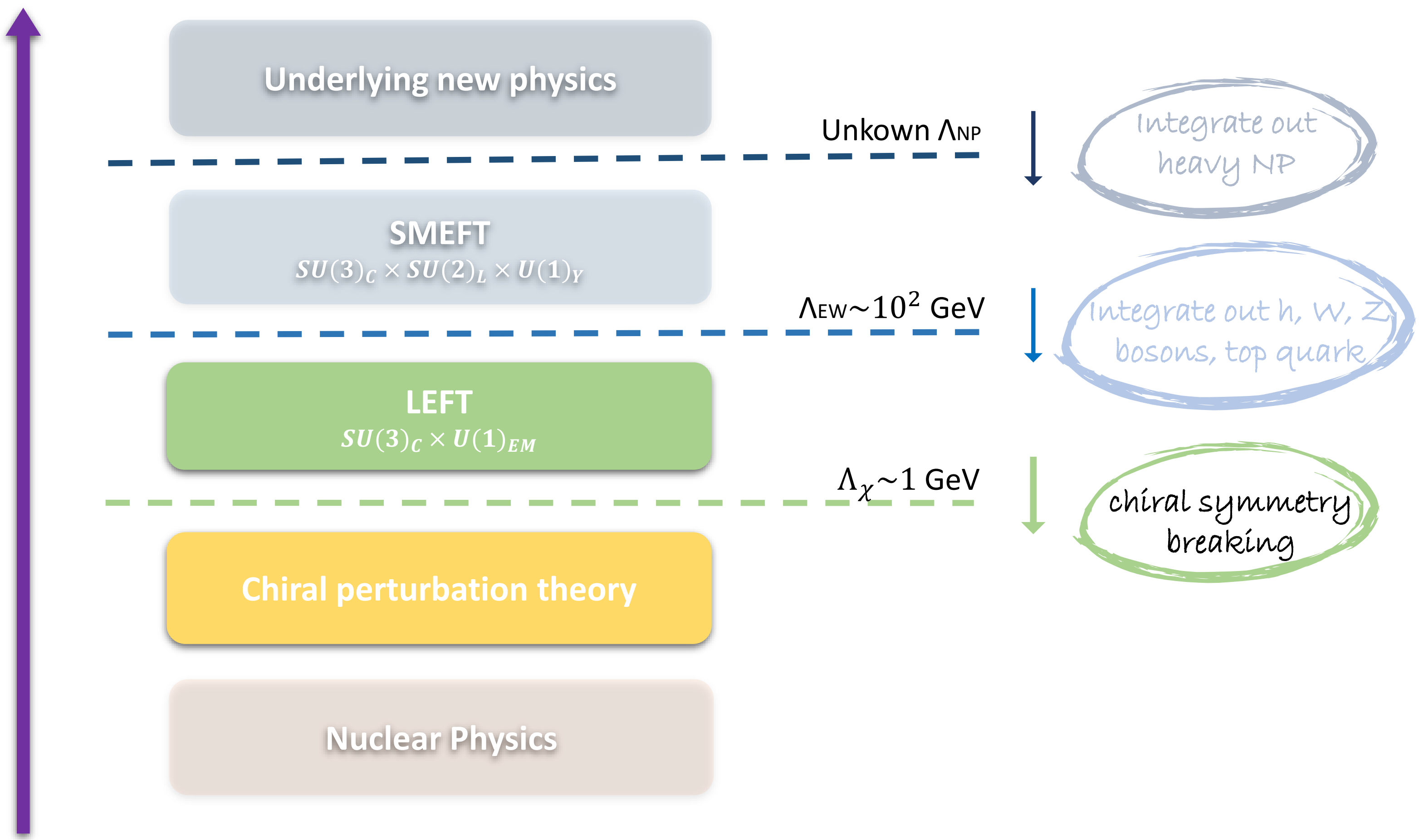
1-loop QCD RGEs in LEFT



- Motivation: tame the large logs from perturbative expansion
- The field strength renormalization & the operation renormalization (operator mixing effect)
- The dominant contributions are from the 1-loop QCD renormalization
- The renormalization group equations: $16\pi^2 \mu \frac{dC_d}{d\mu} = \hat{\gamma} C_d$, $\hat{\gamma}$ as the anomalous dimension matrix
- We calculated the RG equations of Wilson coefficients with dim-reg and $\overline{MS}$ renormalization scheme under the R_ξ gauge
- ξ independent as a check for the calculation

Matching to χPT from LEFT

A general picture of EFT approach for LNV processes



Chiral perturbation theory

Pseudo-Nambu-Goldstone (PNG) boson

$$\xi = \sqrt{\Sigma} = \exp\left[i\pi^a\lambda^a/2F_0\right]$$

- Chiral symmetry: $G = SU(3)_L \times SU(3)_R$
  - ✓ Spontaneously breaking (vaccum):

    quark condensates $\langle 0|\bar{q}q|0\rangle$ induces $SU(3)_L \times SU(3)_R \longrightarrow SU(3)_V$
  - ✓ Explicitly breaking (Lagrangian): quark masses → Spurion analysis

These symmetry structures must be captured by χPT!

$$q_{L,a} \to L_a^p q_{L,p}, \quad \overline{q_R}^b \to \overline{q_R}^p \left(R^\dagger\right)_p^b, \quad q_{R,a} \to R_a^p q_{R,p}, \quad \overline{q_L}^b \to \overline{q_L}^p \left(L^\dagger\right)_p^b$$

$$\xi \to L\xi U^\dagger = U\xi R^\dagger$$

- Mesonic χPT = all possible, G invariant operators constructed by $\Sigma(\xi)$, derivative ∂_μ and spurion field $\mathcal{M}$, and ordered by number of derivatives $\mathcal{O}(p^n)$, e.g., at the lowest order,

$$\mathcal{L}_{p^2} = \frac{F_0^2}{4}\mathrm{Tr}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger) + \frac{F_0^2}{4}(2B_0)\mathrm{Tr}(M^\dagger\Sigma^\dagger + \Sigma M)$$

Matching to χPT from LEFT

LEFT

$SU(3)_C \times U(1)_{EM}$

In terms of quarks, gluons, leptons

$\Lambda_\chi \sim 1$ GeV

chiral symmetry breaking

Chiral perturbation theory

In terms of mesons, leptons

- Non-perturbative matching: guiding by chiral symmetry
- The operators come with unknown constants (LECs)
- Chiral logarithms to estimate the NLO corrections

Matching to χPT from LEFT

$$\xi = \sqrt{\Sigma} = \exp\left[i\pi^a\lambda^a/2F_0\right]$$

The external sources method (for dim-6 and -7 operators in LEFT)

- The QCD Lagrangian with external sources

$$\mathcal{L} = \mathcal{L}_{\rm QCD} + \overline{q_L} l_\mu \gamma^\mu q_L + \overline{q_R} r_\mu \gamma^\mu q_R + \left(\overline{q_R}(s+ip)q_L + \overline{q_L}(t_l^{\mu\nu}\sigma_{\mu\nu})q_R + \text{h.c.}\right),$$

where $l_\mu, r_\mu, s, p, t_l^{\mu\nu}, t_r^{\mu\nu} = t_l^{\mu\nu\dagger}$ are external sources, and $\chi = 2B(s-ip)$

- Identify the external sources to $K^- \to \pi^+$ transition,

$$(l^\mu)_{ui} = -2\sqrt{2}G_F V_{ui}(\overline{\ell_\alpha}\gamma^\mu P_L \nu_\alpha) + C^{LL,V}_{ui\alpha\beta}(\overline{\ell_\alpha}\gamma^\mu P_R \nu^{\rm C}_\beta) + C^{LL,VD}_{ui\alpha\beta}(\overline{\ell_\alpha} i \overleftrightarrow{D}^\mu P_R \nu^{\rm C}_\beta),$$
$$(l^\mu)_{ui} = C^{RR,V}_{ui\alpha\beta}(\overline{\ell_\alpha}\gamma^\mu P_R \nu^{\rm C}_\beta) + C^{RR,VD}_{ui\alpha\beta}(\overline{\ell_\alpha} i \overleftrightarrow{D}^\mu P_R \nu^{\rm C}_\beta),$$
$$(\chi^\dagger)_{ui} = 2BC^{RL,S}_{ui\alpha\beta}(\overline{\ell_\alpha} P_R \nu^{\rm C}_\beta), \quad (\chi)_{ui} = 2BC^{LR,S}_{ui\alpha\beta}(\overline{\ell_\alpha} P_R \nu^{\rm C}_\beta),$$
$$(t_l^{\mu\nu})_{ui} = C^{LR,T}_{ui\alpha\beta}(\overline{\ell_\alpha}\sigma^{\mu\nu} P_R \nu^{\rm C}_\beta) + C^{LR,TD}_{ui\alpha\beta}(\overline{\ell_\alpha}\gamma^{[\mu}\overleftrightarrow{D}^{\nu]} P_R \nu^{\rm C}_\beta), \quad (t_r^{\mu\nu})_{ui} = C^{RL,TD}_{ui\alpha\beta}(\overline{\ell_\alpha}\gamma^{[\mu}\overleftrightarrow{D}^{\nu]} P_R \nu^{\rm C}_\beta),$$

- To linear term of external sources and LO in χPT, i.e., $\mathcal{O}(p^2)$ chiral Lagrangian

$$\mathscr{L}^{(2)}_{\chi\rm PT} = \frac{F_0^2}{4}\mathrm{Tr}\left(D_\mu\Sigma(D^\mu\Sigma)^\dagger\right) + \frac{F_0^2}{4}\mathrm{Tr}\left(\chi\Sigma^\dagger + \Sigma\chi^\dagger\right),$$

where $D_\mu\Sigma = \partial_\mu\Sigma - il_\mu\Sigma + i\Sigma r_\mu$.

Matching to χPT from LEFT

$$\xi = \sqrt{\Sigma} = \exp\left[i\pi^a\lambda^a/2F_0\right]$$

The method of spurion analysis (for dim-9 operators in LEFT)

- Take the quark level operator (irrep. under G) as

$$\mathcal{O} = T_{cd}^{ab}(\overline{q_{X_1}^c}\Gamma_1 q_{Y_1,a})(\overline{q_{X_2}^d}\Gamma_2 q_{Y_2,b}),$$

  Require $\mathcal{O}$ to be invariant under G $\Longrightarrow$ treat T_{cd}^{ab} as a spurion field with a proper transformation law under G

- Construct the corresponding hadronic operators by T_{cd}^{ab} together with the NGB matrix ξ,…, and require the resulting operators to be invariant under G

- For each independent operator, accompany an unknow LEC

The above procedures can be finished by the following simple replacement

- The LO matching

$$q_{L,a} \to \xi_a{}^{\alpha},\ \overline{q_L}^a \to \xi^{\dagger}_{\alpha}{}^a,\ q_{R,a} \to \xi^{\dagger}_a{}^{\alpha},\ \overline{q_R}^a \to \xi_{\alpha}{}^a,$$

- NLO or NNLO matching

$$q_{L,a} \to ((D_\mu\xi^\dagger)^\dagger)_a{}^\alpha,\ \overline{q_L}^a \to (D_\mu\xi^\dagger)_\alpha{}^a,\ q_{R,a} \to (D_\mu\xi)_a^{\dagger\alpha},\ \overline{q_R}^a \to (D_\mu\xi)_\alpha{}^a,$$
$$q_{L,a} \to (M^\dagger\xi^\dagger)_a{}^\alpha,\ \overline{q_L}^a \to (\xi M)_\alpha{}^a,\ q_{R,a} \to (M\xi)_a{}^\alpha,\ \overline{q_R}^a \to (\xi^\dagger M^\dagger)_\alpha{}^a,$$

Matching to χPT from LEFT

$$\xi = \sqrt{\Sigma} = \exp\left[i\pi^a\lambda^a/2F_0\right]$$

The method of spurion analysis (for dim-9 operators in LEFT)

Notation	Quark operator	chiral irrep	Hadronic operator
$\mathcal{O}_{udus}^{LLLL,S/P}$	$(\overline{u_L}\gamma^\mu d_L)[\overline{u_L}\gamma_\mu s_L](j/j_5)$	$27_L \times 1_R$	$\frac{5}{12}g_{27\times1}F_0^4(\Sigma i\partial_\mu\Sigma^\dagger)_2^1(\Sigma i\partial^\mu\Sigma^\dagger)_3^1$
$\mathcal{O}_{udus}^{LRLR,S/P}$	$(\overline{u_L}d_R)[\overline{u_L}s_R](j/j_5)$	$\bar{6}_L \times 6_R$	$-g_{\bar{6}\times6}^a\frac{F_0^4}{4}(\Sigma^\dagger)_2^1(\Sigma^\dagger)_3^1$
$\tilde{\mathcal{O}}_{udus}^{LRLR,S/P}$	$(\overline{u_L}d_R][\overline{u_L}s_R)(j/j_5)$	$\bar{6}_L \times 6_R$	$-g_{\bar{6}\times6}^b\frac{F_0^4}{4}(\Sigma^\dagger)_2^1(\Sigma^\dagger)_3^1$
$\mathcal{O}_{udus}^{LRLL,A}$	$(\overline{u_L}d_R)[\overline{u_L}\gamma^\mu s_L]j_{\mu5}$	$\overline{15}_L \times 3_R$	$-g_{\overline{15}\times3}^a\frac{F_0^4}{4}(\Sigma i\partial_\mu\Sigma^\dagger)_3^1(\Sigma^\dagger)_2^1$
$\tilde{\mathcal{O}}_{udus}^{LRLL,A}$	$(\overline{u_L}d_R][\overline{u_L}\gamma^\mu s_L)j_{\mu5}$	$\overline{15}_L \times 3_R$	$-g_{\overline{15}\times3}^b\frac{F_0^4}{4}(\Sigma i\partial_\mu\Sigma^\dagger)_3^1(\Sigma^\dagger)_2^1$
$\mathcal{O}_{usud}^{LRLL,A}$	$(\overline{u_L}s_R)[\overline{u_L}\gamma^\mu d_L]j_{\mu5}$	$\overline{15}_L \times 3_R$	$-g_{\overline{15}\times3}^c\frac{F_0^4}{4}(\Sigma i\partial_\mu\Sigma^\dagger)_2^1(\Sigma^\dagger)_3^1$
$\tilde{\mathcal{O}}_{usud}^{LRLL,A}$	$(\overline{u_L}s_R][\overline{u_L}\gamma^\mu d_L)j_{\mu5}$	$\overline{15}_L \times 3_R$	$-g_{\overline{15}\times3}^d\frac{F_0^4}{4}(\Sigma i\partial_\mu\Sigma^\dagger)_2^1(\Sigma^\dagger)_3^1$
$\mathcal{O}_{udus}^{LRRL,S/P}$	$(\overline{u_L}d_R)[\overline{u_R}s_L](j/j_5)$	$8_L \times 8_R$	$g_{8\times8}^a\frac{F_0^4}{4}(\Sigma^\dagger)_2^1(\Sigma)_3^1$
$\tilde{\mathcal{O}}_{udus}^{LRRL,S/P}$	$(\overline{u_L}d_R][\overline{u_R}s_L)(j/j_5)$	$8_L \times 8_R$	$g_{8\times8}^b\frac{F_0^4}{4}(\Sigma^\dagger)_2^1(\Sigma)_3^1$
...	...	...	...

- g_is which can be determined from matrix elements of $\pi^- \to \pi^+$, $K^+ \to \pi^+\pi^0$ and $K^0 \leftrightarrow \overline{K}^0$ by chiral symmetry and LQCD [1805.02634, 1806.02780]:

$$g_{27\times1} = 0.38 \pm 0.08,\ g_{8\times8}^a = 5.5 \pm 2\ \mathrm{GeV}^2,\ g_{8\times8}^b = 1.55 \pm 0.65\ \mathrm{GeV}^2.$$

Matching to SMEFT from LEFT

SMEFT

$SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\mathcal{L} \supset C_i^{(5)} \mathcal{O}_i^{(5)} + C_i^{(7)} \mathcal{O}_i^{(7)}$$

$\Lambda_{EW} \sim 10^2$ GeV

LEFT

$SU(3)_C \times U(1)_{EM}$

$$\mathcal{L} \supset C_i'^{(3)} \mathcal{O}_i'^{(3)} + C_i'^{(6)} \mathcal{O}_i'^{(6)} + C_i'^{(7)} \mathcal{O}_i'^{(7)} + C_i'^{(9)} \mathcal{O}_i'^{(9)}$$

- Assuming that there are no new particles with a mass of order Λ_{EW} or below
- It will simplifies the structures of LEFT

Matching to SMEFT from LEFT

- The dim-5 Weinberg operator and the relevant basis of dim-7 operators in SMEFT

$$\mathcal{O}_5 = \varepsilon_{ij}\varepsilon_{mn}(\overline{L^{C,i}}L^m)H^jH^n,$$

[Phys. Rev. Lett. 43 (1979) 1566] [JHEP 11 (2016) 043]

ψ^2H^4	$\mathcal{O}_{LH} = \varepsilon_{ij}\varepsilon_{mn}(\overline{L^{C,i}}L^m)H^jH^n(H^\dagger H)$	ψ^4H	$\mathcal{O}_{\bar{e}LLLH} = \varepsilon_{ij}\varepsilon_{mn}(\bar{e}L^i)(\overline{L^{C,j}}L^m)H^n$
ψ^2H^3D	$\mathcal{O}_{LeHD} = \varepsilon_{ij}\varepsilon_{mn}(\overline{L^{C,i}}\gamma_\mu e)H^j(H^m iD^\mu H^n)$		$\mathcal{O}_{\bar{d}QLLH1} = \varepsilon_{ij}\varepsilon_{mn}(\bar{d}Q^i)(\overline{L^{C,j}}L^m)H^n$
ψ^2H^2X	$\mathcal{O}_{LHB} = g_1\varepsilon_{ij}\varepsilon_{mn}(\overline{L^{C,i}}\sigma_{\mu\nu}L^m)H^jH^nB^{\mu\nu}$		$\mathcal{O}_{\bar{d}QLLH2} = \varepsilon_{ij}\varepsilon_{mn}(\bar{d}\sigma_{\mu\nu}Q^i)(\overline{L^{C,j}}\sigma^{\mu\nu}L^m)H^n$
	$\mathcal{O}_{LHW} = g_2\varepsilon_{ij}(\varepsilon\tau^I)_{mn}(\overline{L^{C,i}}\sigma_{\mu\nu}L^m)H^jH^nW^{I\mu\nu}$		$\mathcal{O}_{\bar{d}uLeH} = \varepsilon_{ij}(\bar{d}\gamma_\mu u)(\overline{L^{C,i}}\gamma^\mu e)H^j$
$\psi^2H^2D^2$	$\mathcal{O}_{LDH1} = \varepsilon_{ij}\varepsilon_{mn}(\overline{L^{C,i}}\overleftrightarrow{D}_\mu L^j)(H^mD^\mu H^n)$		$\mathcal{O}_{\bar{Q}uLLH} = \varepsilon_{ij}(\bar{Q}u)(\overline{L^C}L^i)H^j$
	$\mathcal{O}_{LDH2} = \varepsilon_{im}\varepsilon_{jn}(\overline{L^{C,i}}L^j)(D_\mu H^mD^\mu H^n)$	ψ^4D	$\mathcal{O}_{\bar{d}uLDL} = \varepsilon_{ij}(\bar{d}\gamma_\mu u)(\overline{L^{C,i}}i\overleftrightarrow{D}^\mu L^j)$

- Mathcing at the EW scale

Dim	Operators	Matching at the electroweak scale $\Lambda_{\rm EW}$
MM: dim-3	$\mathscr{L}_{\rm M} = -\frac{1}{2}m_{\alpha\beta}\overline{\nu^C_\alpha}\nu_\beta$	$m_{\alpha\beta} = -v^2C^{\alpha\beta*}_{LH5} - \frac{1}{2}v^4C^{\alpha\beta*}_{LH}$
LD: dim 6	$\mathcal{O}^{RL,S}_{pr\alpha\beta} = (\overline{u^p_R}d^r_L)(\overline{l_{L\alpha}}\nu^C_\beta)$	$C^{RL,S}_{pr\alpha\beta} = \frac{v}{\sqrt{2}}V_{wr}C^{wp\alpha\beta*}_{\bar{Q}uLLH}$
	$\mathcal{O}^{LR,S}_{pr\alpha\beta} = (\overline{u^p_L}d^r_R)(\overline{l_{L\alpha}}\nu^C_\beta)$	$C^{LR,S}_{pr\alpha\beta} = \frac{v}{\sqrt{2}}C^{rp\alpha\beta*}_{\bar{d}QLLH1}$
	$\mathcal{O}^{LL,V}_{pr\alpha\beta} = (\overline{u^p_L}\gamma_\mu d^r_L)(\overline{l_{R\alpha}}\gamma^\mu\nu^C_\beta)$	$C^{LL,V}_{pr\alpha\beta} = \frac{v}{\sqrt{2}}V_{pr}C^{\beta\alpha*}_{LeHD}$
	$\mathcal{O}^{RR,V}_{pr\alpha\beta} = (\overline{u^p_R}\gamma_\mu d^r_R)(\overline{l_{R\alpha}}\gamma^\mu\nu^C_\beta)$	$C^{RR,V}_{pr\alpha\beta} = \frac{v}{\sqrt{2}}C^{rp\beta\alpha*}_{\bar{d}uLeH}$
	$\mathcal{O}^{LR,T}_{pr\alpha\beta} = (\overline{u^p_L}\sigma_{\mu\nu}d^r_R)(\overline{l_{L\alpha}}\sigma^{\mu\nu}\nu^C_\beta)$	$C^{LR,T}_{pr\alpha\beta} = \frac{v}{\sqrt{2}}C^{rp\alpha\beta*}_{\bar{d}QLLH2}$
LD: dim 7	$\mathcal{O}^{LL,VD}_{pr\alpha\beta} = (\overline{u^p_L}\gamma_\mu d^r_L)(\overline{l_{L\alpha}}i\overleftrightarrow{D}^\mu\nu^C_\beta)$	$C^{LL,VD}_{pr\alpha\beta} = -V_{pr}\left(4C^{\beta\alpha*}_{LHW} + 2C^{\alpha\beta*}_{LDH1}\right)$
	$\mathcal{O}^{RR,VD}_{pr\alpha\beta} = (\overline{u^p_R}\gamma_\mu d^r_R)(\overline{l_{L\alpha}}i\overleftrightarrow{D}^\mu\nu^C_\beta)$	$C^{RR,VD}_{pr\alpha\beta} = 2C^{rp\alpha\beta*}_{\bar{d}uLDL}$
SD: dim 9	$\mathcal{O}^{LLLL,S/P}_{prst,\alpha\beta} = (\overline{u^p_L}\gamma^\mu d^r_L)[\overline{u^s_L}\gamma_\mu d^t_L]j^{\alpha\beta}_{(5)}$	$C^{LLLL,S/P}_{prst,\alpha\beta} = -2\sqrt{2}G_FV_{pr}V_{st}\left(C^{\{\alpha\beta\}*}_{LHW} + C^{\alpha\beta*}_{LDH1} + \frac{1}{2}C^{\alpha\beta*}_{LDH2}\right)$
	$\mathcal{O}^{LRRL,S/P}_{prst,\alpha\beta} = (\overline{u^p_L}d^r_R)[\overline{u^s_R}d^t_L]j^{\alpha\beta}_{(5)}$	$C^{LRRL,S/P}_{prst,\alpha\beta} = 0$
	$\tilde{\mathcal{O}}^{LRRL,S/P}_{prst,\alpha\beta} = (\overline{u^p_L}d^r_R][\overline{u^s_R}d^t_L)j^{\alpha\beta}_{(5)}$	$\tilde{C}^{LRRL\ S/P}_{prst,\alpha\beta} = -4\sqrt{2}G_FV_{pt}C^{rs\alpha\beta*}_{\bar{d}uLDL}$

Master formula for the branching ratio in SMEFT

$$\frac{\mathcal{B}(e^-e^-)}{\mathrm{GeV}^6} = \frac{1.7\times10^{-33}}{\mathrm{GeV}^6}\frac{|m_{ee}|^2}{\mathrm{eV}^2} + 80\,|\mathcal{Y}^{ee}_{K1}|^2 + 4.3\,|\mathcal{Y}^{ee}_{\pi 1}|^2$$
$$+10^{-3}\times\left(48\,|\mathcal{X}^{ee}_1|^2 + 45\,|\mathcal{Y}^{ee}_{K2}|^2 + 2.4\,|\mathcal{Y}^{ee}_{\pi 2}|^2\right)$$
$$+10^{-8}\times\left(29\,|\mathcal{Y}^{ee}_{K3}|^2 + 23\,|\mathcal{X}^{ee}_2|^2 + 1.6\,|\mathcal{Y}^{ee}_{\pi 3}|^2\right) + \mathrm{int.},$$

$$\frac{\mathcal{B}(\mu^-\mu^-)}{\mathrm{GeV}^6} = \frac{4.5\times10^{-34}}{\mathrm{GeV}^6}\frac{|m_{\mu\mu}|^2}{\mathrm{eV}^2} + 16\,\left|\mathcal{Y}^{\mu\mu}_{K1}\right|^2 + 2.2\,|\mathcal{Y}^{\mu\mu}_{\pi 1}|^2$$
$$+10^{-3}\times\left(17\,|\mathcal{X}^{\mu\mu}_1|^2 + 19\,\left|\mathcal{Y}^{\mu\mu}_{K2}\right|^2 + |\mathcal{Y}^{\mu\mu}_{\pi 2}|^2\right)$$
$$+10^{-9}\times\left(67\,|\mathcal{X}^{\mu\mu}_2|^2 + 49\,\left|\mathcal{Y}^{\mu\mu}_{K3}\right|^2 + 6.6\,|\mathcal{Y}^{\mu\mu}_{\pi 3}|^2\right) + \mathrm{int.},$$

$$\frac{\mathcal{B}(e^-\mu^-)}{\mathrm{GeV}^6} = \frac{2.1\times10^{-33}}{\mathrm{GeV}^6}\frac{|m_{e\mu}|^2}{\mathrm{eV}^2} + 26\,\left|\mathcal{Y}^{\mu e}_{K1}\right|^2 + 17\,\left|\mathcal{Y}^{e\mu}_{K1}\right|^2 + 2\,|\mathcal{Y}^{e\mu}_{\pi 1}|^2 + 1.4\,|\mathcal{Y}^{\mu e}_{\pi 1}|^2$$
$$+10^{-3}\times\left(61\,|\mathcal{X}^{e\mu}_1|^2 + 35\,\left|\mathcal{Y}^{\mu e}_{K2}\right|^2 + 24\,\left|\mathcal{Y}^{e\mu}_{K2}\right|^2 + 1.9\,|\mathcal{Y}^{e\mu}_{\pi 2}|^2 + 1.3\,|\mathcal{Y}^{\mu e}_{\pi 2}|^2\right)$$
$$+10^{-9}\times\left(280\,|\mathcal{X}^{e\mu}_2|^2 + 110\,\left|\mathcal{Y}^{e\mu}_{K3}\right|^2 + 55\,\left|\mathcal{Y}^{\mu e}_{K3}\right|^2 + 6.7\,|\mathcal{Y}^{\mu e}_{\pi 3}|^2 + 5.7\,|\mathcal{Y}^{e\mu}_{\pi 3}|^2\right) + \mathrm{int.},$$

$m_\nu \leq \mathcal{O}(eV)$

- The dominant contribution is from the LD (neutrino exchange) !

Lower bounds (GeV) on NP scale from exp. data

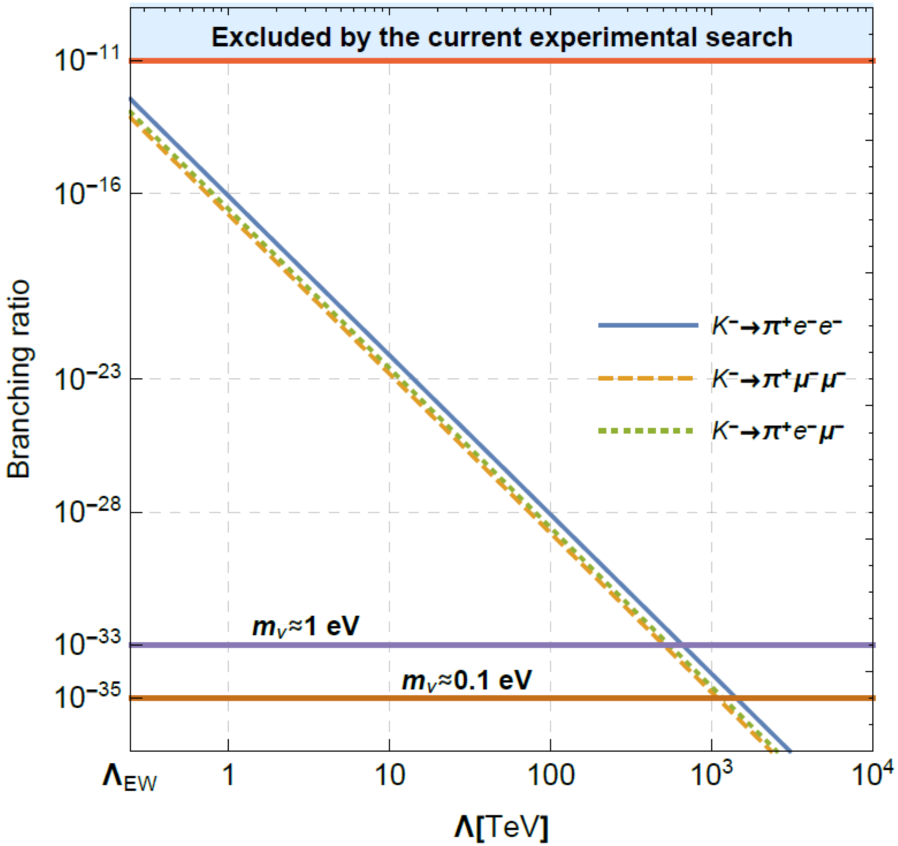
$K^- \to \pi^+ e^- e^-$		$K^- \to \pi^+ \mu^- \mu^-$		$K^- \to \pi^+ e^- \mu^-$			
names	bounds	names	bounds	names	bounds	names	bounds
$\lvert \mathscr{Y}_{K1}^{ee} \rvert^{-\frac{1}{3}}$	84.5	$\lvert \mathscr{Y}_{K1}^{\mu\mu} \rvert^{-\frac{1}{3}}$	85.1	$\lvert \mathscr{Y}_{K1}^{\mu e} \rvert^{-\frac{1}{3}}$	61.1	$\lvert \mathscr{Y}_{K1}^{e\mu} \rvert^{-\frac{1}{3}}$	56.9
$\lvert \mathscr{Y}_{\pi1}^{ee} \rvert^{-\frac{1}{3}}$	51.9	$\lvert \mathscr{Y}_{\pi1}^{\mu\mu} \rvert^{-\frac{1}{3}}$	61.2	$\lvert \mathscr{Y}_{\pi1}^{e\mu} \rvert^{-\frac{1}{3}}$	39.8	$\lvert \mathscr{Y}_{\pi1}^{\mu e} \rvert^{-\frac{1}{3}}$	37.5
$\lvert \mathscr{X}_{1}^{ee} \rvert^{-\frac{1}{3}}$	24.5	$\lvert \mathscr{X}_{1}^{\mu\mu} \rvert^{-\frac{1}{3}}$	32.3	$\lvert \mathscr{X}_{1}^{e\mu} \rvert^{-\frac{1}{3}}$	22.3		
$\lvert \mathscr{Y}_{K2}^{ee} \rvert^{-\frac{1}{3}}$	24.3	$\lvert \mathscr{Y}_{K2}^{\mu\mu} \rvert^{-\frac{1}{3}}$	27.7	$\lvert \mathscr{Y}_{K2}^{\mu e} \rvert^{-\frac{1}{3}}$	20.3	$\lvert \mathscr{Y}_{K2}^{e\mu} \rvert^{-\frac{1}{3}}$	19.1
$\lvert \mathscr{Y}_{\pi2}^{ee} \rvert^{-\frac{1}{3}}$	14.9	$\lvert \mathscr{Y}_{\pi2}^{\mu\mu} \rvert^{-\frac{1}{3}}$	17	$\lvert \mathscr{Y}_{\pi2}^{e\mu} \rvert^{-\frac{1}{3}}$	12.5	$\lvert \mathscr{Y}_{\pi2}^{\mu e} \rvert^{-\frac{1}{3}}$	11.7
$\lvert \mathscr{X}_{2}^{ee} \rvert^{-\frac{1}{3}}$	3.2	$\lvert \mathscr{X}_{2}^{\mu\mu} \rvert^{-\frac{1}{3}}$	3.4	$\lvert \mathscr{X}_{2}^{e\mu} \rvert^{-\frac{1}{3}}$	2.9		
$\lvert \mathscr{Y}_{K3}^{ee} \rvert^{-\frac{1}{3}}$	3.3	$\lvert \mathscr{Y}_{K3}^{\mu\mu} \rvert^{-\frac{1}{3}}$	3.2	$\lvert \mathscr{Y}_{K3}^{e\mu} \rvert^{-\frac{1}{3}}$	2.6	$\lvert \mathscr{Y}_{K3}^{\mu e} \rvert^{-\frac{1}{3}}$	2.2
$\lvert \mathscr{Y}_{\pi3}^{ee} \rvert^{-\frac{1}{3}}$	2	$\lvert \mathscr{Y}_{\pi3}^{\mu\mu} \rvert^{-\frac{1}{3}}$	2.3	$\lvert \mathscr{Y}_{\pi3}^{\mu e} \rvert^{-\frac{1}{3}}$	1.5	$\lvert \mathscr{Y}_{\pi3}^{e\mu} \rvert^{-\frac{1}{3}}$	1.5

- Looser than those from nuclear $0\nu\beta\beta$, $\Lambda_{NP} > \mathcal{O}(1)\ TeV$
    - The much smaller data samples

BUT

- The second generation of leptons and quarks which are complementary to the study of $0\nu\beta\beta$

Predictions



- New bound on the branching ratio with $\Lambda_{NP} > 1\ TeV$:

$$\mathcal{B}(e^- e^-) < 8.0 \times 10^{-17},$$
$$\mathcal{B}(\mu^- \mu^-) < 1.6 \times 10^{-17},$$
$$\mathcal{B}(e^- \mu^-) < 2.6 \times 10^{-17},$$

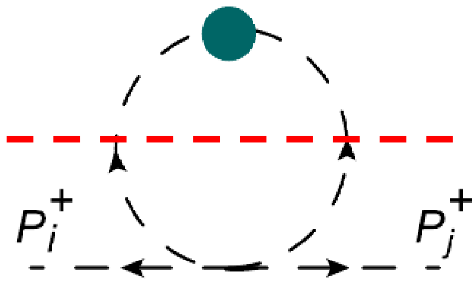
- Comparing with the current exps. :

$$\mathcal{B}_{\rm exp}(e^- e^-) < 2.2 \times 10^{-10},$$
$$\mathcal{B}_{\rm exp}(\mu^- \mu^-) < 4.2 \times 10^{-11},$$
$$\mathcal{B}_{\rm exp}(e^- \mu^-) < 5 \times 10^{-10},$$

- Neutrino mass contributions are severely suppressed
- Several orders of magnitude smaller than the current experimental upper bounds.

Other processes

- LNV τ decays, $\tau^- \to P_i^- P_j^- l^+$ **[2102.03491, Liao-Ma-Wang]**
  - Involve the third generation lepton
  - Dispersion relation



- LNV nuclear processes, e.g. $\mu^- X \to e^+(\mu^+) X'$ **[In preparation, Fan-Liao-Wang]**
  - Chiral effective theory involving two nucleons [See other experts' talk]
  - Similar with the $0\nu\beta\beta$ in series of EFTs [Cirigliano-Dekens-Mereghetti-de Vries-… series of papers]
- LNV B, D decays **[In preparation, Liao-Ma-Wang-Yu]**
  - QCD sum rules & Heavy quark effective theory & more?

Summary

- We studied the LNV process in the series of EFTs
- Matching and running are done between different EFTs

- These studies are complementary to $0\nu\beta\beta$
- We systematically include the potential LNV sources
- The uncertainties can be systematically estimated

Thanks for you attention!

Backup Slides

Operators in LEFT

Relevant dim-9 operator basis

- **Some remarks:**
  - There are flavor symmetries in quark sector

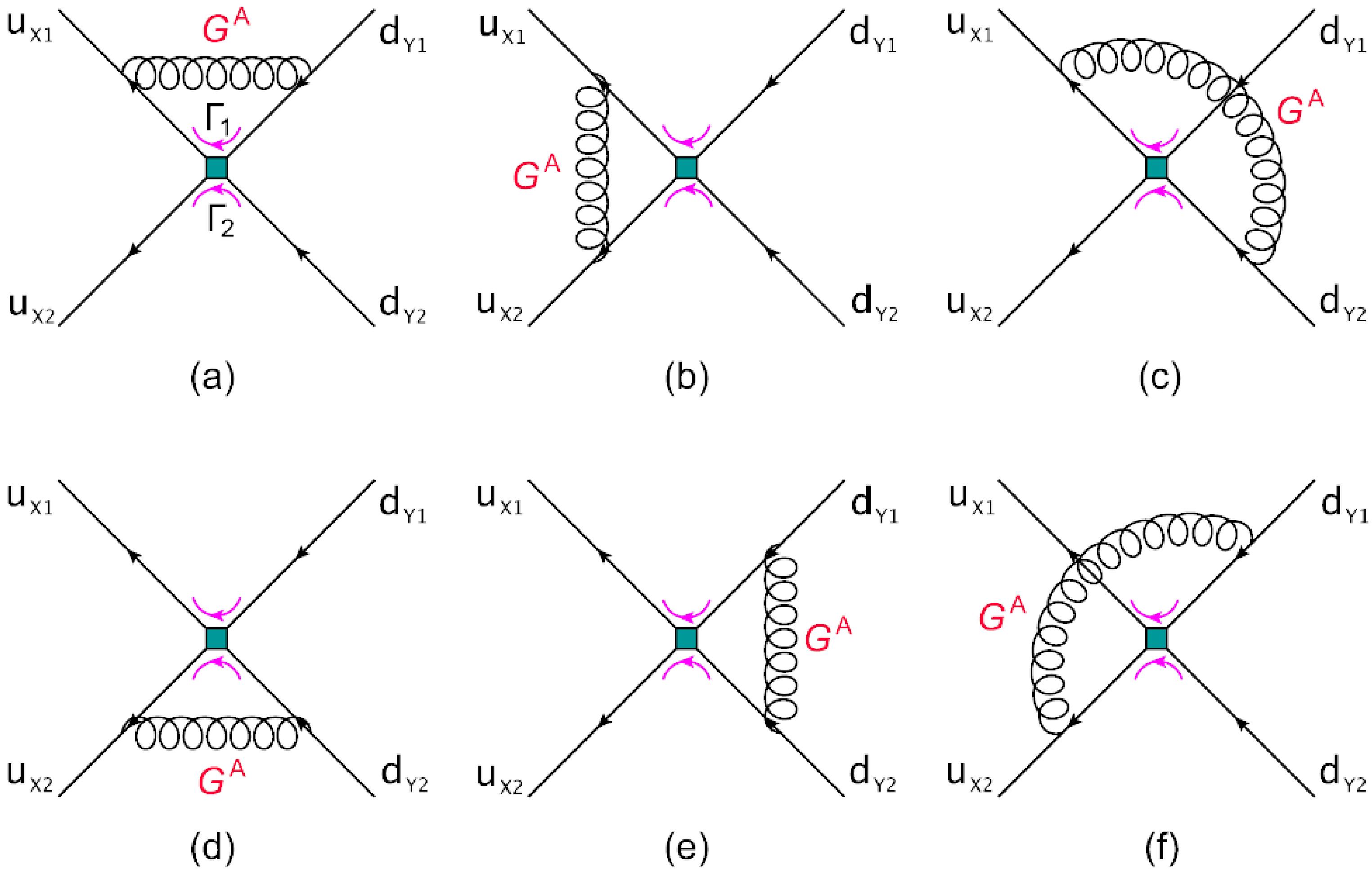
$$\mathcal{O}_{prst}^{LLLL,\,S/P} = \mathcal{O}_{stpr}^{LLLL,\,S/P},\ \mathcal{O}_{prst}^{LLLL,\,T} = -\mathcal{O}_{stpr}^{LLLL,\,T},\ \tilde{\mathcal{O}}_{prst}^{LLLL,\,T} = -\tilde{\mathcal{O}}_{stpr}^{LLLL,\,T} \ldots$$

  - Compared with the basis of N. Quintero [1606.03477], tensor lepton current and color exchanged operators are new
  - Our results (5886) are confirmed by the Hilbert series method [1512.03433]
  - When consider the sub-basis related with $0\nu\beta\beta$ process, we can get the relation between our basis and others[1511.03945, 1606.04549]

$$\begin{pmatrix} \mathcal{O}_1^{LL} \\ \mathcal{O}_2^{LL} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -4-\frac{8}{N} & -8 \end{pmatrix} \begin{pmatrix} \mathcal{O}_{2RL} \\ \mathcal{O}_{2RL}^{\lambda} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -4 & -8 \end{pmatrix} \begin{pmatrix} \mathcal{O}_{udud}^{RLRL,\,S/P} \\ \tilde{\mathcal{O}}_{udud}^{RLRL,\,S/P} \end{pmatrix}$$

$$\vdots$$

One-loop QCD running effects for dim-9 operators



The final renormalization group running equations

$$\mu \frac{d}{d\mu} \begin{pmatrix} C^{LLLL,S/P}_{prst} \\ C^{LLLL,S/P}_{ptsr} \end{pmatrix} = -\frac{\alpha_s}{2\pi} \begin{pmatrix} \frac{3}{N} & -3 \\ -3 & \frac{3}{N} \end{pmatrix} \begin{pmatrix} C^{LLLL,S/P}_{prst} \\ C^{LLLL,S/P}_{ptsr} \end{pmatrix} ,$$

$$\mu \frac{d}{d\mu} \begin{pmatrix} C^{LRLR,S/P}_{prst} \\ C^{LRLR,S/P}_{ptsr} \\ \tilde{C}^{LRLR,S/P}_{prst} \\ \tilde{C}^{LRLR,S/P}_{ptsr} \end{pmatrix} = -\frac{\alpha_s}{2\pi} \begin{pmatrix} \frac{2}{N} + 6C_F & -4 & 2 & \frac{2}{N} - 4C_F \\ -4 & \frac{2}{N} + 6C_F & \frac{2}{N} - 4C_F & 2 \\ -2 & \frac{4}{N} & -\frac{2}{N} - 2C_F & -2 \\ \frac{4}{N} & -2 & -2 & -\frac{2}{N} - 2C_F \end{pmatrix} \begin{pmatrix} C^{LRLR,S/P}_{prst} \\ C^{LRLR,S/P}_{ptsr} \\ \tilde{C}^{LRLR,S/P}_{prst} \\ \tilde{C}^{LRLR,S/P}_{ptsr} \end{pmatrix} ,$$

$$\mu \frac{d}{d\mu} \begin{pmatrix} C^{LRLL,A}_{prst} \\ C^{LRLL,A}_{srpt} \\ \tilde{C}^{LRLL,A}_{prst} \\ \tilde{C}^{LRLL,A}_{srpt} \end{pmatrix} = -\frac{\alpha_s}{2\pi} \begin{pmatrix} \frac{1}{N} + 3C_F & -2 & 1 & \frac{1}{N} - 2C_F \\ -2 & \frac{1}{N} + 3C_F & \frac{1}{N} - 2C_F & 1 \\ -1 & \frac{2}{N} & -\frac{1}{N} - C_F & -1 \\ \frac{2}{N} & -1 & -1 & -\frac{1}{N} - C_F \end{pmatrix} \begin{pmatrix} C^{LRLL,A}_{prst} \\ C^{LRLL,A}_{srpt} \\ \tilde{C}^{LRLL,A}_{prst} \\ \tilde{C}^{LRLL,A}_{srpt} \end{pmatrix} ,$$

$$\mu \frac{d}{d\mu} \begin{pmatrix} C^{LRRR,A}_{prst} \\ C^{LRRR,A}_{ptsr} \\ \tilde{C}^{LRRR,A}_{prst} \\ \tilde{C}^{LRRR,A}_{ptsr} \end{pmatrix} = -\frac{\alpha_s}{2\pi} \begin{pmatrix} \frac{1}{N} + 3C_F & -2 & 1 & \frac{1}{N} - 2C_F \\ -2 & \frac{1}{N} + 3C_F & \frac{1}{N} - 2C_F & 1 \\ -1 & \frac{2}{N} & -\frac{1}{N} - C_F & -1 \\ \frac{2}{N} & -1 & -1 & -\frac{1}{N} - C_F \end{pmatrix} \begin{pmatrix} C^{LRRR,A}_{prst} \\ C^{LRRR,A}_{ptsr} \\ \tilde{C}^{LRRR,A}_{prst} \\ \tilde{C}^{LRRR,A}_{prst} \end{pmatrix} ,$$

$$\mu \frac{d}{d\mu} \begin{pmatrix} C^{LRRL,S/P}_{prst} \\ \tilde{C}^{LRRL,S/P}_{prst} \end{pmatrix} = -\frac{\alpha_s}{2\pi} \begin{pmatrix} 6C_F & 3 \\ 0 & -\frac{3}{N} \end{pmatrix} \begin{pmatrix} C^{LRRL,S/P}_{prst} \\ \tilde{C}^{LRRL,S/P}_{ptsr} \end{pmatrix} , \cdots ,$$

The QCD running effect from Λ_{EW} to Λ_χ

- For dim-6 scalar and tensor operators,

$$C^S(\Lambda_\chi) = 1.656 C^S(\Lambda_{\mathrm{EW}}),\ C^S \in \{C^{RL,S}_{pr\alpha\beta}, C^{LR,S}_{pr\alpha\beta}\},$$
$$C^{LR,T}_{pr\alpha\beta}(\Lambda_\chi) = 0.845 C^{LR,T}_{pr\alpha\beta}(\Lambda_{\mathrm{EW}}).$$

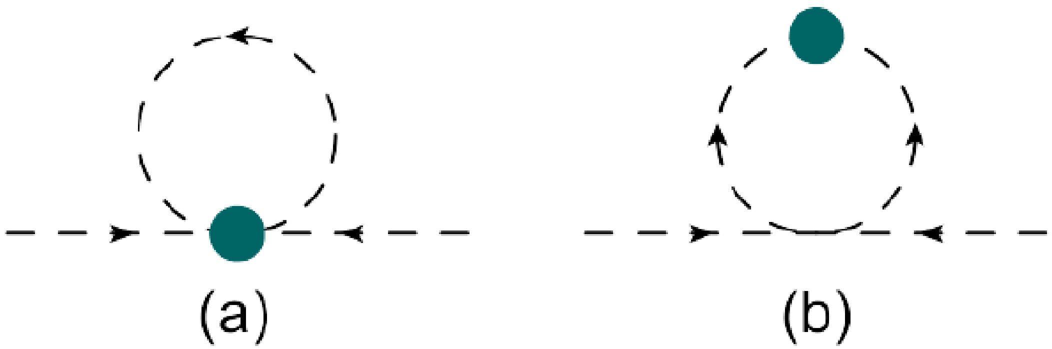
- The other dim-6 and -7 operators involve a quark vector current and thus do not run due to the QCD Ward identity.

- For dim-9 operators,

$$C^{LLLL,S/P}_{uiuj}(\Lambda_\chi) = 0.78 C^{LLLL,S/P}_{uiuj}(\Lambda_{\mathrm{EW}}),$$
$$\tilde{C}^{LRRL,S/P}_{uiuj}(\Lambda_\chi) = 0.88 \tilde{C}^{LRRL,S/P}_{uiuj}(\Lambda_{\mathrm{EW}}),$$
$$C^{LRRL,S/P}_{uiuj}(\Lambda_\chi) = 0.62 \tilde{C}^{LRRL,S/P}_{uiuj}(\Lambda_{\mathrm{EW}}).$$

Chiral logarithm

- In order to estimate the next leading order effects, we calculated the nonanalytic terms in 1-loop diagrams (a) and (b),



- We can get the 34%, 32.5% and 31.3% 1-loop corrections relative to tree level results on $\mathscr{M}_{27\times 1}$, $\mathscr{M}_{6\times\bar{6}}$ and $\mathscr{M}_{8\times 8}$, respectively.

Chiral Lagarnagian

- The resulting effective Lagrangian for LD contributions

$$\mathscr{L}^{(2)}_{\chi\mathrm{PT}} \supset F_0\Big[G_F\left(V_{ud}\partial_\mu\pi^- + V_{us}\partial_\mu K^-\right)\left(\overline{l_{L\alpha}}\gamma^\mu\nu_\alpha\right) + iB\left(c^{\alpha\beta}_{\pi 1}\pi^- + c^{\alpha\beta}_{K1}K^-\right)\left(\overline{l_{L\alpha}}\nu^C_\beta\right)$$
$$-\left(c^{\alpha\beta}_{\pi 2}\partial_\mu\pi^- + c^{\alpha\beta}_{K2}\partial_\mu K^-\right)\left(\overline{l_{R\alpha}}\gamma^\mu\nu^C_\beta\right) - \left(c^{\alpha\beta}_{\pi 3}\partial_\mu\pi^- + c^{\alpha\beta}_{K3}\partial_\mu K^-\right)\left(\overline{l_{L\alpha}}i\overleftrightarrow{D}^\mu\nu^C_\beta\right)\Big],$$

$$c^{\alpha\beta}_{P_i 1} = \frac{\sqrt{2}}{2}\left(C^{RL,S}_{ui\alpha\beta} - C^{LR,S}_{ui\alpha\beta}\right), \quad c^{\alpha\beta}_{P_i 2} = \frac{\sqrt{2}}{4}\left(C^{LL,V}_{ui\alpha\beta} - C^{RR,V}_{ui\alpha\beta}\right), \quad c^{\alpha\beta}_{P_i 3} = \frac{\sqrt{2}}{4}\left(C^{LL,VD}_{ui\alpha\beta} - C^{RR,VD}_{ui\alpha\beta}\right),$$

- The resulting effective Lagrangian for SD contributions

$$\mathscr{L}_{K^-\to\pi^+l^-l^-} = \frac{1}{2}K^-\pi^-\left[c_1\left(\bar{l}l^C\right) + c_2\left(\bar{l}\gamma_5 l^C\right)\right] + \frac{1}{2}\left[c_3\partial^\mu K^-\pi^- + c_4\partial^\mu\pi^- K^-\right]\left(\bar{l}\gamma_\mu\gamma_5 l^C\right)$$
$$+\frac{1}{2}\partial^\mu K^-\partial_\mu\pi^-\left[c_5\left(\bar{l}l^C\right) + c_6\left(\bar{l}\gamma_5 l^C\right)\right],$$

Where c_is are combinations of the Wilson coefficients and the low energy constants (LECs) g_is which can be determined by chiral symmetry and LQCD [1805.02634, 1806.02780]:

$$g_{27\times 1} = 0.38 \pm 0.08,\ g^a_{8\times 8} = 5.5 \pm 2\ \mathrm{GeV}^2,\ g^b_{8\times 8} = 1.55 \pm 0.65\ \mathrm{GeV}^2.$$

Lower bounds (GeV) on NP scale from LNV tau decays

$\tau^+ \to e^- \pi^+ \pi^+$		$\tau^+ \to e^- K^+ K^+$		$\tau^+ \to e^- K^+ \pi^+$			
names	bounds	names	bounds	names	bounds	names	bounds
$\lvert\mathscr{Y}^{e\tau}_{\pi 1}\rvert^{-\frac{1}{3}}$	15.8	$\lvert\mathscr{Y}^{\tau e}_{K1}\rvert^{-\frac{1}{3}}$	6.4	$\lvert\mathscr{Y}^{\tau e}_{K1}\rvert^{-\frac{1}{3}}$	10.9	$\lvert\mathscr{Y}^{e\tau}_{K1}\rvert^{-\frac{1}{3}}$	10.7
$\lvert\mathscr{Y}^{\tau e}_{\pi 1}\rvert^{-\frac{1}{3}}$	14.8	$\lvert\mathscr{Y}^{e\tau}_{K1}\rvert^{-\frac{1}{3}}$	6.0	$\lvert\mathscr{Y}^{e\tau}_{\pi 1}\rvert^{-\frac{1}{3}}$	8.8	$\lvert\mathscr{Y}^{\tau e}_{\pi 1}\rvert^{-\frac{1}{3}}$	6.7
$\lvert\mathscr{Y}^{e\tau}_{\pi 2}\rvert^{-\frac{1}{3}}$	6.8	$\lvert\mathscr{X}^{\tau e}_{1,KK}\rvert^{-\frac{1}{3}}$	3.8	$\lvert\mathscr{Y}^{\tau e}_{K2}\rvert^{-\frac{1}{3}}$	5.5	$\lvert\mathscr{Y}^{e\tau}_{K2}\rvert^{-\frac{1}{3}}$	5.0
$\lvert\mathscr{Y}^{\tau e}_{\pi 2}\rvert^{-\frac{1}{3}}$	6.8	$\lvert\mathscr{Y}^{\tau e}_{K2}\rvert^{-\frac{1}{3}}$	2.9	$\lvert\mathscr{X}^{\tau e}_{1,K\pi}\rvert^{-\frac{1}{3}}$	4.0		
$\lvert\mathscr{X}^{\tau e}_{1,\pi\pi}\rvert^{-\frac{1}{3}}$	5.5	$\lvert\mathscr{Y}^{e\tau}_{K2}\rvert^{-\frac{1}{3}}$	2.9	$\lvert\mathscr{Y}^{e\tau}_{\pi 2}\rvert^{-\frac{1}{3}}$	3.4	$\lvert\mathscr{Y}^{\tau e}_{\pi 2}\rvert^{-\frac{1}{3}}$	3.1
$\lvert\mathscr{X}^{\tau e}_{2}\rvert^{-\frac{1}{3}}$	1.8	$\lvert\mathscr{Y}^{\tau e}_{K3}\rvert^{-\frac{1}{3}}$	0.7	$\lvert\mathscr{Y}^{\tau e}_{K3}\rvert^{-\frac{1}{3}}$	1.3	$\lvert\mathscr{Y}^{e\tau}_{K3}\rvert^{-\frac{1}{3}}$	0.9
$\lvert\mathscr{Y}^{\tau e}_{\pi 3}\rvert^{-\frac{1}{3}}$	1.6	$\lvert\mathscr{Y}^{e\tau}_{K3}\rvert^{-\frac{1}{3}}$	0.5	$\lvert\mathscr{X}^{\tau e}_{2}\rvert^{-\frac{1}{3}}$	1.0		
$\lvert\mathscr{Y}^{e\tau}_{\pi 3}\rvert^{-\frac{1}{3}}$	1.0	$\lvert\mathscr{X}^{\tau e}_{2}\rvert^{-\frac{1}{3}}$	0.4	$\lvert\mathscr{Y}^{\tau e}_{\pi 3}\rvert^{-\frac{1}{3}}$	0.7	$\lvert\mathscr{Y}^{e\tau}_{\pi 3}\rvert^{-\frac{1}{3}}$	0.6

$\tau^+ \to \mu^- \pi^+ \pi^+$		$\tau^+ \to \mu^- K^+ K^+$		$\tau^+ \to \mu^- K^+ \pi^+$			
names	bounds	names	bounds	names	bounds	names	bounds
$\lvert\mathscr{Y}^{\mu\tau}_{\pi 1}\rvert^{-\frac{1}{3}}$	13.7	$\lvert\mathscr{Y}^{\tau\mu}_{K1}\rvert^{-\frac{1}{3}}$	6.0	$\lvert\mathscr{Y}^{\tau\mu}_{K1}\rvert^{-\frac{1}{3}}$	10.1	$\lvert\mathscr{Y}^{\mu\tau}_{K1}\rvert^{-\frac{1}{3}}$	9.8
$\lvert\mathscr{Y}^{\tau\mu}_{\pi 1}\rvert^{-\frac{1}{3}}$	13.0	$\lvert\mathscr{Y}^{\mu\tau}_{K1}\rvert^{-\frac{1}{3}}$	5.5	$\lvert\mathscr{Y}^{\mu\tau}_{\pi 1}\rvert^{-\frac{1}{3}}$	7.9	$\lvert\mathscr{Y}^{\tau\mu}_{\pi 1}\rvert^{-\frac{1}{3}}$	6.2
$\lvert\mathscr{Y}^{\mu\tau}_{\pi 2}\rvert^{-\frac{1}{3}}$	5.9	$\lvert\mathscr{X}^{\tau\mu}_{1,KK}\rvert^{-\frac{1}{3}}$	3.6	$\lvert\mathscr{Y}^{\tau\mu}_{K2}\rvert^{-\frac{1}{3}}$	5.1	$\lvert\mathscr{Y}^{\mu\tau}_{K2}\rvert^{-\frac{1}{3}}$	4.6
$\lvert\mathscr{Y}^{\tau\mu}_{\pi 2}\rvert^{-\frac{1}{3}}$	5.9	$\lvert\mathscr{Y}^{\tau\mu}_{K2}\rvert^{-\frac{1}{3}}$	2.7	$\lvert\mathscr{X}^{\tau\mu}_{1,K\pi}\rvert^{-\frac{1}{3}}$	3.7		
$\lvert\mathscr{X}^{\tau\mu}_{1,\pi\pi}\rvert^{-\frac{1}{3}}$	4.9	$\lvert\mathscr{Y}^{\mu\tau}_{K2}\rvert^{-\frac{1}{3}}$	2.7	$\lvert\mathscr{Y}^{\mu\tau}_{\pi 2}\rvert^{-\frac{1}{3}}$	3.1	$\lvert\mathscr{Y}^{\tau\mu}_{\pi 2}\rvert^{-\frac{1}{3}}$	2.8
$\lvert\mathscr{X}^{\tau\mu}_{2}\rvert^{-\frac{1}{3}}$	1.5	$\lvert\mathscr{Y}^{\tau\mu}_{K3}\rvert^{-\frac{1}{3}}$	0.7	$\lvert\mathscr{Y}^{\tau\mu}_{K3}\rvert^{-\frac{1}{3}}$	1.1	$\lvert\mathscr{Y}^{\mu\tau}_{K3}\rvert^{-\frac{1}{3}}$	0.8
$\lvert\mathscr{Y}^{\tau\mu}_{\pi 3}\rvert^{-\frac{1}{3}}$	1.4	$\lvert\mathscr{X}^{\tau\mu}_{2}\rvert^{-\frac{1}{3}}$	0.4	$\lvert\mathscr{X}^{\tau\mu}_{2}\rvert^{-\frac{1}{3}}$	0.9		
$\lvert\mathscr{Y}^{\mu\tau}_{\pi 3}\rvert^{-\frac{1}{3}}$	1.0	$\lvert\mathscr{Y}^{\mu\tau}_{K3}\rvert^{-\frac{1}{3}}$	0.4	$\lvert\mathscr{Y}^{\mu\tau}_{\pi 3}\rvert^{-\frac{1}{3}}$	0.6	$\lvert\mathscr{Y}^{\tau\mu}_{\pi 3}\rvert^{-\frac{1}{3}}$	0.5

- Looser than bounds from LNV kaon decays

Dispersion relation

- Chiral expansion: in derivatives of the NGBs and in powers of light quark masses
- The invariant mass of $P_i P_j$ system can be considerable to Λ_χ for tau decay

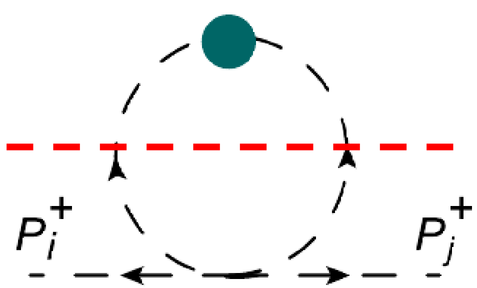
Dispersive framework

Based on analyticity and unitarity

Note: only deal with the local interactions

$$\mathcal{M}^{P_iP_j}_{27\times 1}(s) \quad = \quad \langle P_i^+(q_1)P_j^+(q_2)|(\overline{u_L}\gamma^\mu d_L^i)[\overline{u_L}\gamma_\mu d_L^j]|0\rangle = -(q_1\cdot q_2)F^{P_iP_j}_{27\times 1}(s)(1+\delta_{ij}),$$

$$\mathrm{Im}\,F^{\pi\pi}_{g_n}(s) \quad = \quad \frac{2\lambda^{1/2}_{\pi\pi}(s)}{s}F^{\pi\pi}_{g_n}(s)[f_0^2(s)]^*\theta(s-s_{\pi\pi}),$$



$$F^{P_iP_j}_{27\times 1}(s) = F_{27\times 1}(0)\Omega(s),$$

$$\Omega(s) = \exp\left[\frac{s}{\pi}\int_{s_{P_iP_j}}^{\infty} ds'\,\frac{\delta_F(s)}{s'(s'-s)}\right], \qquad F_{27\times 1}(0) = \frac{5}{6}F_0^2 g_{27\times 1},$$