Neutrinoless Double Beta Decay and Sterile Neutrinos

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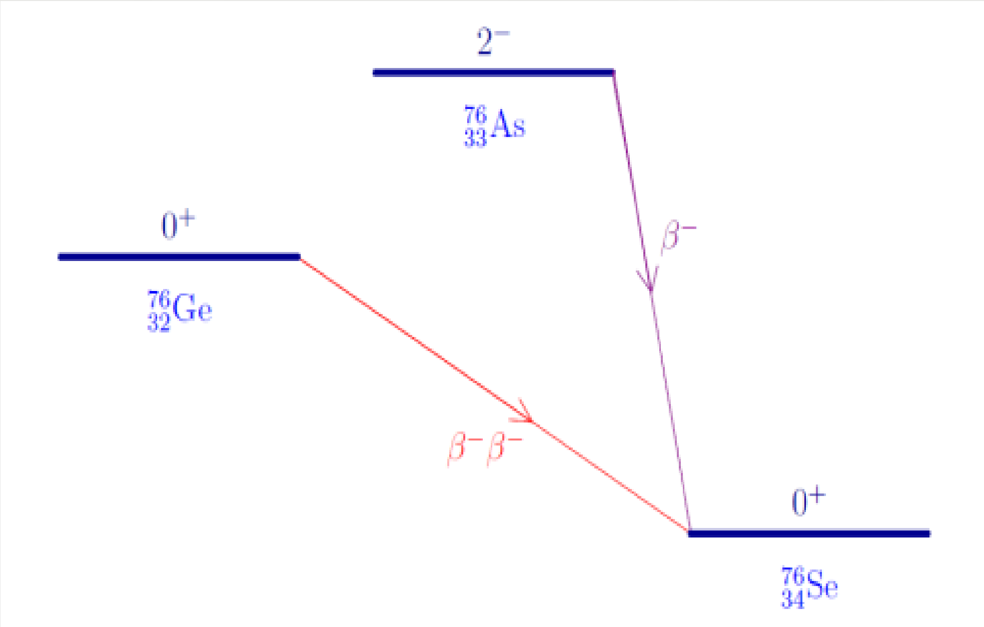
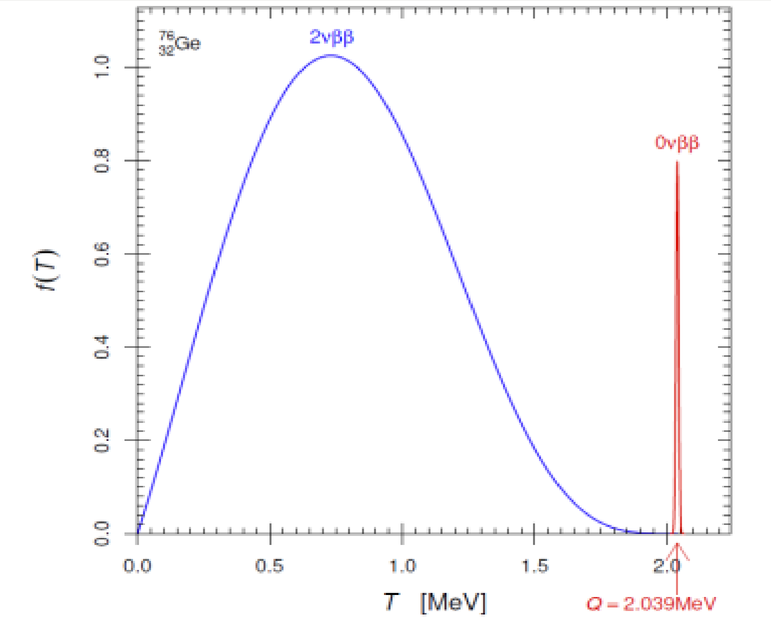
$0\nu 2\beta$-decay





$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

- **Standard Interpretation:**
  mediated by light massive Majorana neutrinos
- **Nonstandard Interpretations: sterile neutrinos and beyond**

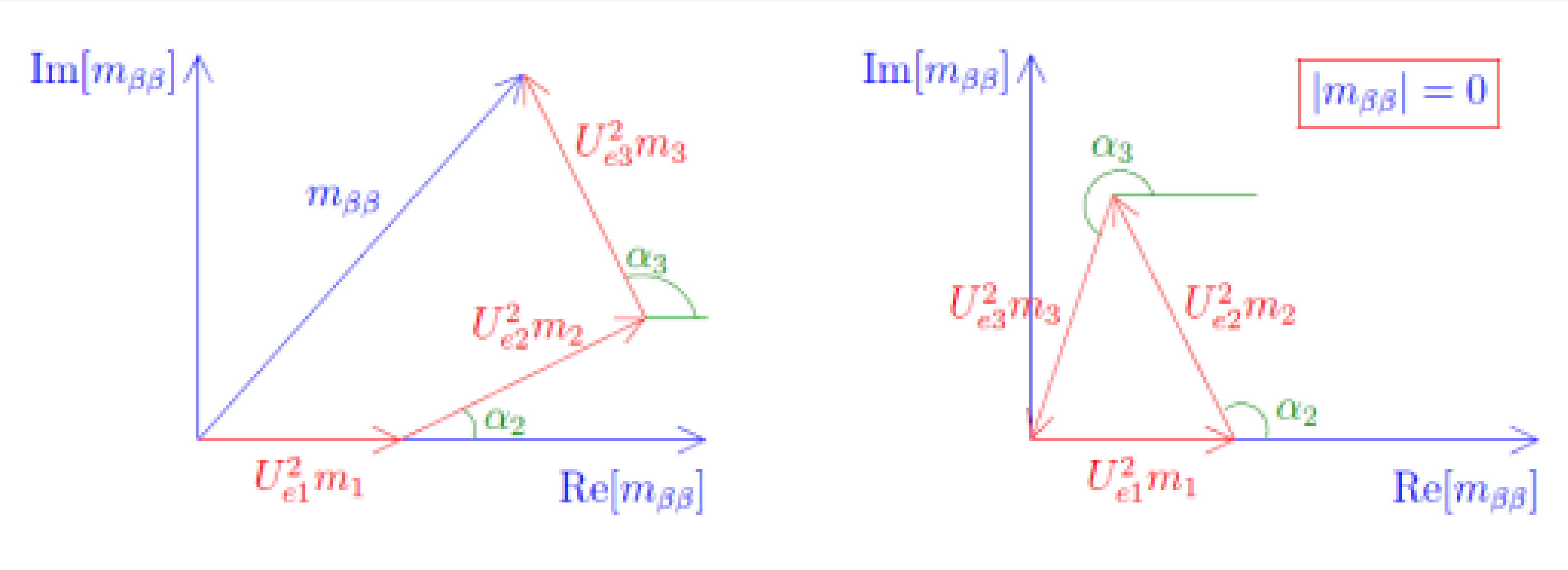
Recent reviews: 1902.04097, 1601.07512, 1411.4791, … …

Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

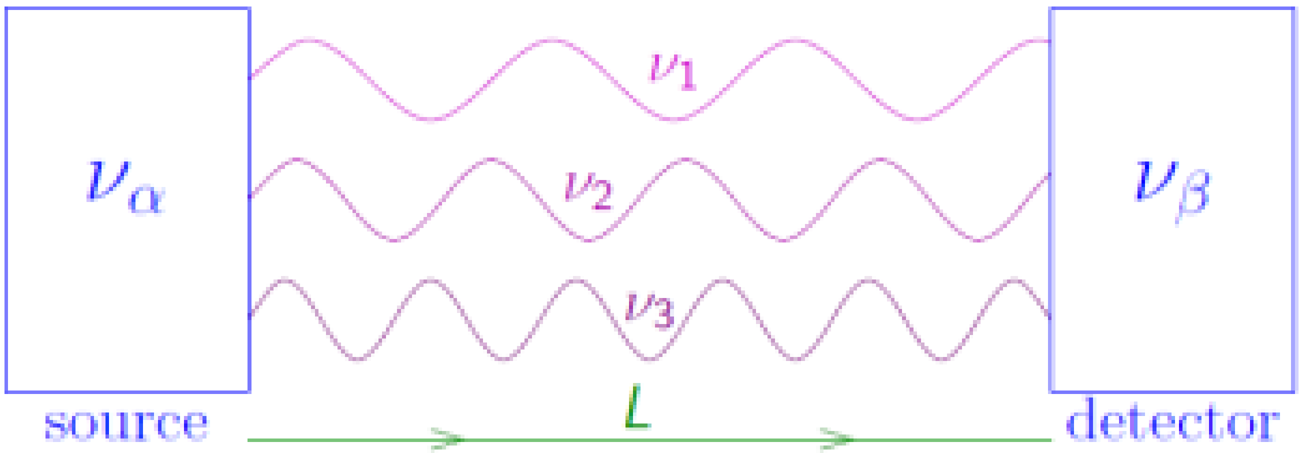
$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2(\lambda_3 - \delta_{13})$$



- **7 out of 9 parameters of light Majorana neutrinos !**
- **Neutrino oscillation and non-oscillation measurements contribute to the prediction of $m_{\beta\beta}$!**

Neutrino oscillations

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U_{\alpha 1}\,|\nu_1\rangle + U_{\alpha 2}\,|\nu_2\rangle + U_{\alpha 3}\,|\nu_3\rangle$$



$$|\nu(t>0)\rangle = U_{\alpha 1}\,e^{-iE_1 t}\,|\nu_1\rangle + U_{\alpha 2}\,e^{-iE_2 t}\,|\nu_2\rangle + U_{\alpha 3}\,e^{-iE_3 t}\,|\nu_3\rangle \neq |\nu_\alpha\rangle$$

$$E_k^2 = p^2 + m_k^2 \qquad\qquad t = L$$

$$P_{\nu_\alpha\to\nu_\beta}(L) = |\langle\nu_\beta|\nu(L)\rangle|^2 = \sum_{k,j} U_{\beta k}\,U_{\alpha k}^*\,U_{\beta j}^*\,U_{\alpha j}\,\exp\left(-i\frac{\Delta m_{kj}^2 L}{2E}\right)$$

The oscillation probabilities depend on U and $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$

Three-neutrino mixing framework

Standard Parameterization of Mixing Matrix (as CKM)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

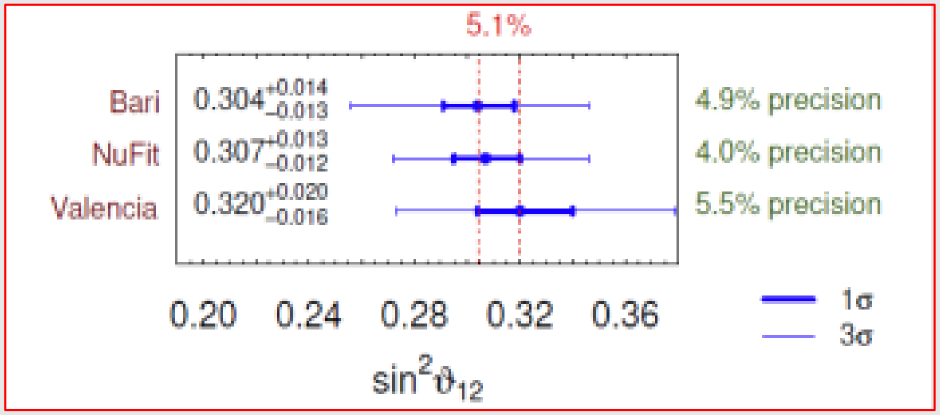
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

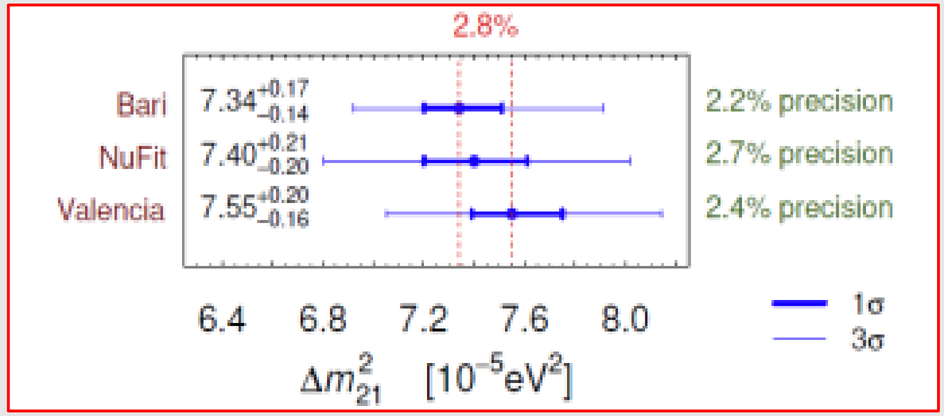
$c_{ab} \equiv \cos\vartheta_{ab}$ $s_{ab} \equiv \sin\vartheta_{ab}$ $0 \leq \vartheta_{ab} \leq \frac{\pi}{2}$ $0 \leq \delta_{13}, \lambda_{21}, \lambda_{31} < 2\pi$

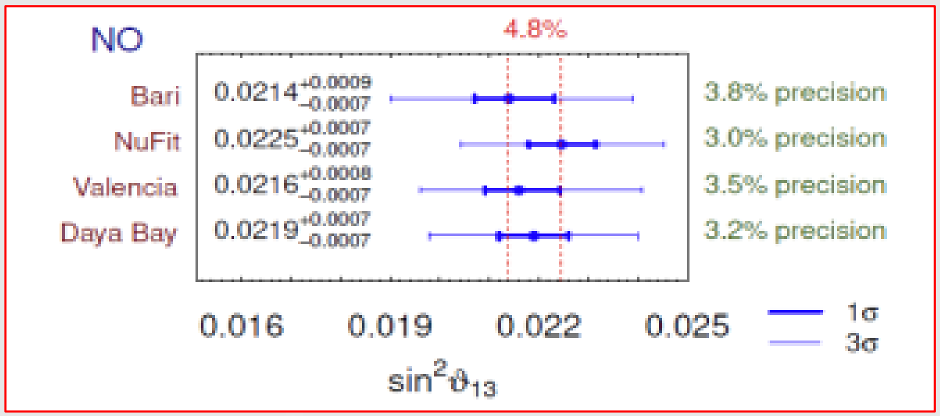
OSCILLATION PARAMETERS
- 3 Mixing Angles: ϑ_{12}, ϑ_{23}, ϑ_{13}
- 1 CPV Dirac Phase: δ_{13}
- 2 independent $\Delta m^2_{kj} \equiv m^2_k - m^2_j$: Δm^2_{21}, Δm^2_{31}

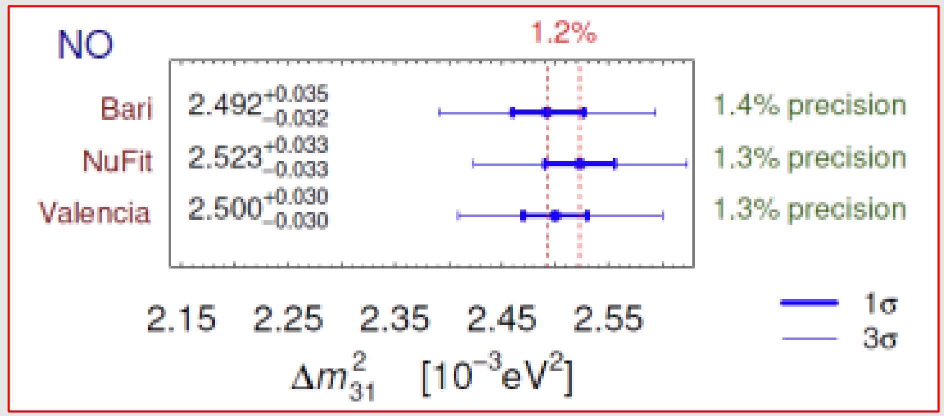
2 CPV Majorana Phases: λ_{21}, λ_{31} $\Longleftrightarrow$ $|\Delta L| = 2$ processes

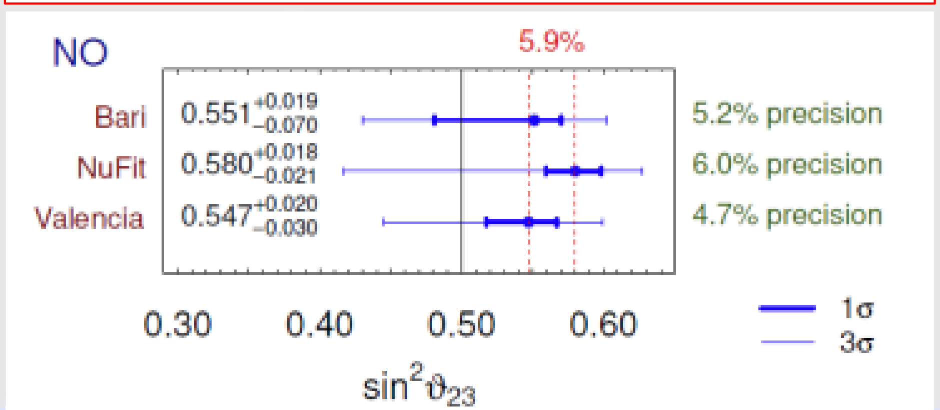
Latest oscillation parameters







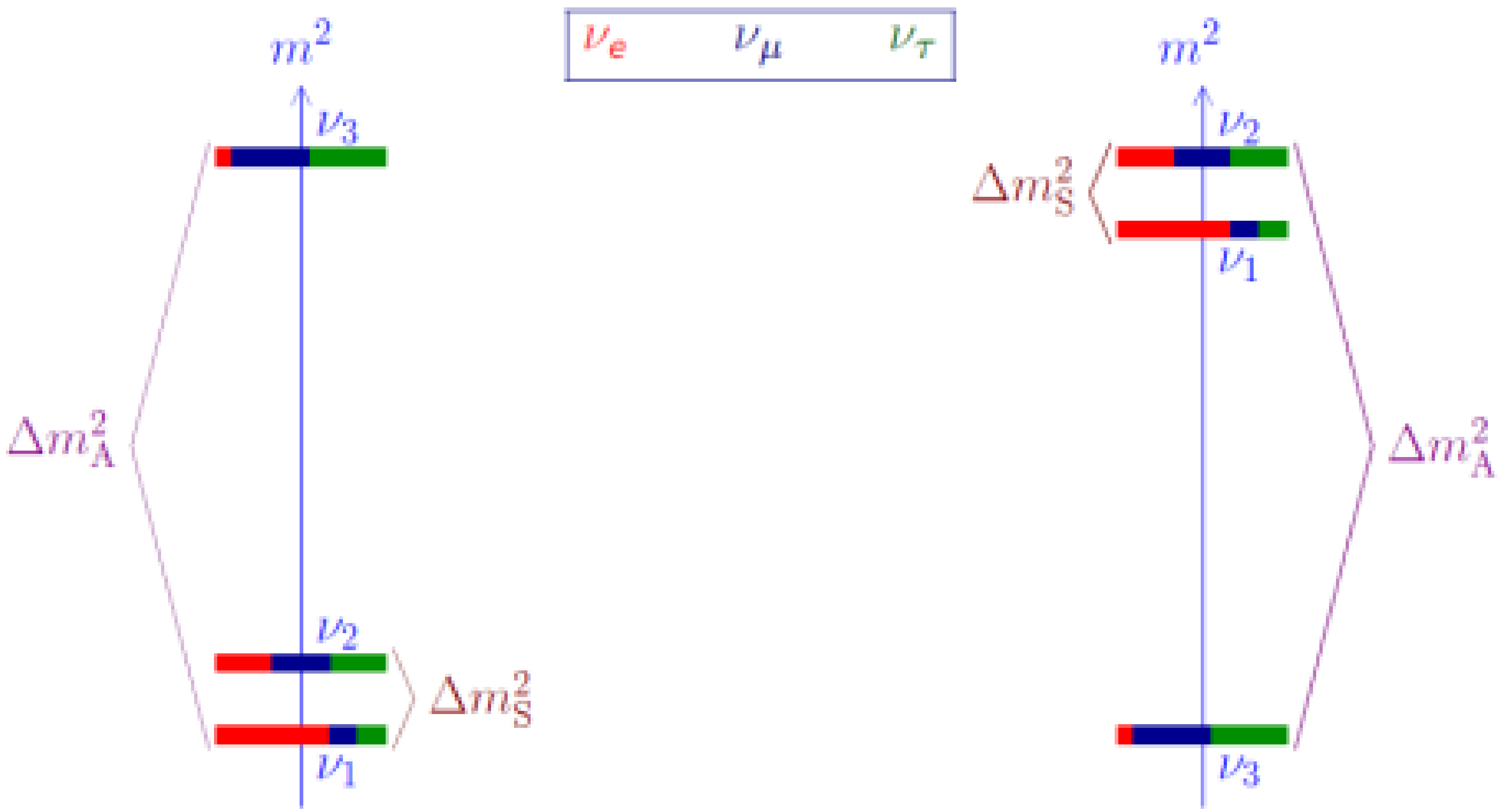




- **5 parameters: measured with rather high accuracy**
- **4 parameters:**

  relevant to $m_{\beta\beta}$

Neutrino mass spectrum



Normal Ordering

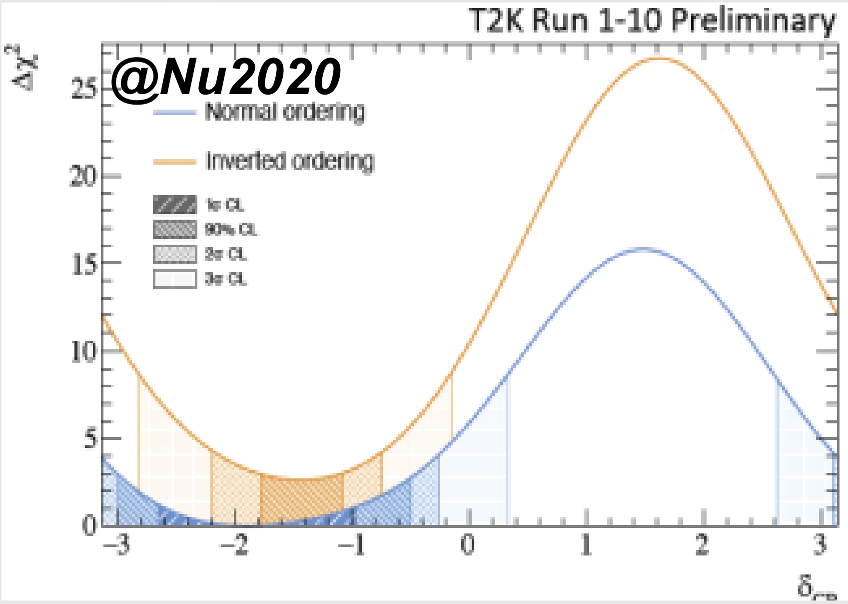
$\Delta m^2_{31} > \Delta m^2_{32} > 0$

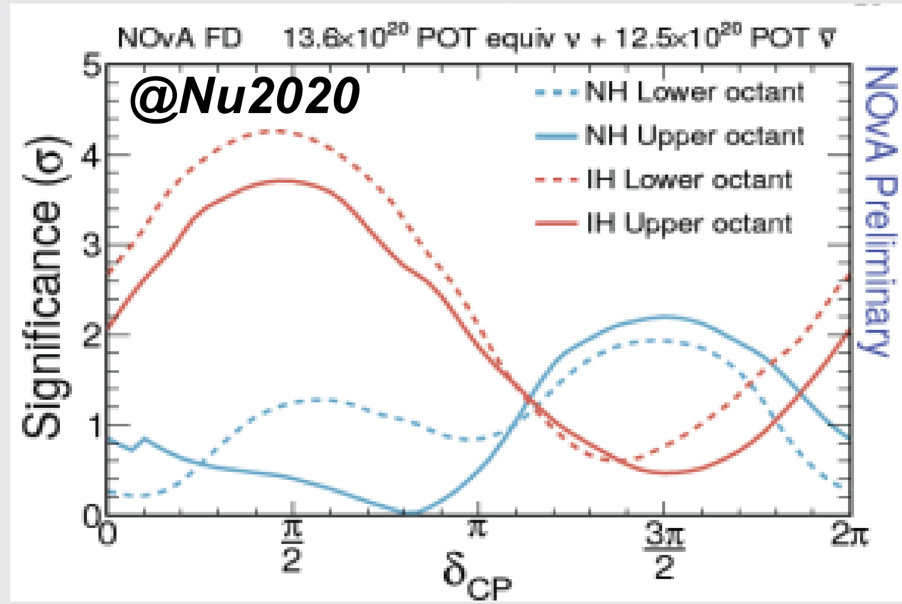
Inverted Ordering

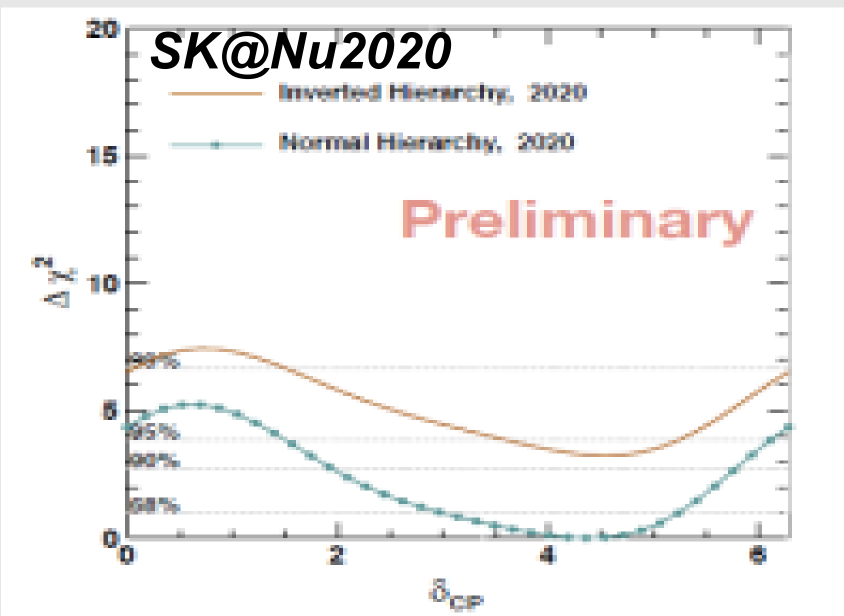
$\Delta m^2_{32} < \Delta m^2_{31} < 0$

absolute scale is not determined by neutrino oscillation data

Neutrino mass ordering (circa 2021)





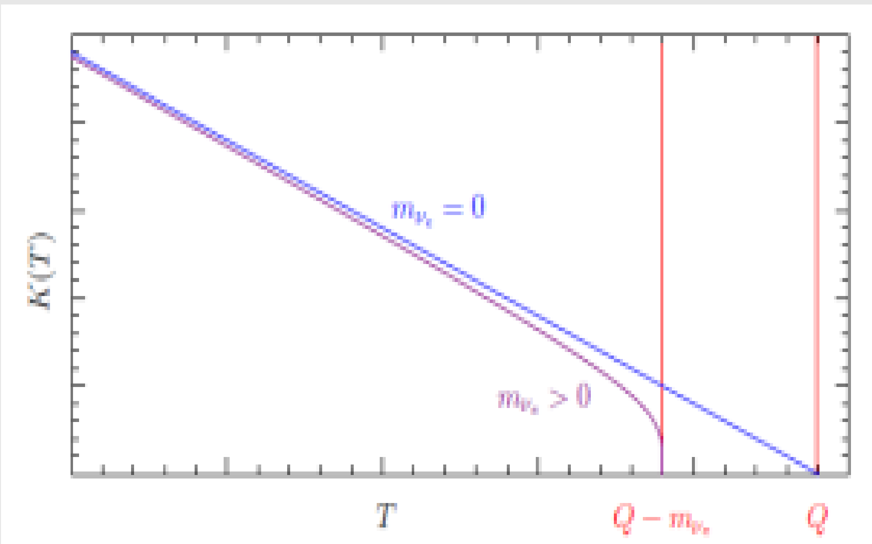


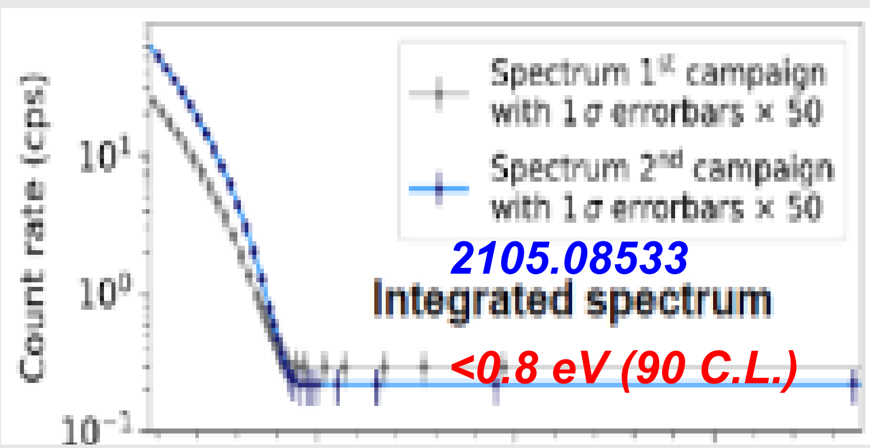
Near Future:

- **T2K & NOvA & SuperK**
- **JUNO (reactors): 2022**
- **PINGU (ORCA): 202x?**
- **DUNE (HyperK): ~2027?**

Neutrino mass scale from beta-decay

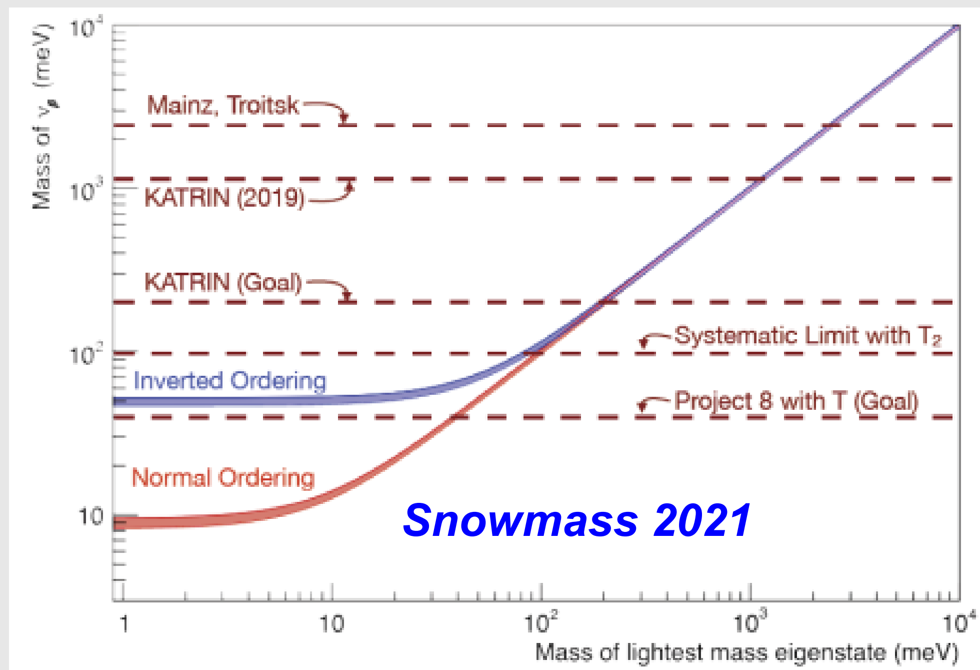
$$^{3}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + e^{-} + \bar{\nu}_e$$





Future Prospect: $m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$

- **KATRIN: 200 meV**
- **Systematic limit: ~100 meV**
- **Project 8: 40 meV**



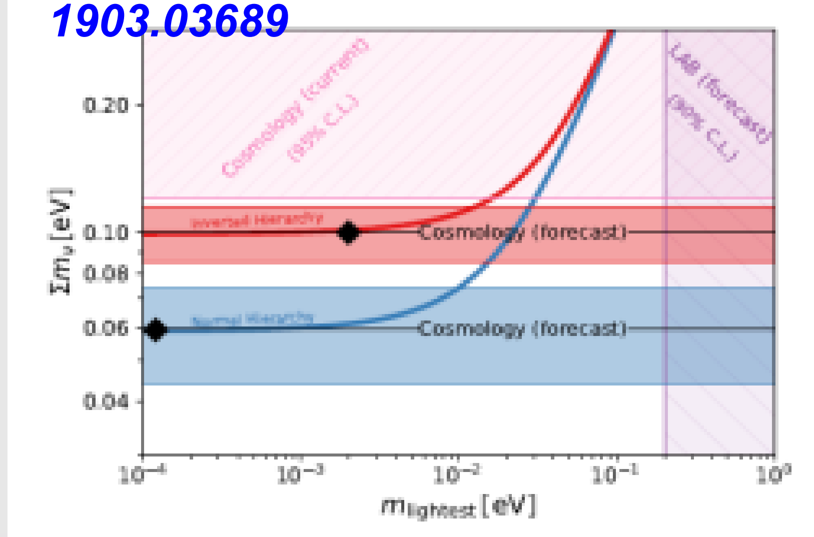
Neutrino mass scale from cosmology

PDG 2020	Model	95% CL (eV)	Ref.
CMB alone			
Pl18[TT+lowE]	ΛCDM+$\sum m_\nu$	< 0.54	[16]
Pl18[TT,TE,EE+lowE]	ΛCDM+$\sum m_\nu$	< 0.26	[16]
CMB + probes of background evolution			
Pl18[TT+lowE] + BAO	ΛCDM+$\sum m_\nu$	< 0.16	[16]
Pl18[TT,TE,EE+lowE] + BAO	ΛCDM+$\sum m_\nu$	< 0.13	[16]
Pl18[TT,TE,EE+lowE]+BAO	ΛCDM+$\sum m_\nu$+5 params.	< 0.515	[18]
CMB + LSS			
Pl18[TT+lowE+lensing]	ΛCDM+$\sum m_\nu$	< 0.44	[16]
Pl18[TT,TE,EE+lowE+lensing]	ΛCDM+$\sum m_\nu$	< 0.24	[16]
CMB + probes of background evolution + LSS			
Pl18[TT+lowE+lensing] + BAO	ΛCDM+$\sum m_\nu$	< 0.13	[16]
Pl18[TT,TE,EE+lowE+lensing] + BAO	ΛCDM+$\sum m_\nu$	< 0.12	[16]
Pl18[TT,TE,EE+lowE+lensing] + BAO+Pantheon	ΛCDM+$\sum m_\nu$	< 0.11	[16]

Cosmology: sum of neutrino masses

- **Data sets and model dependence**
- **Current best limit: ~120 meV**
- **Future projection → 60 meV**

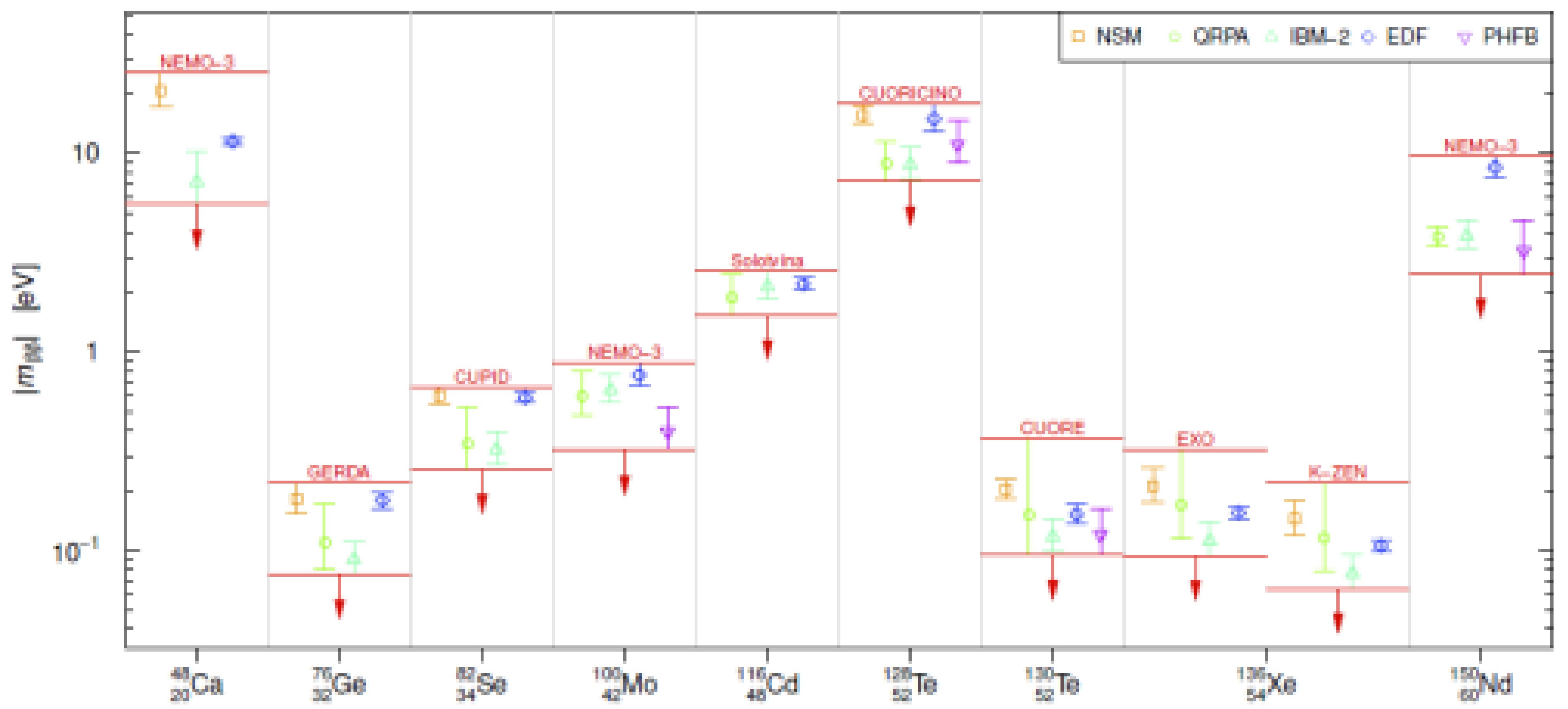
1903.03689



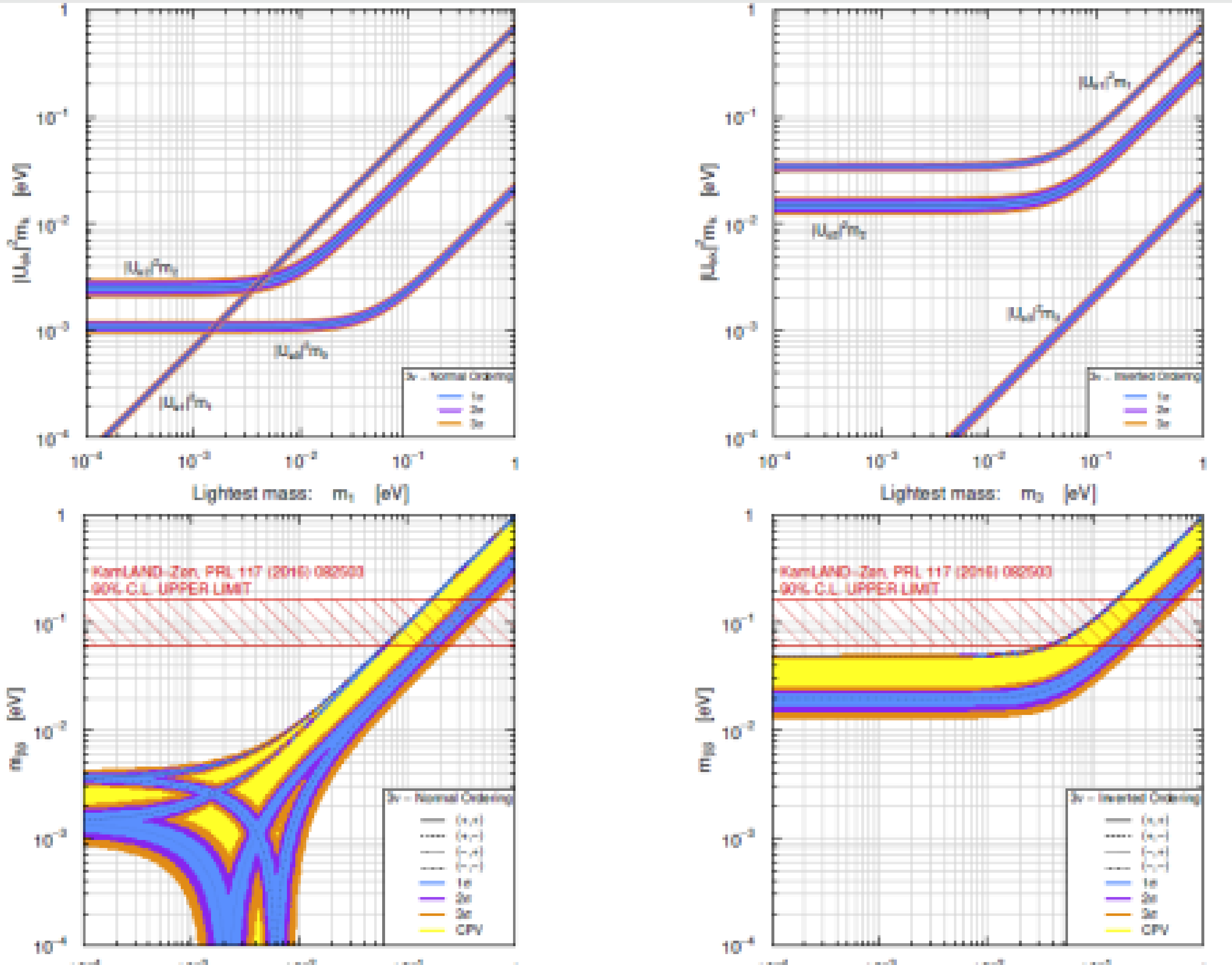
Neutrino mass scale from 0ν2β-decay

0ν2β-decay: effective neutrino mass limit as in 2021

An updated plot of 1411.4791



$m_{\beta\beta}$: Decomposition



1505.00978

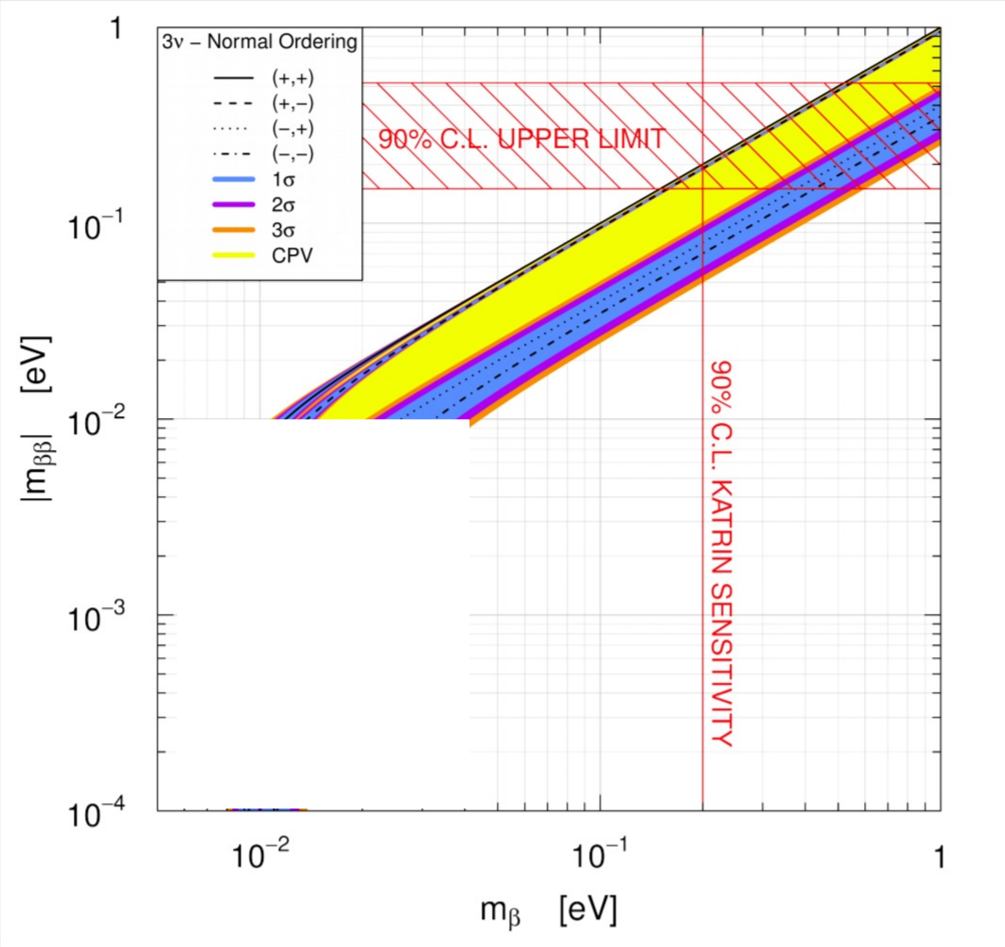
Three different regions:

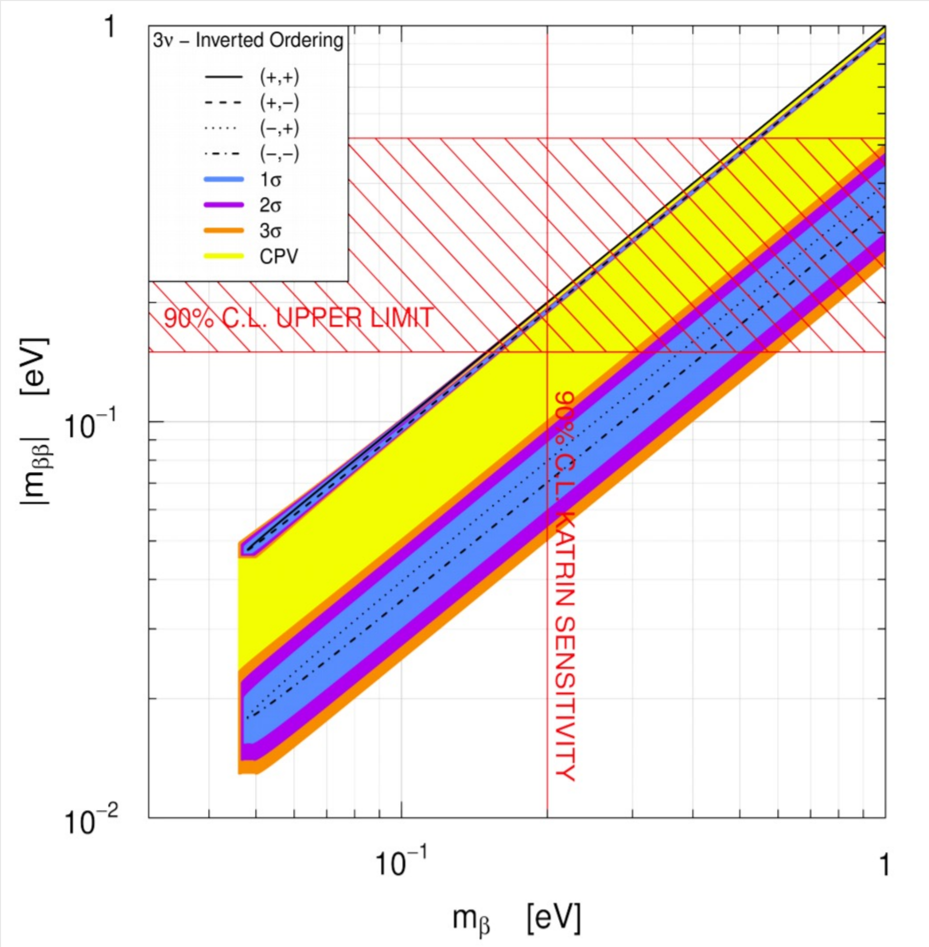
- **QD:** $m_{1/3}$>10 meV
- **Hierarchical:** $m_{1/3}$<1 meV
- **Cancelation:** [1, 10] meV

I: Quasi-Degenerate region

$$|m_{\beta\beta}| \simeq m_\nu \sqrt{1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2}}$$

➢ **Extraction of the CP phase by comparing with beta decay or cosmology probe**



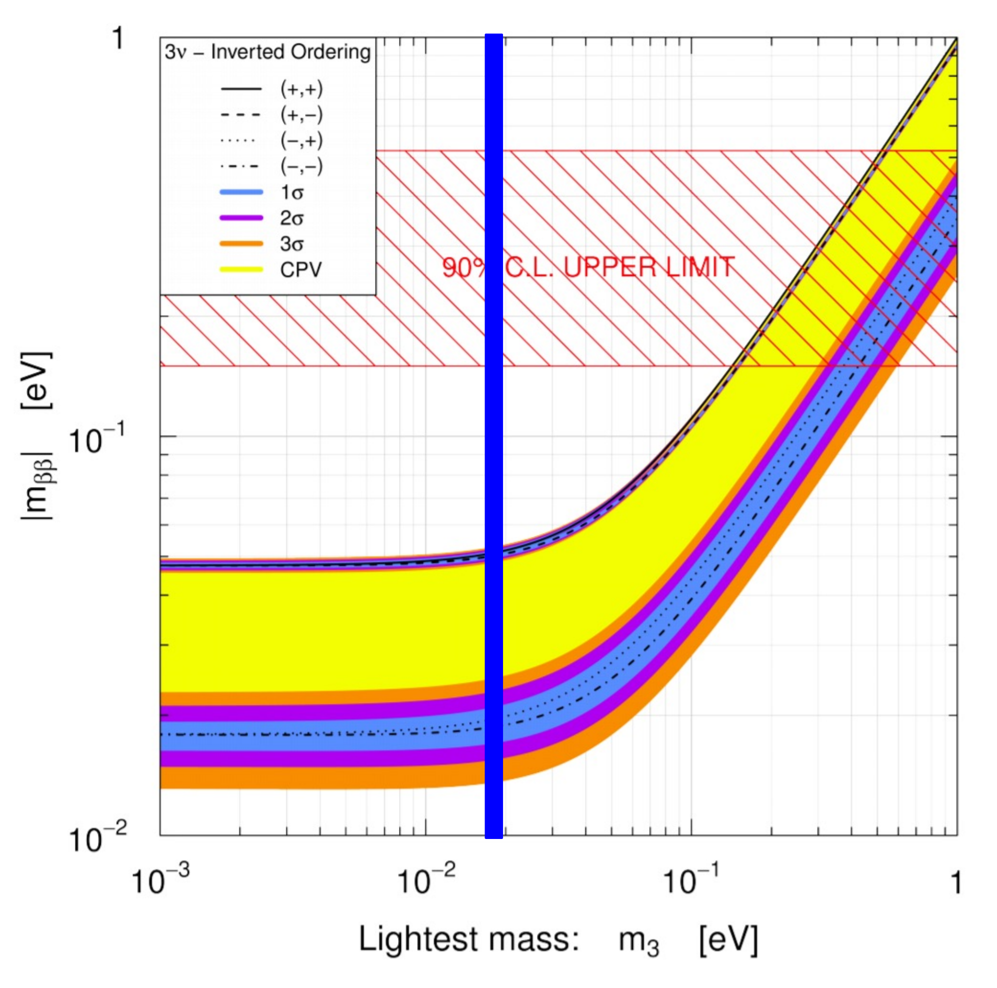


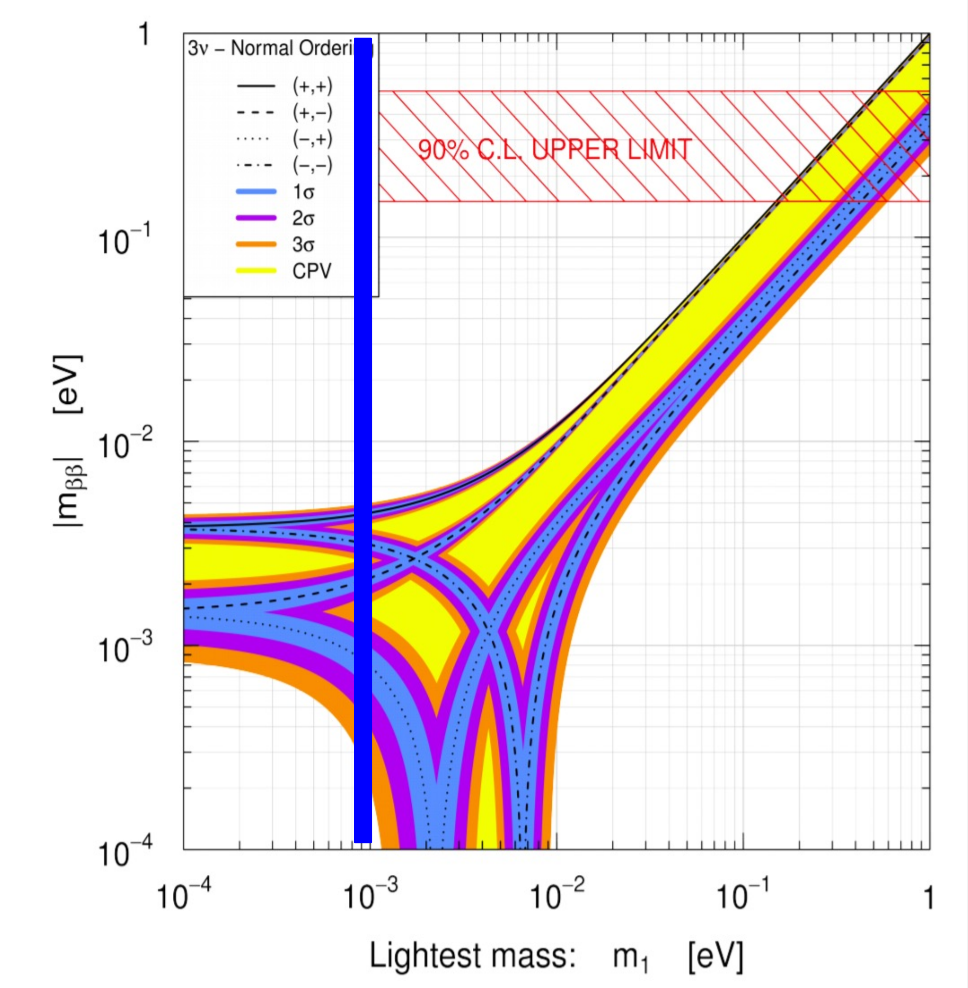
II: Hierarchical Region

- **Independent of the absolute neutrino masses (NO & IO)**

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m_A^2 (1 - s_{2\vartheta_{12}}^2 s_{\alpha_2}^2)}$$

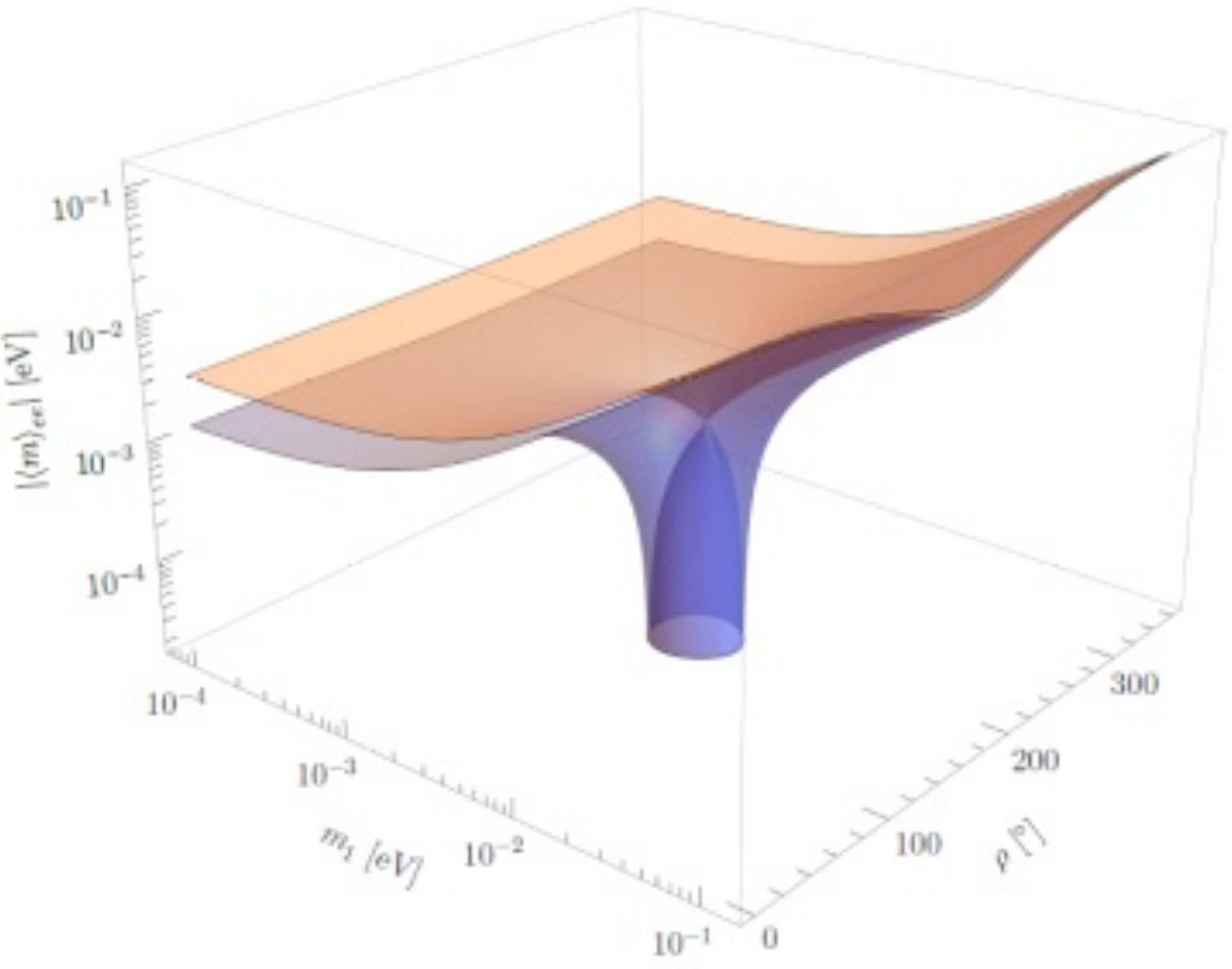
$$|m_{\beta\beta}| \simeq |s_{12}^2 \sqrt{\Delta m_S^2} + e^{i\alpha} s_{13}^2 \sqrt{\Delta m_A^2}|$$



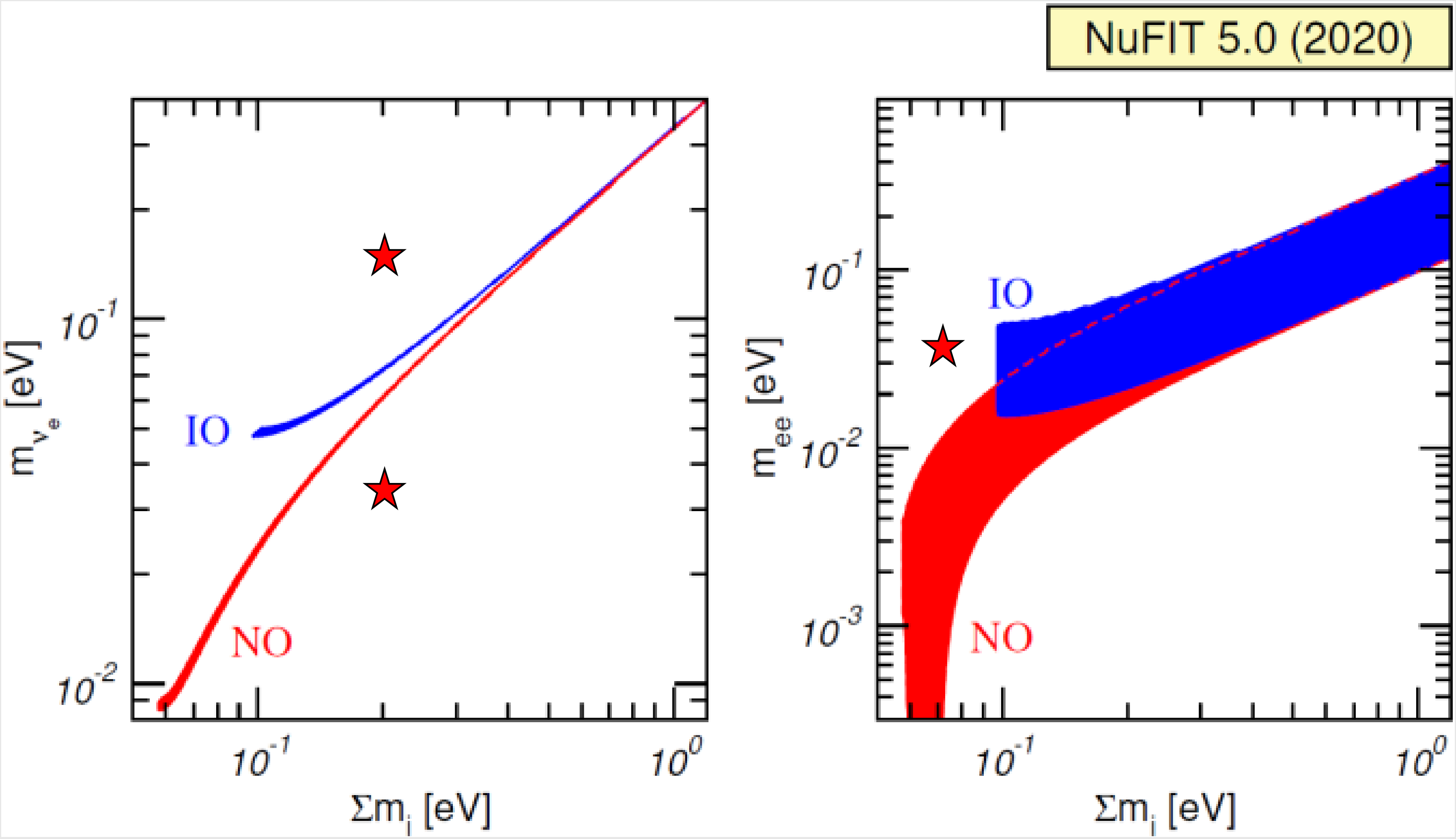


III: Cancelation region

Xing & Zhao, 1612.08538: **The critical threshold point is just ~1 meV !**



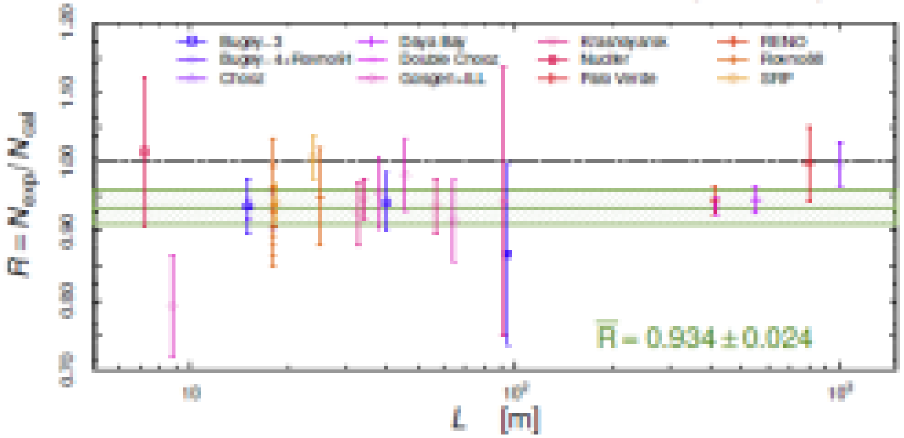
Mass probe correlation



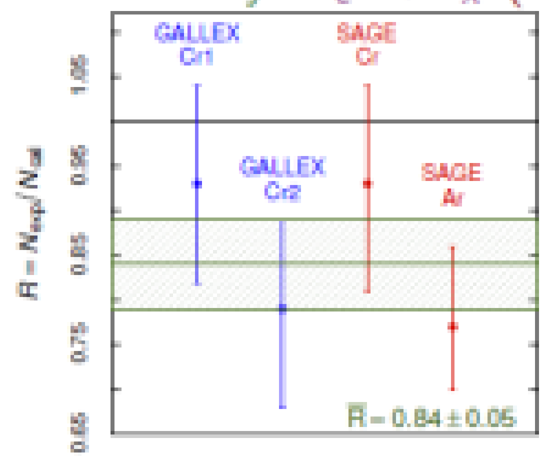
- **What is the interpretation if out of the standard region?**

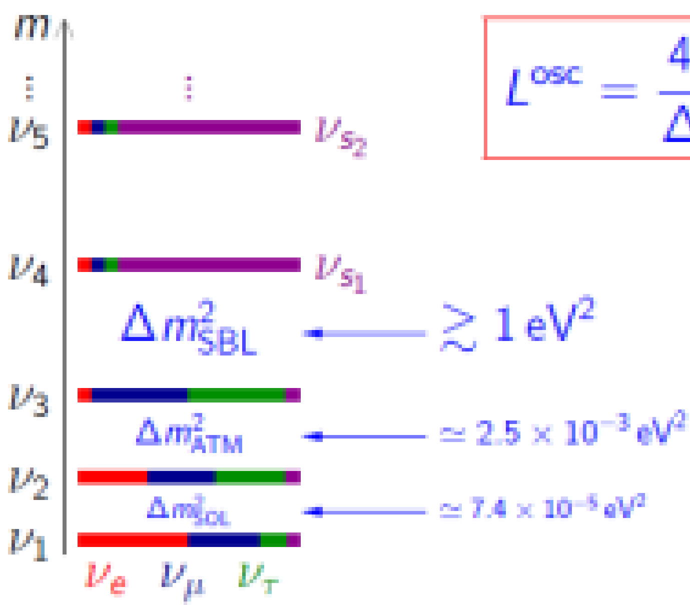
Short baseline oscillations: Anomalies?

Reactor Anomaly: $\bar{\nu}_e \to \bar{\nu}_x$ $(\sim 3\sigma)$



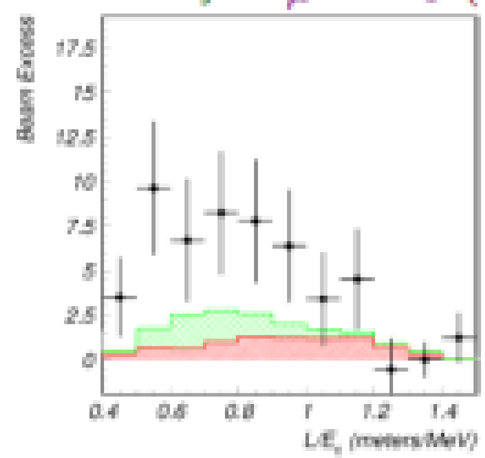
Gallium Anomaly: $\nu_e \to \nu_x$ $(\sim 3\sigma)$





$$L^{osc} = \frac{4\pi E}{\Delta m^2}$$

LSND Anomaly: $\bar{\nu}_\mu \to \bar{\nu}_e$ $(\sim 4\sigma)$

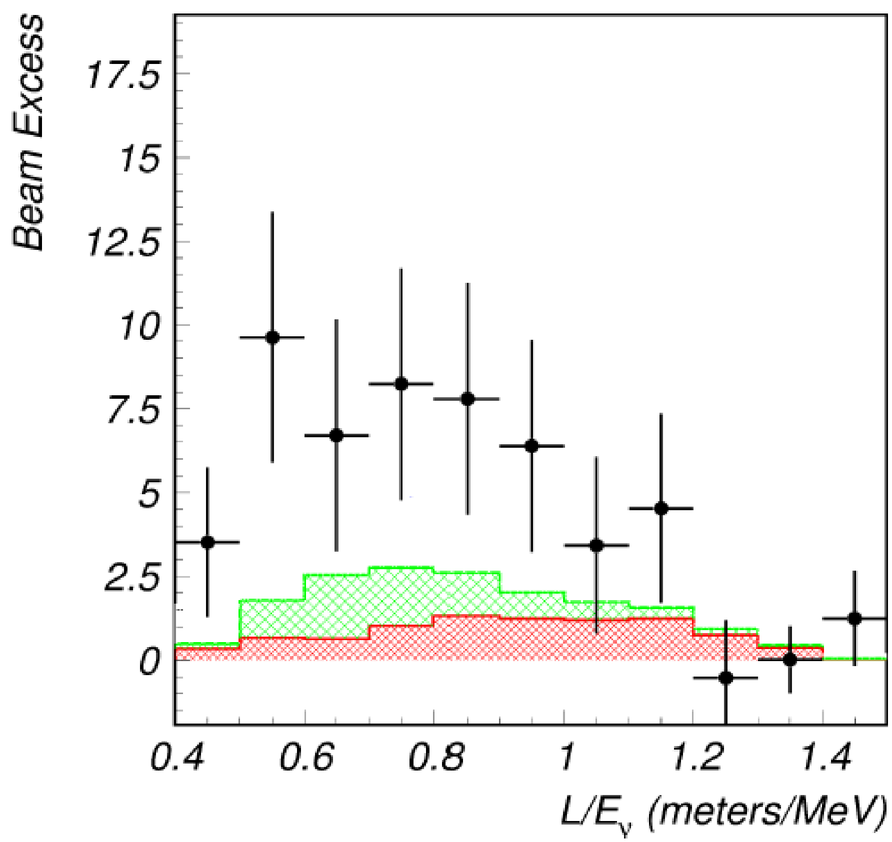


Minimal perturbation of 3ν mixing: effective 3+1 with $|U_{e4}|, |U_{\mu 4}|, |U_{\tau 4}| \ll 1$

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\qquad$ $20\,\text{MeV} \leq E \leq 52.8\,\text{MeV}$



- Well-known and pure source of $\bar{\nu}_\mu$

$$p + \text{target} \to \pi^+ \xrightarrow{\text{at rest}} \mu^+ + \nu_\mu$$

$$\mu^+ \xrightarrow[\text{at rest}]{} e^+ + \nu_e + \bar{\nu}_\mu$$

$$L \simeq 30\,\text{m}$$

$$\bar{\nu}_e + p \to n + e^+$$

Well-known detection process of $\bar{\nu}_e$

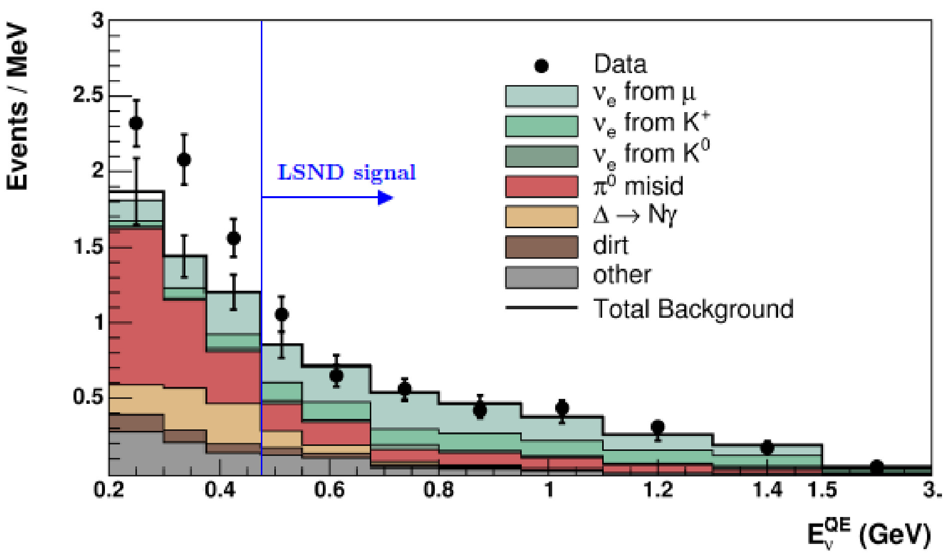
- $\approx 3.8\sigma$ excess
- But signal not seen by KARMEN at $L \simeq 18\,\text{m}$ with the same method

[PRD 65 (2002) 112001]

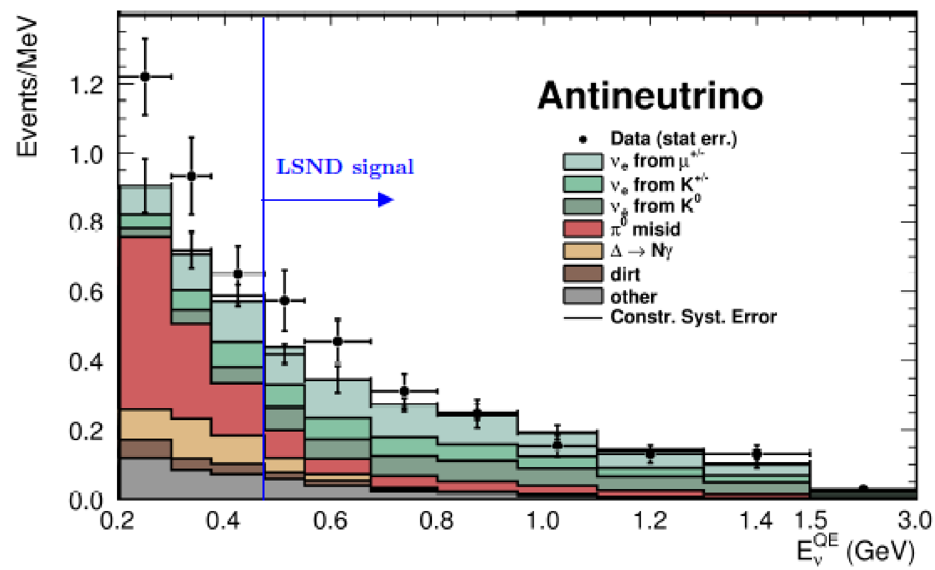
MiniBooNE

$L \simeq 541\,\mathrm{m}$ $\quad$ $200\,\mathrm{MeV} \leq E \lesssim 3\,\mathrm{GeV}$

$\nu_\mu \to \nu_e$ [PRL 102 (2009) 101802]



$\bar{\nu}_\mu \to \bar{\nu}_e$ [PRL 110 (2013) 161801]



Purpose: **check LSND signal with different L&E, but the same L/E**

~4-5σ excess **in the Low energy range: unidentified backgrounds?**

Future test with MicroBooNE ? Soon within this year!

Gallium anomaly

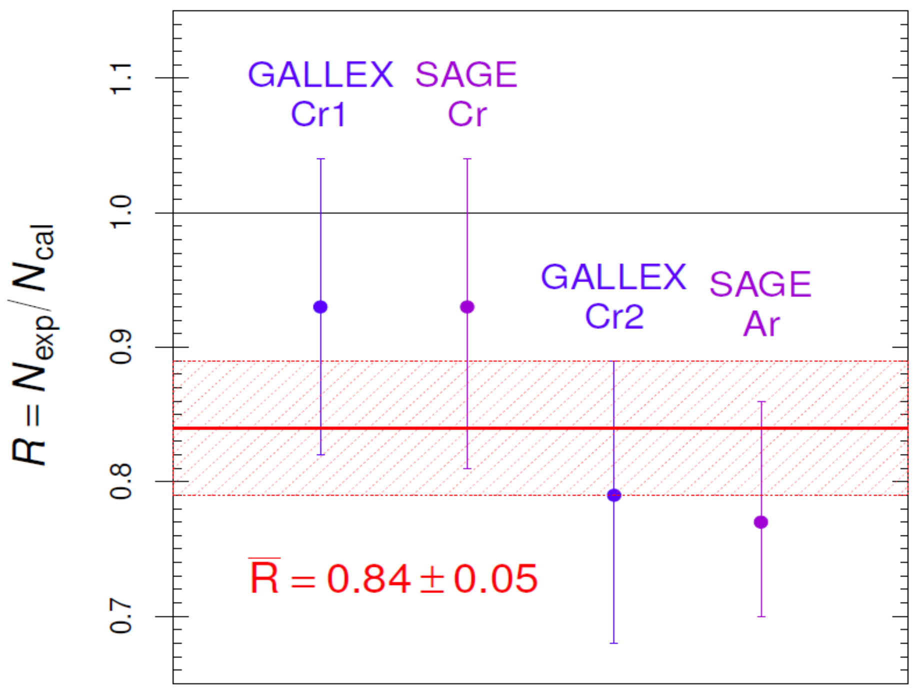
SAGE, PRC (2006); PRC (2009); Laveder et al. (2007), etc.

Gallium Radioactive Source Experiments: GALLEX and SAGE

Test of Solar Neutrino Detection

Detection Process: $\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-$

ν_e Sources: $e^- + {}^{51}\mathrm{Cr} \to {}^{51}\mathrm{V} + \nu_e$ $\quad e^- + {}^{37}\mathrm{Ar} \to {}^{37}\mathrm{Cl} + \nu_e$



$\langle L \rangle_{\mathrm{GALLEX}} = 1.9\,\mathrm{m}$ $\quad \langle L \rangle_{\mathrm{SAGE}} = 0.6\,\mathrm{m}$

- **~2.9σ deficit**

Neutrino energies: ~0.8 MeV

$$\Delta m^2_{\mathrm{SBL}} \gtrsim 1\,\mathrm{eV}^2 \gg \Delta m^2_{\mathrm{ATM}} \gg \Delta m^2_{\mathrm{SOL}}$$

- **Anomaly supported by new cross section measurement**

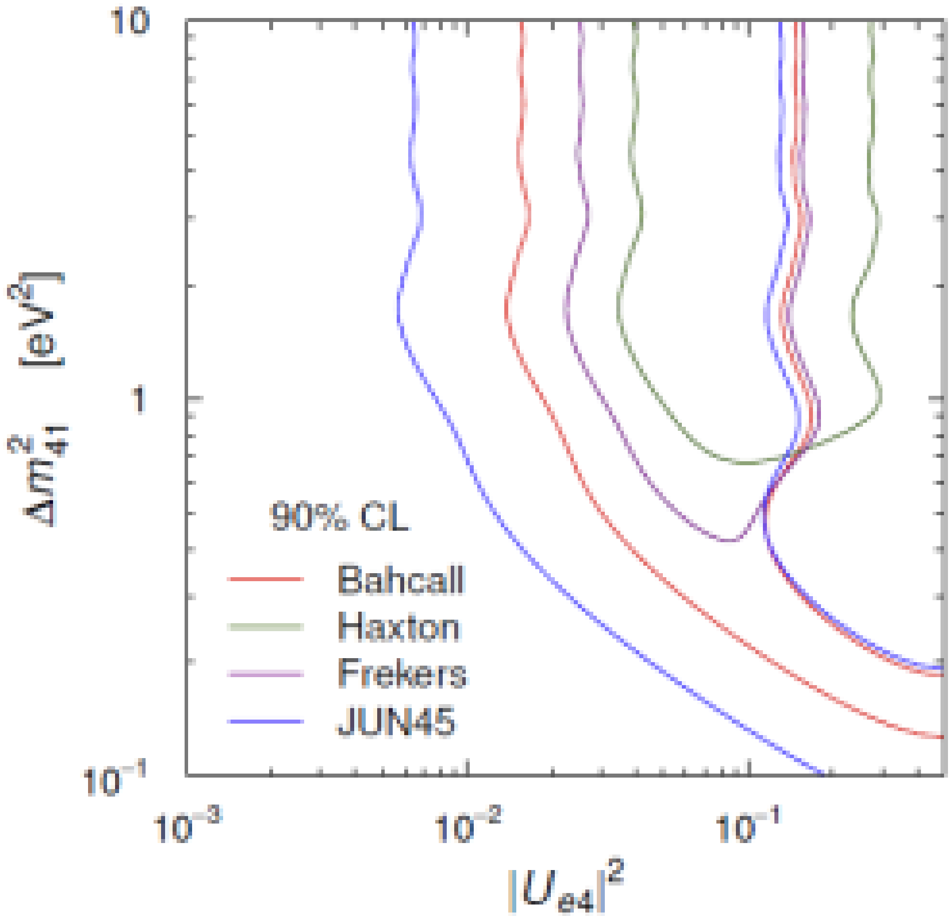
$$^{3}\mathrm{He} + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + {}^{3}\mathrm{H}$$

Frekers et al., PLB 706 (2011) 134

Gallium anomaly: shell model calculations ?

- New JUN45 shell-model calculation of the cross section of

$$\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$$



1906.10980
Kostensalo, Suhonen, Giunti, Srivastava

Cross sections in units of 10^{-45} cm^2:

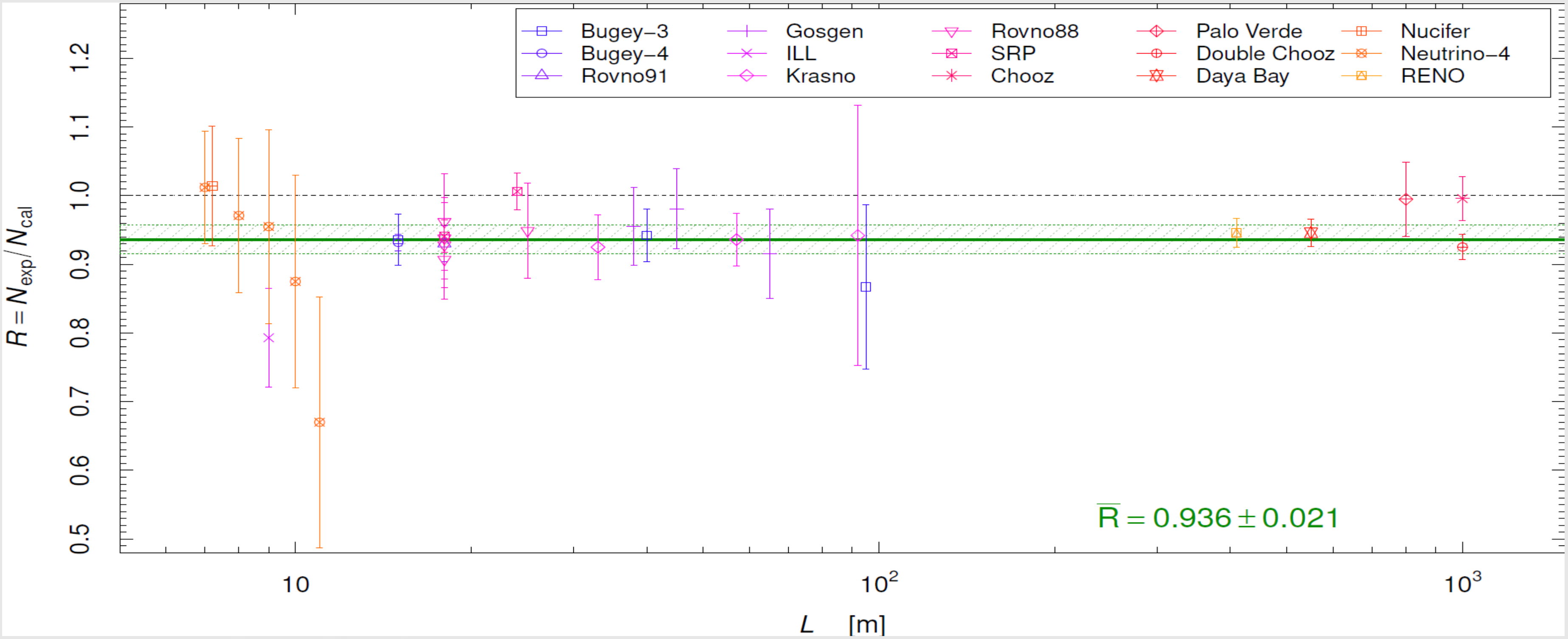
	$\sigma(^{51}\text{Cr})$	$\sigma(^{37}\text{Ar})$
Bahcall	5.81 ± 0.16	7.00 ± 0.21
Haxton	6.39 ± 0.65	7.72 ± 0.81
Frekers	5.92 ± 0.11	7.15 ± 0.14
JUN45	5.67 ± 0.06	6.80 ± 0.08

- The statistical significance of the gallium anomaly is reduced from 2.9σ (Frekers) to 2.3σ (JUN45).

Reactor Antineutrino Anomaly

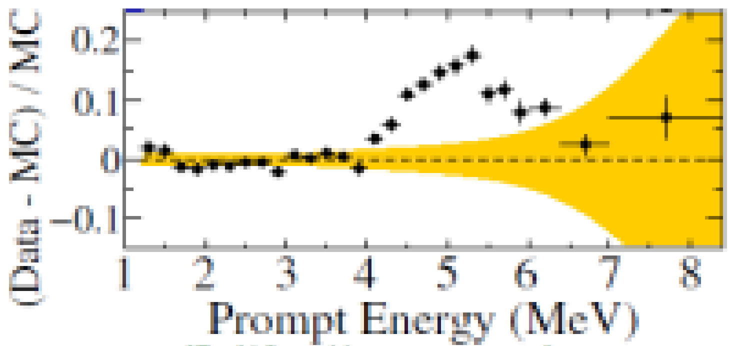
[Mention et al, PRD 83 (2011) 073006]

New reactor $\bar{\nu}_e$ fluxes [Mueller et al, PRC 83 (2011) 054615; Huber, PRC 84 (2011) 024617]

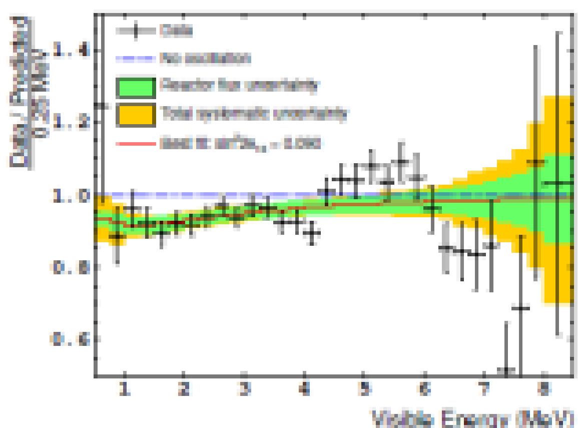


- **Discrepancy between theory and measurements**
- **~2.8σ deficit (depending on the theoretical flux uncertainty)**
- **Nominal theoretical uncertainty from the model (Saclay+Huber) ~ 2.5%**

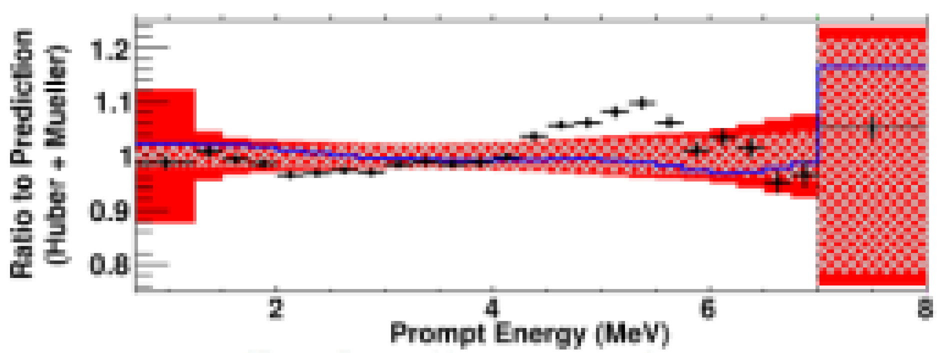
Challenge I: 5 MeV bump



[RENO, arXiv:1511.05849]



[Double Chooz, arXiv:1406.7763]



[Daya Bay, arXiv:1508.04233]

- Cannot be explained by neutrino oscillations (SBL oscillations are averaged in RENO, DC, DB).
- If it is due to a theoretical miscalculation of the spectrum, it can have opposite effects on the anomaly:

  [see: Berryman, Huber, arXiv:1909.09267]

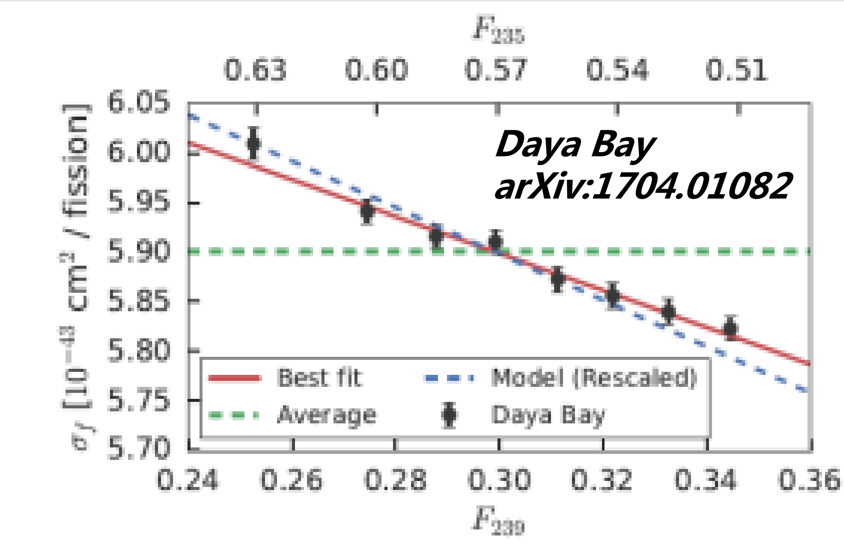
  - If it is a 4-6 MeV excess it increases the anomaly: new HKSS flux calculation

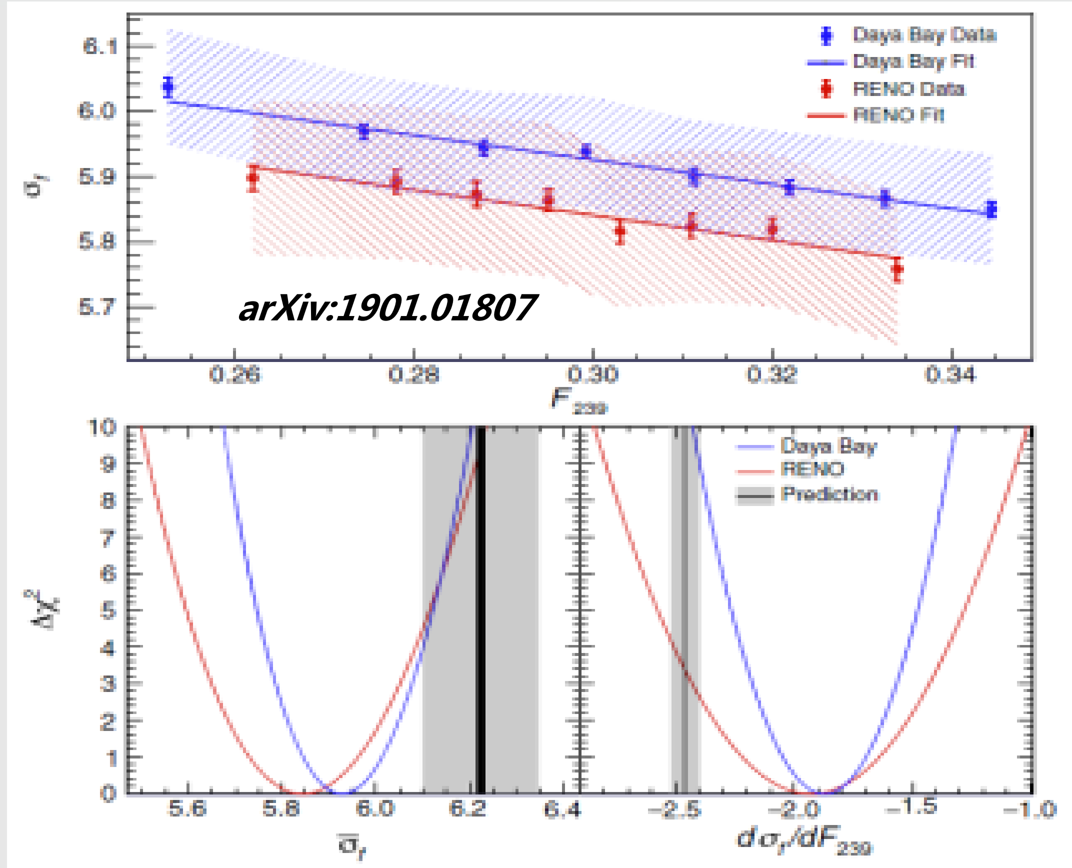
    [Hayen, Kostensalo, Severijns, Suhonen, arXiv:1908.08302]

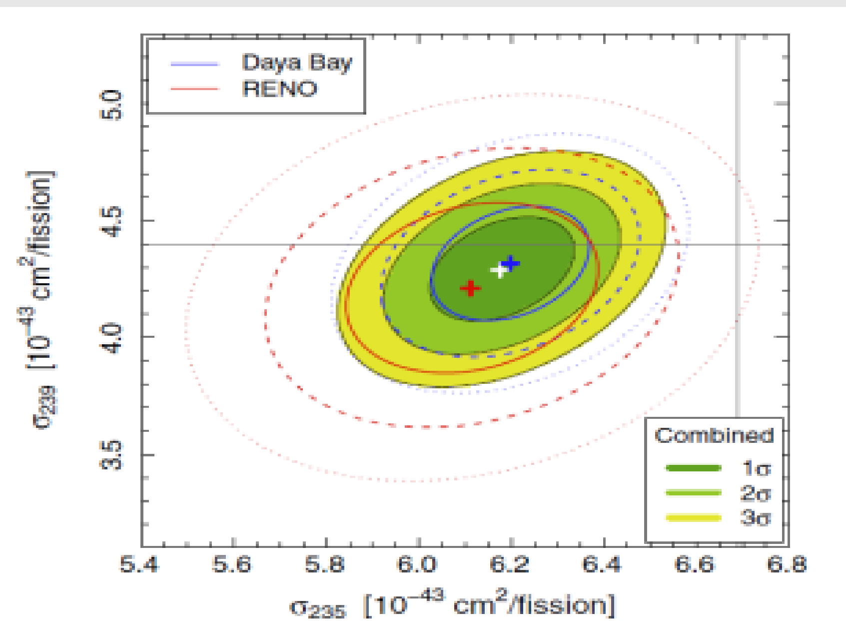
  - If it is a 1-4 MeV suppression it decreases the anomaly: new EF flux calculation

    [Estienne, Fallot, et al, arXiv:1904.09358]

Challenge II: fuel evolution

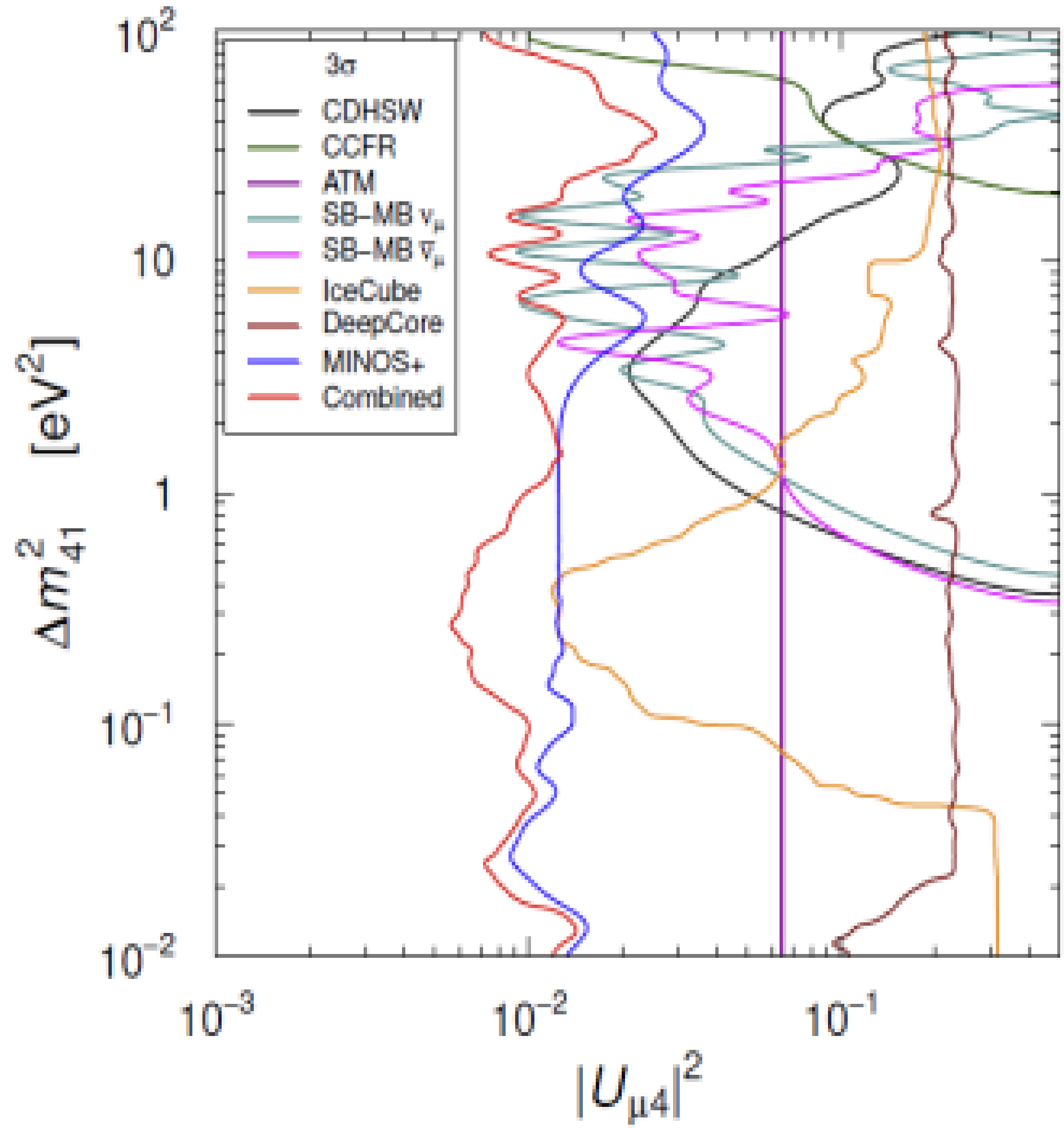






- **Reactor rates as function of the fuel fraction: A new information (slope)!**
- **Inconsistent with model prediction at around 3σ**

Global fits: muon disappearance

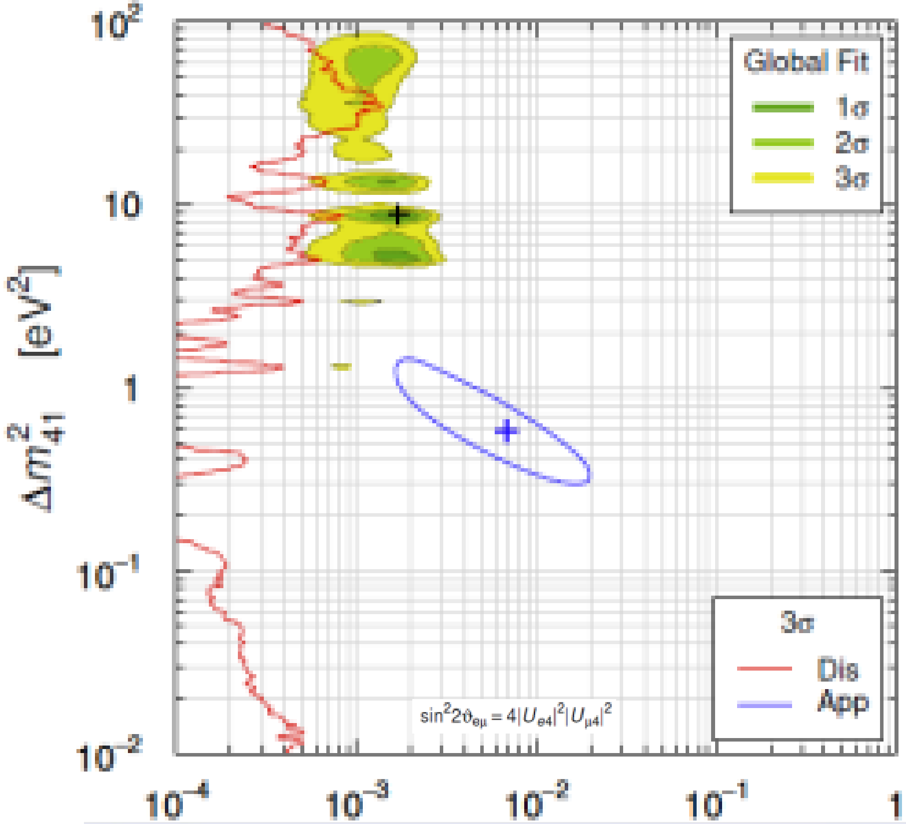


Appearance v.s. Disappearance: Tension

ν_e DIS
$\sin^2 2\vartheta_{ee} \simeq 4|U_{e4}|^2$

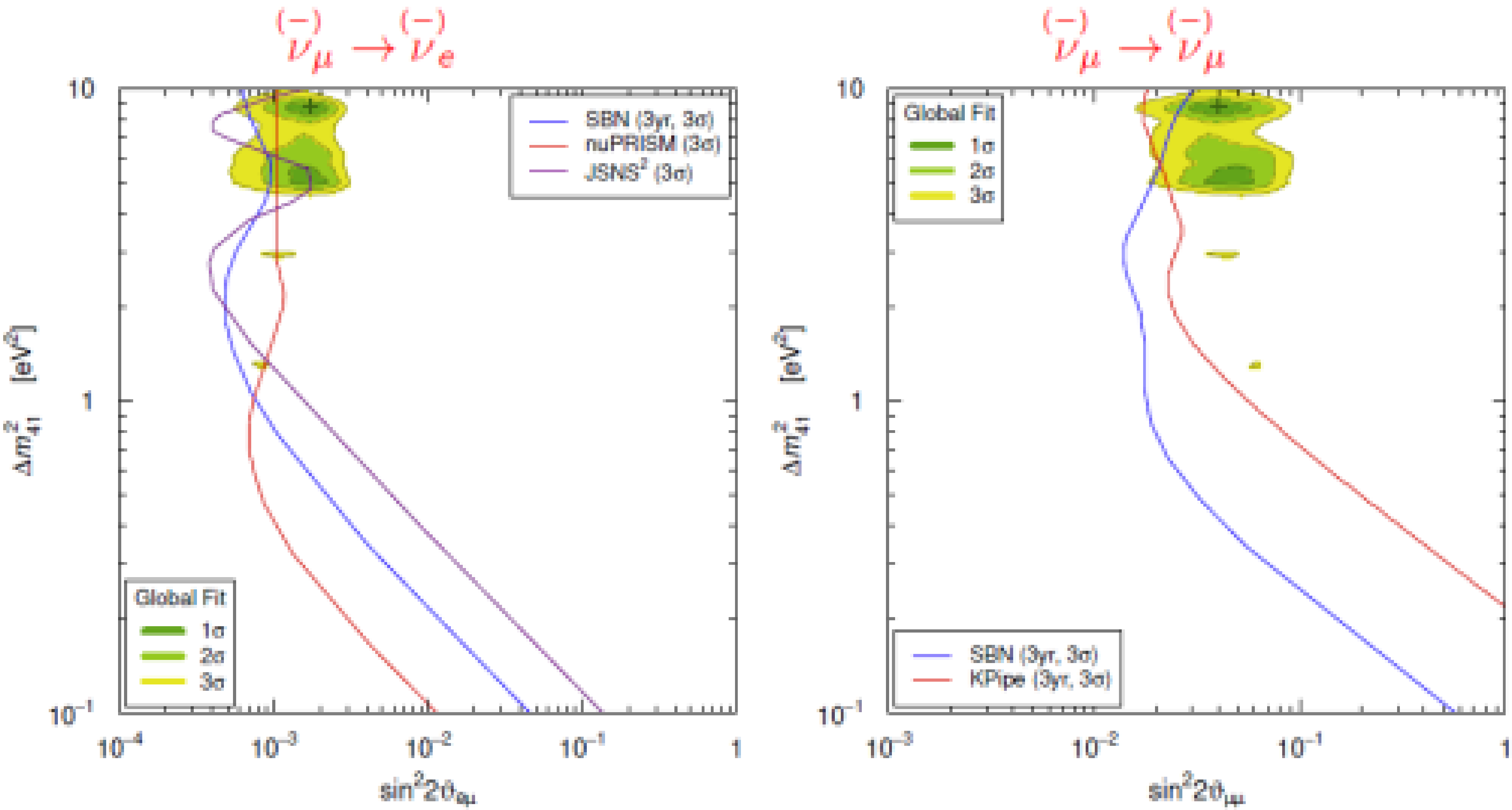
ν_μ DIS
$\sin^2 2\vartheta_{\mu\mu} \simeq 4|U_{\mu4}|^2$

$\nu_\mu \to \nu_e$ APP
$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4}\sin^2 2\vartheta_{ee}\sin^2 2\vartheta_{\mu\mu}$



- $\nu_\mu \to \nu_e$ is quadratically suppressed!
- Global Fit:
  $\chi^2/\text{NDF} = 843.6/794$
  $\text{GoF} = 11\%$
  $\chi^2_{\text{PG}}/\text{NDF}_{\text{PG}} = 46.7/2$
  $\text{GoF}_{\text{PG}} = 7 \times 10^{-11}$ ← ☹
- Similar tension in
  $3+2, \quad 3+3, \quad \ldots, \quad 3+N_s$

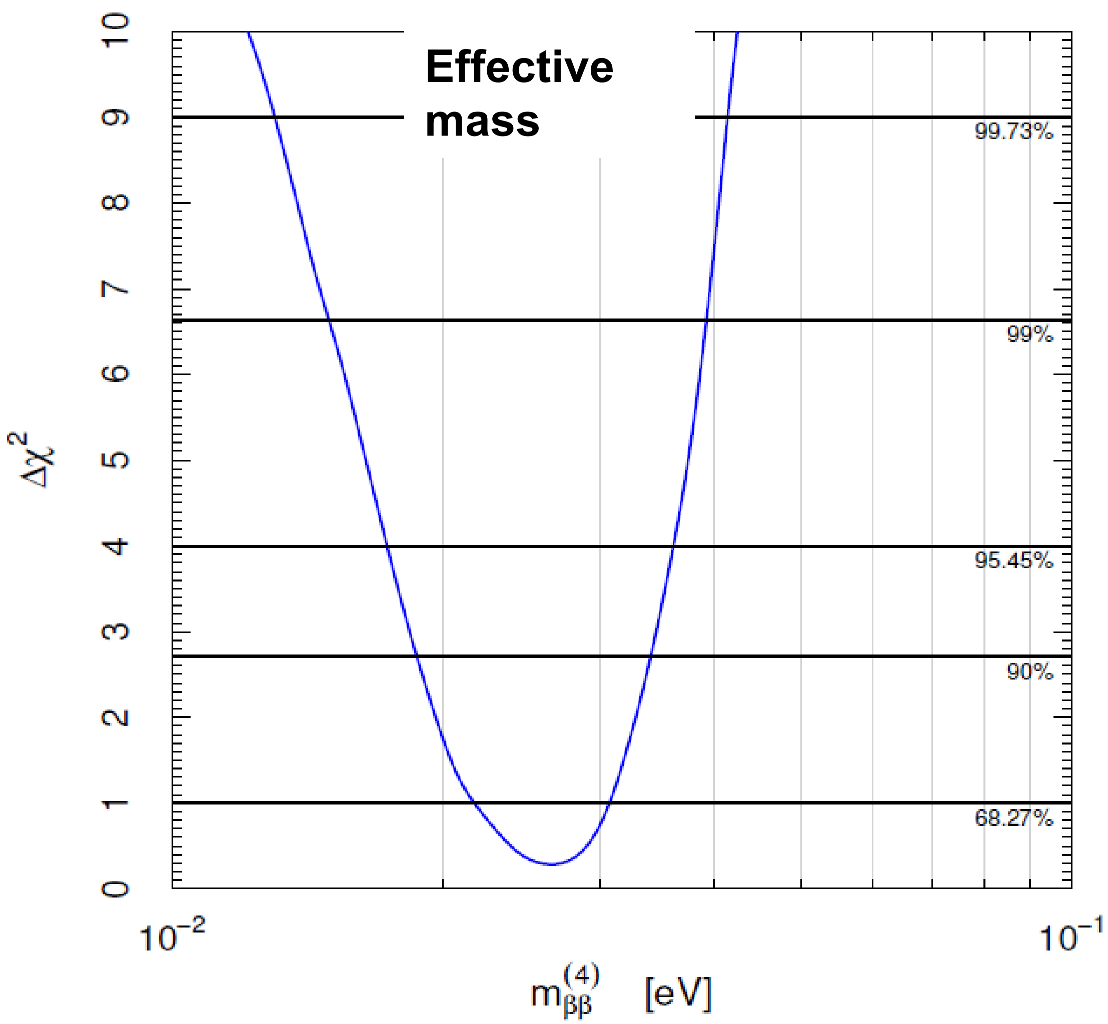
Future Test ?



$\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$
SBN (3yr, 3σ)
nuPRISM (3σ)
JSNS² (3σ)
Global Fit
1σ
2σ
3σ
Δm^2_{41} [eV²]
$\sin^2 2\theta_{e\mu}$
$\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu$
Global Fit
1σ
2σ
3σ
SBN (3yr, 3σ)
KPipe (3yr, 3σ)
Δm^2_{41} [eV²]
$\sin^2 2\theta_{\mu\mu}$

$0\nu 2\beta$-decay: the effective mass

$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_{21}}\, m_2 + |U_{e3}|^2\, e^{i\alpha_{31}}\, m_3 + |U_{e4}|^2\, e^{i\alpha_{41}}\, m_4$$



$$m_{\beta\beta}^{(k)} = |U_{ek}|^2 m_k$$

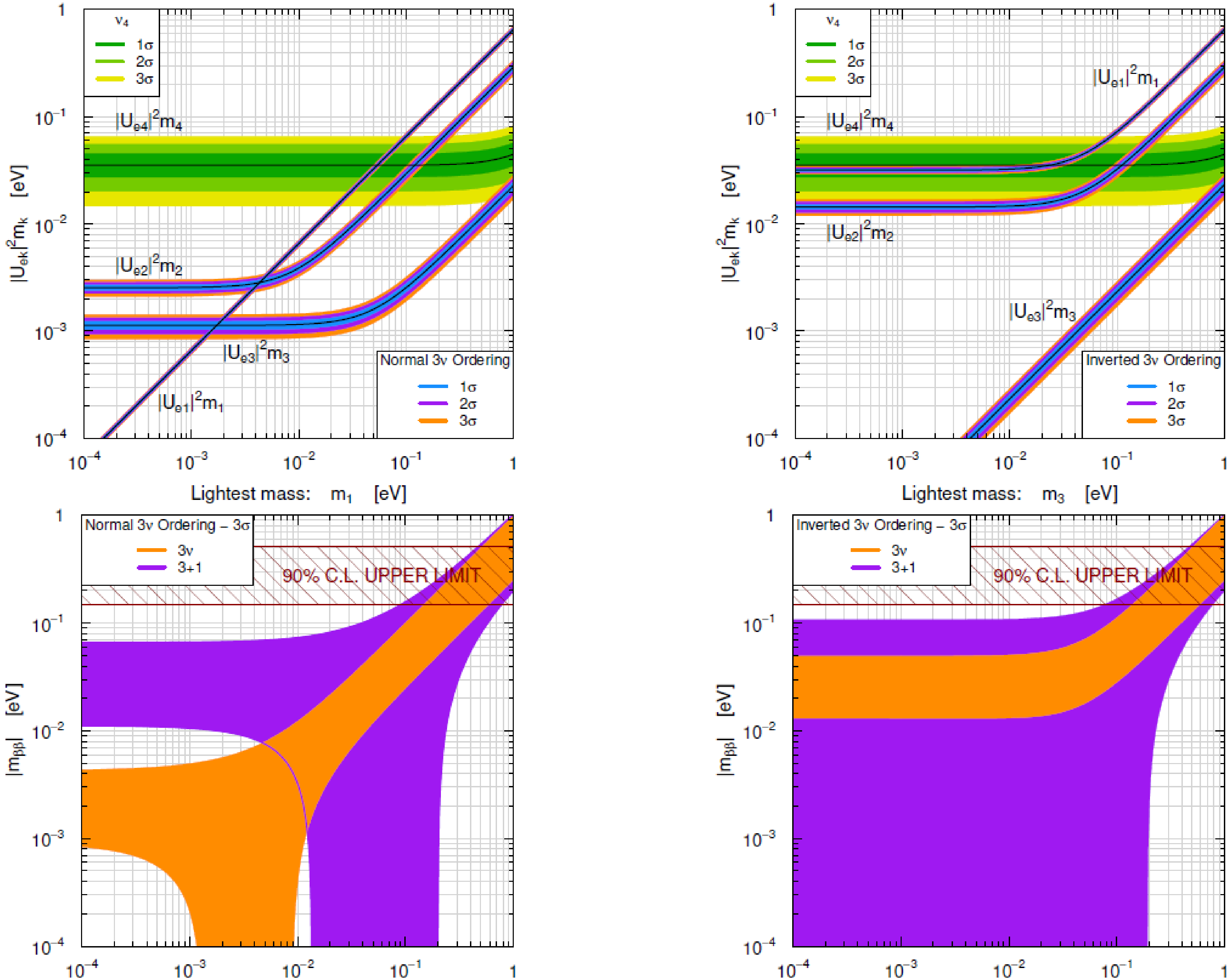
$$m_1 \ll m_4$$

$$\Downarrow$$

$$m_{\beta\beta}^{(4)} \simeq |U_{e4}|^2 \sqrt{\Delta m_{41}^2}$$

warning:
possible cancellation
with $m_{\beta\beta}^{(3\nu)}$

[Barry, Rodejohann, Zhang, JHEP 07 (2011) 091]
[Li, Liu, PLB 706 (2012) 406]
[Rodejohann, JPG 39 (2012) 124008]
[Girardi, Meroni, Petcov, JHEP 1311 (2013) 146]
[CG, Zavanin, JHEP 07 (2015) 171]



ν4
1σ
2σ
3σ
$|U_{e4}|^2 m_4$
$|U_{e2}|^2 m_2$
$|U_{e3}|^2 m_3$
$|U_{e1}|^2 m_1$
Normal 3ν Ordering
Inverted 3ν Ordering
$|U_{ek}|^2 m_k$ [eV]
Lightest mass: m_1 [eV]
Lightest mass: m_3 [eV]
Normal 3ν Ordering – 3σ
Inverted 3ν Ordering – 3σ
3ν
3+1
90% C.L. UPPER LIMIT
$|m_{\beta\beta}|$ [eV]

Test with $0\nu2\beta$

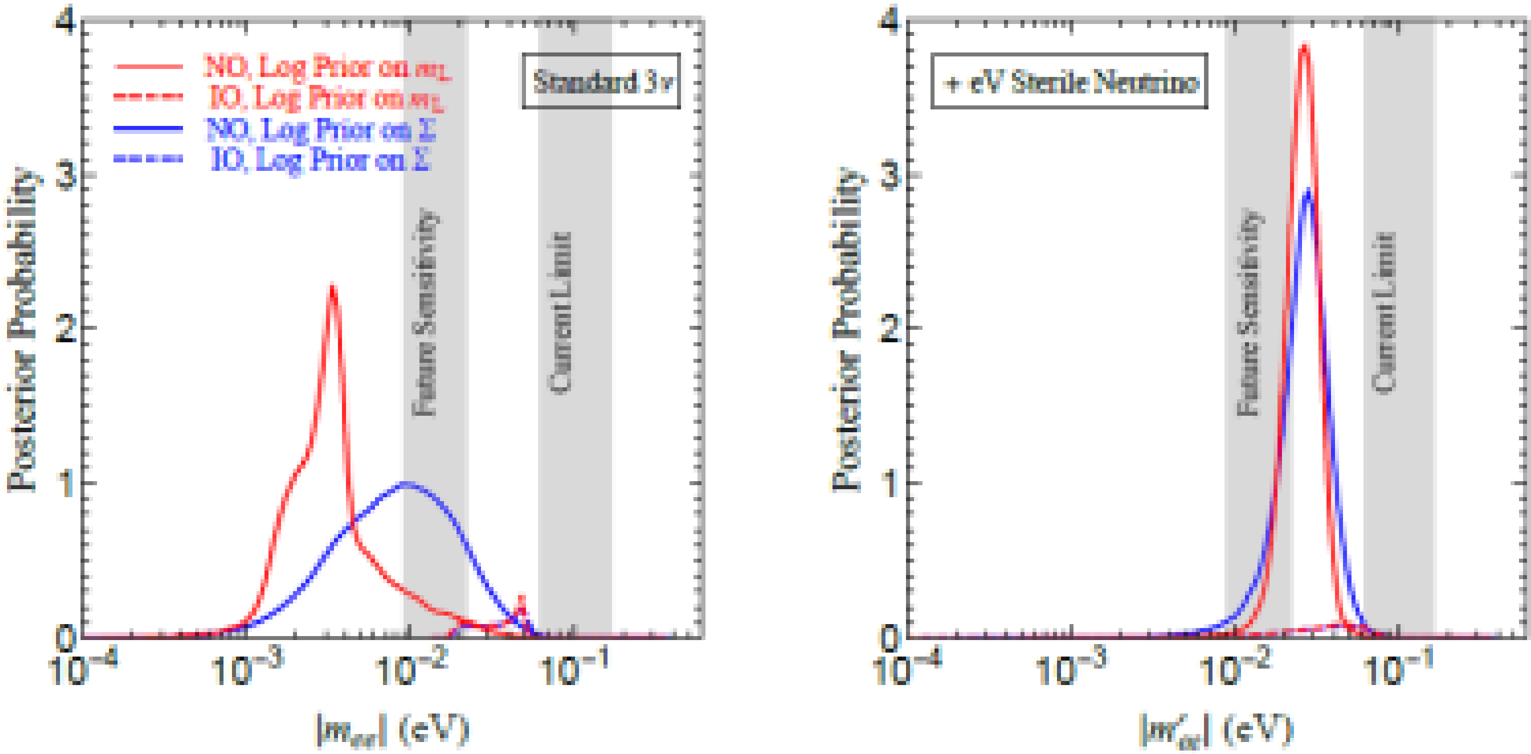
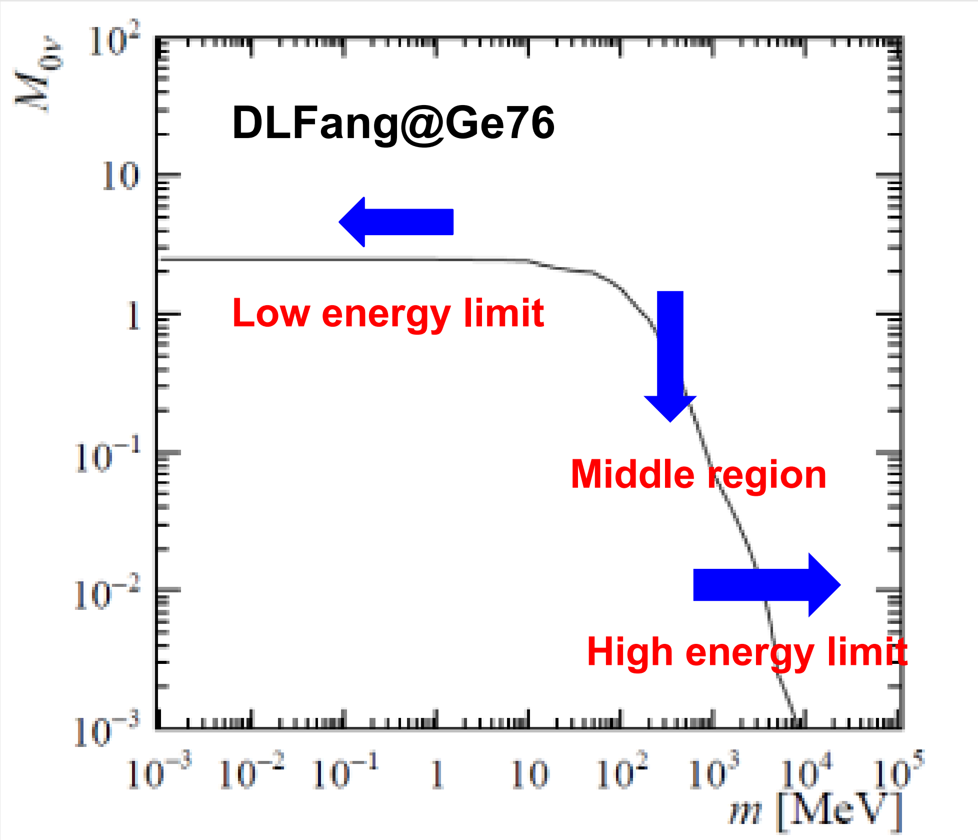


Figure 5: The posterior probability densities of $|m_{ee}|$ or $|m'_{ee}|$ for different choices of models and priors. The standard 3ν results are presented in the left panel, while those with the sterile neutrino in the right panel. The posteriors with the logarithmic prior on m_L (the logarithmic prior on Σ) are plotted as the red (blue) solid curves in the NO case, but as the dashed curves in the IO case.

Huang & Zhou: 1902.03839

→can be tested in next-generation experiments!

Heavy sterile from seesaw ?



$$\langle m \rangle_{ee} \equiv \left| \sum_{i=1}^{3} V_{ei}^2 m_i \right| = \left| \sum_{k=1}^{n} R_{ek}^2 M_k \right|$$

$$\left| \sum_{k=1}^{n} \frac{R_{ek}^2}{M_k} \right| < 5 \times 10^{-8}\ \mathrm{GeV}^{-1} \quad \Longleftrightarrow \quad \left| \sum_{k=1}^{n} R_{ek}^2 M_k \right| < 0.23\ \mathrm{eV}$$

Different mass regions:

- **Low energy limit:** same as light massive neutrinos
- **High energy limit:**

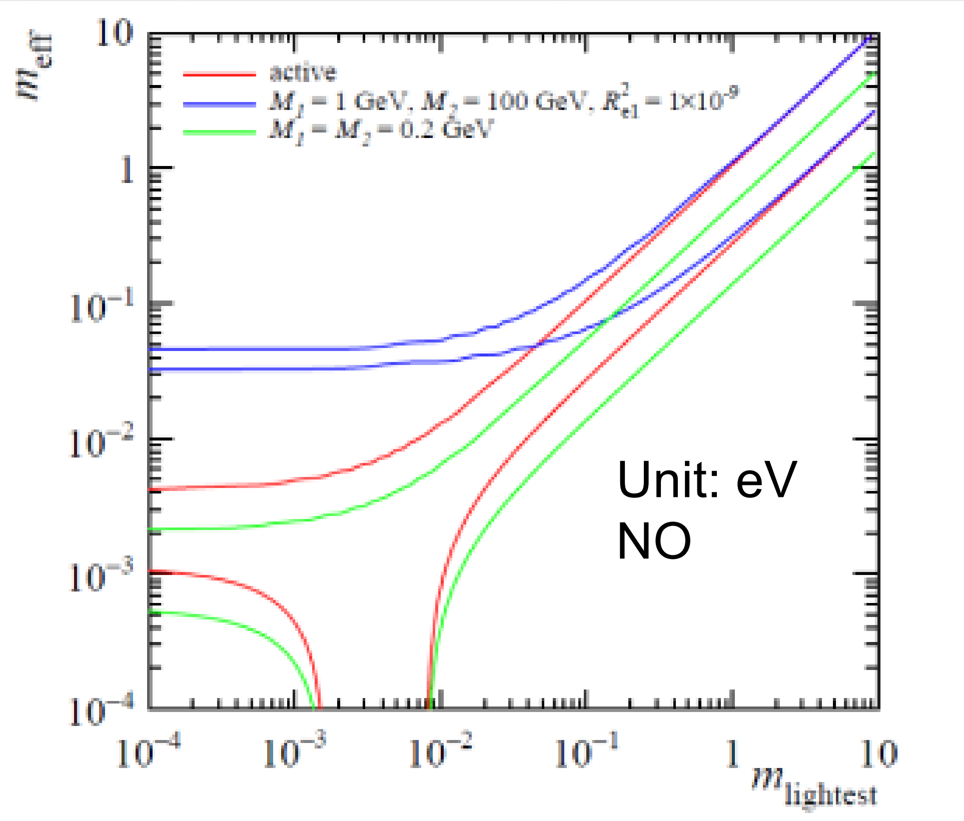
  Xing, 0907.3014

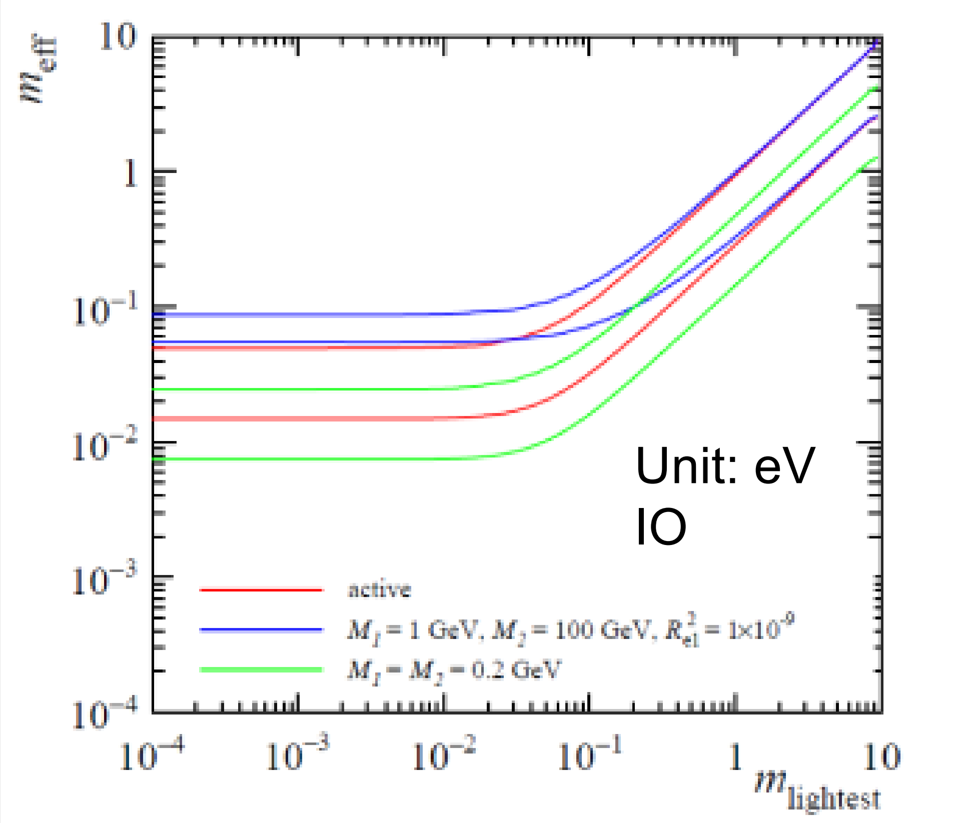
Contribution will be negligible if the seesaw relation is required.

- **Middle region:**

Could be compatible when apply the seesaw relation

Sterile neutrinos from seesaw





Effective mass (both light and heavy neutrinos)

Degenerate case: suppression

Hierarchical case: enhancement

What if $0\nu 2\beta$ is observed?

Neutrinoless double beta decay: neutrino nature and masses!

- **After the discovery of $0\nu 2\beta$**

  Distinguishing Mechanisms:

- **Comparison of different mass probes: agreement or not ?**

- **Other contributions: light/heavy sterile neutrinos, and more …**

- **Decay products**

  individual electron energies, angular correlations, spectrum

- **Nuclear aspects**

  multiple isotopes, decay to excited states, 0νECEC,

Thank you !

$$m_{\text{eff}}^{\nu} = \sum_{i=1}^{3} U_{ei}^2 m_i = \left(c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 e^{2i\delta_{12}} m_2 + s_{13}^2 e^{2i\delta_{13}} m_3\right)$$

$$m_{\text{eff}} = m_{\text{eff}}^{\nu} + R_{e1}^2 M_1 f_\beta(M_1) - \left(m_{\text{eff}}^{\nu} + R_{e1}^2 M_1\right) f_\beta(M_2)$$

给定最轻的中微子质量m_{lightset}，和M_1, M_2, R_{e1}^2，并考虑振荡实验所确定的参数误差，可以计算m_{eff}的范围

$\sin^2(\theta_{12}) = 0.307 \pm 0.013$

$\Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5}\ \text{eV}^2$

$\sin^2(\theta_{23}) = 0.547 \pm 0.021$ (Inverted order)

$\sin^2(\theta_{23}) = 0.545 \pm 0.021$ (Normal order)

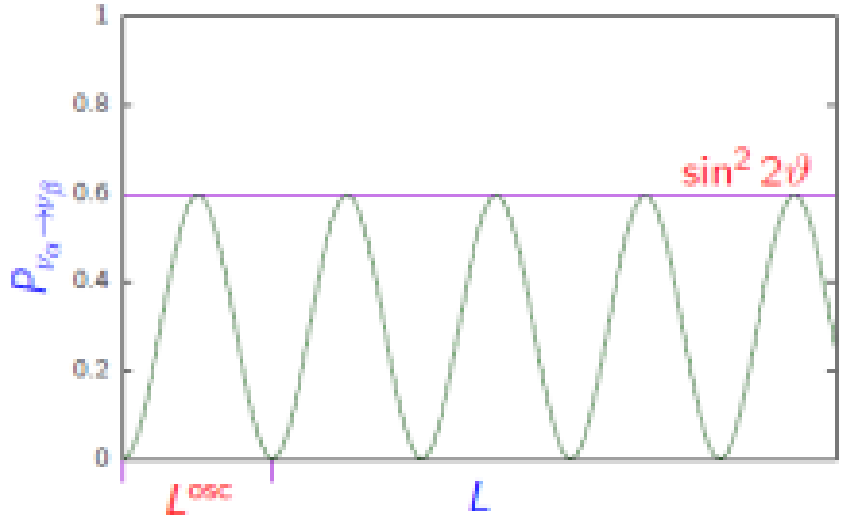
$\Delta m_{32}^2 = (-2.546^{+0.034}_{-0.040}) \times 10^{-3}\ \text{eV}^2$ (Inverted order)

$\Delta m_{32}^2 = (2.453 \pm 0.034) \times 10^{-3}\ \text{eV}^2$ (Normal order)

$\sin^2(\theta_{13}) = (2.18 \pm 0.07) \times 10^{-2}$

Oscillation Types

2ν-mixing: $P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \Longrightarrow L^{osc} = \frac{4\pi E}{\Delta m^2}$



Tiny neutrino masses lead to observable macroscopic oscillation distances!

$$\frac{L}{E} \lesssim \begin{cases} 10 \frac{\text{m}}{\text{MeV}} \left(\frac{\text{km}}{\text{GeV}}\right) & \text{short-baseline experiments} & \Delta m^2 \gtrsim 10^{-1}\,\text{eV}^2 \\ 10^3 \frac{\text{m}}{\text{MeV}} \left(\frac{\text{km}}{\text{GeV}}\right) & \text{long-baseline experiments} & \Delta m^2 \gtrsim 10^{-3}\,\text{eV}^2 \\ 10^4 \frac{\text{km}}{\text{GeV}} & \text{atmospheric neutrino experiments} & \Delta m^2 \gtrsim 10^{-4}\,\text{eV}^2 \\ 10^{11} \frac{\text{m}}{\text{MeV}} & \text{solar neutrino experiments} & \Delta m^2 \gtrsim 10^{-11}\,\text{eV}^2 \end{cases}$$

Neutrino oscillations are the optimal tool to reveal tiny neutrino masses!

Categories of neutrino oscillations-I

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

(SNO, Borexino
Super-Kamiokande
GALLEX/GNO, SAGE
Homestake, Kamiokande)

VLBL Reactor
$\bar{\nu}_e$ disappearance

(KamLAND)

$\rightarrow$

$$\begin{cases} \Delta m^2_S = \Delta m^2_{21} \simeq 7.4 \times 10^{-5}\,\text{eV}^2 \\ \sin^2\vartheta_S = \sin^2\vartheta_{12} \simeq 0.30 \end{cases}$$

Categories of neutrino oscillations-II

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

Atmospheric
$\nu_\mu \to \nu_\tau$
(Super-Kamiokande, Kamiokande, IMB, MACRO, Soudan-2, IceCube, ANTARES)

LBL Accelerator
ν_μ disappearance
(K2K, MINOS, T2K, NOνA)

LBL Accelerator
$\nu_\mu \to \nu_\tau$
(OPERA)

$\to$

$$\begin{cases} \Delta m^2_{\rm A} \simeq |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2 \\ \sin^2\vartheta_{\rm A} = \sin^2\vartheta_{23} \simeq 0.50 \end{cases}$$

Categories of neutrino oscillations-III

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

LBL Accelerator
$\nu_\mu \to \nu_e$ (T2K, MINOS, NOνA)

LBL Reactor
$\bar{\nu}_e$ disappearance (Daya Bay, RENO, Double Chooz)

$\rightarrow$

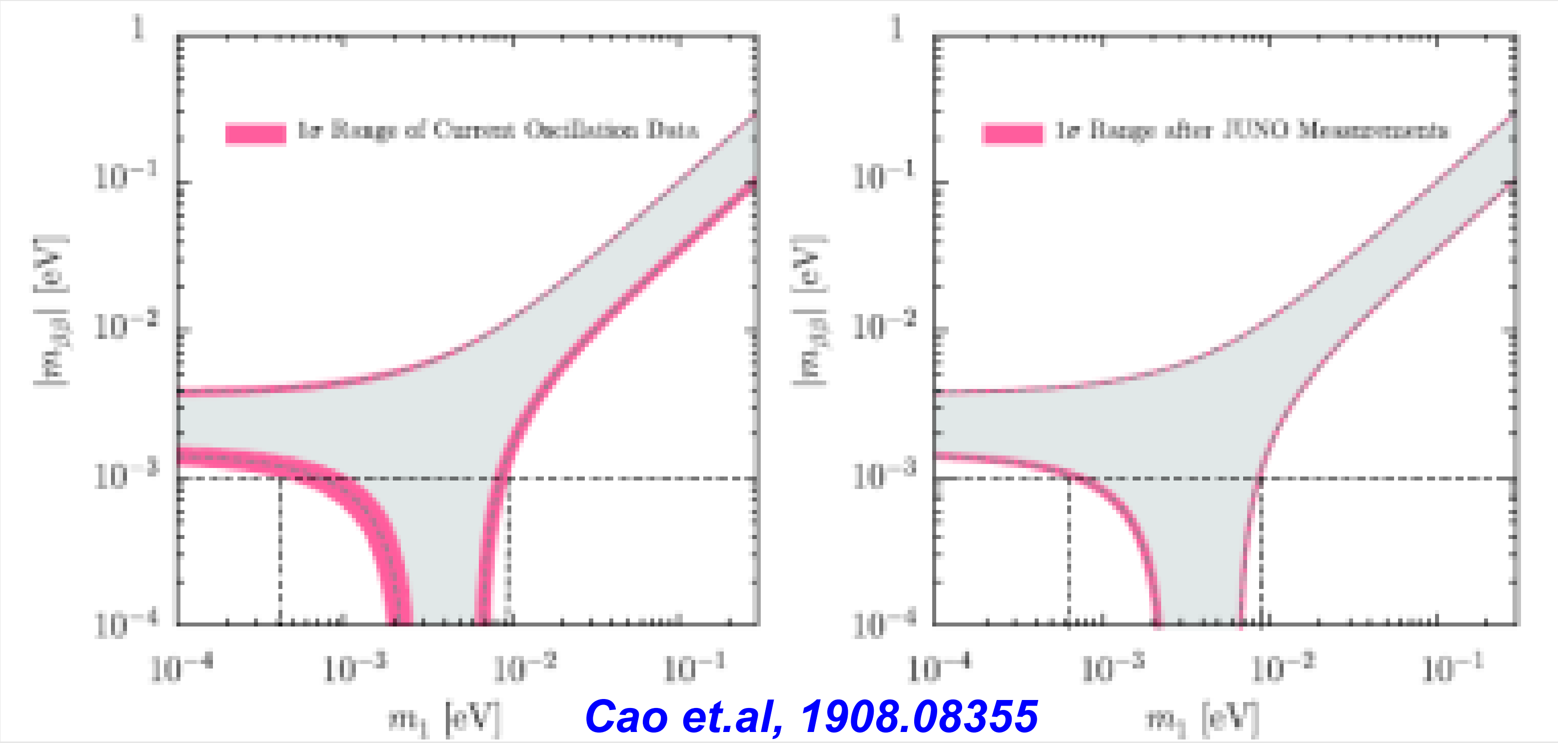
$$\Delta m^2_{\rm A} \simeq |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,{\rm eV}^2$$

$$\sin^2 \vartheta_{13} \simeq 0.022$$

Role of Precision Measurement (JUNO)

JUNO Physics Book: 1507.056131

	Nominal	+ B2B (1%)	+ BG	+ EL (1%)	+ NL (1%)
$\sin^2\theta_{12}$	0.54%	0.60%	0.62%	0.64%	0.67%
Δm^2_{21}	0.24%	0.27%	0.29%	0.44%	0.59%
$\lvert\Delta m^2_{ee}\rvert$	0.27%	0.31%	0.31%	0.35%	0.44%

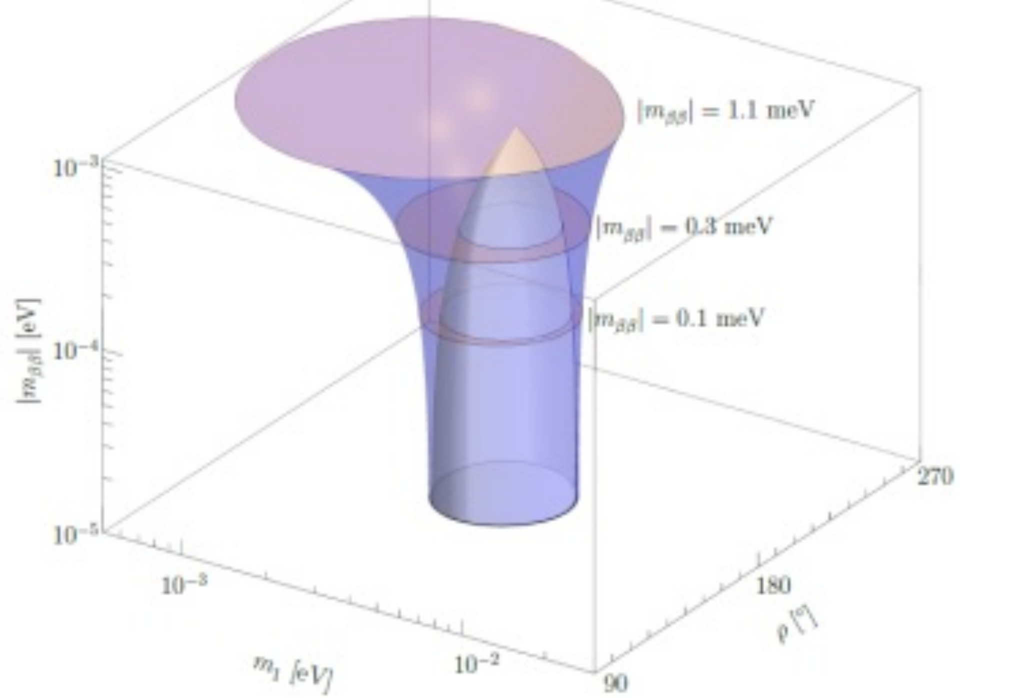


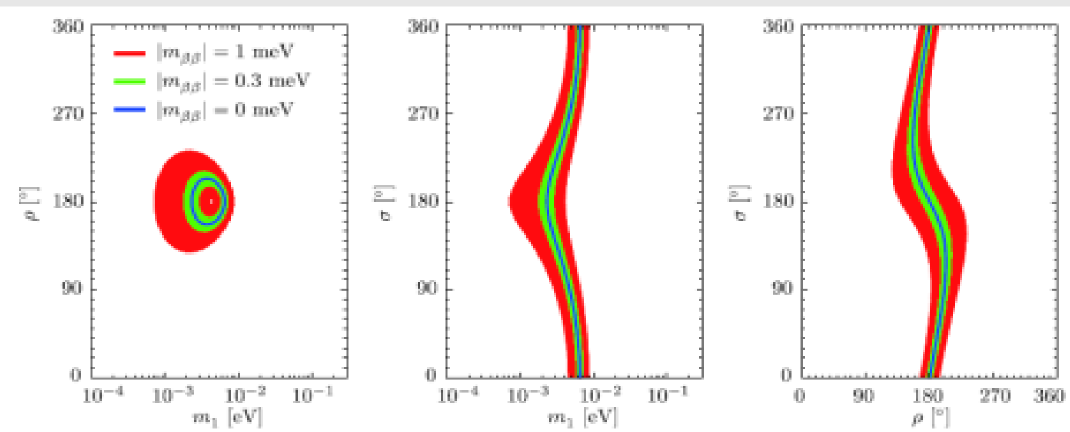
Cao et.al, 1908.08355

- **Precision measurement can eliminate (almost) all the uncertainties from oscillation parameters**

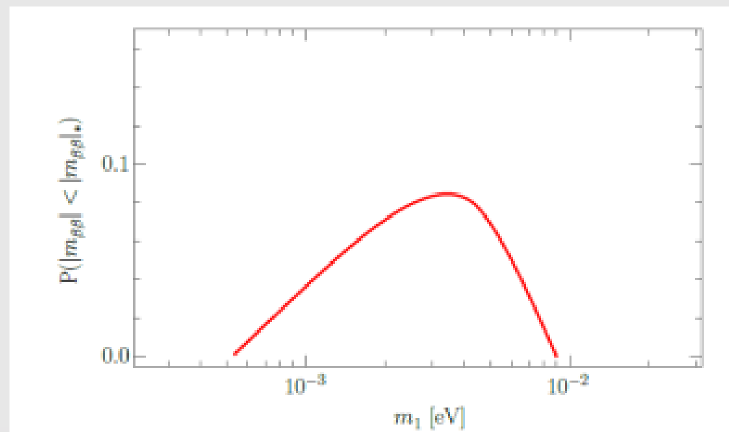
Fine structures

Cao et.al, 1908.08355 $(m_1, \rho, |m_{\beta\beta}|_*) \simeq (4\ \text{meV}, 180^\circ, 1.1\ \text{meV})$.



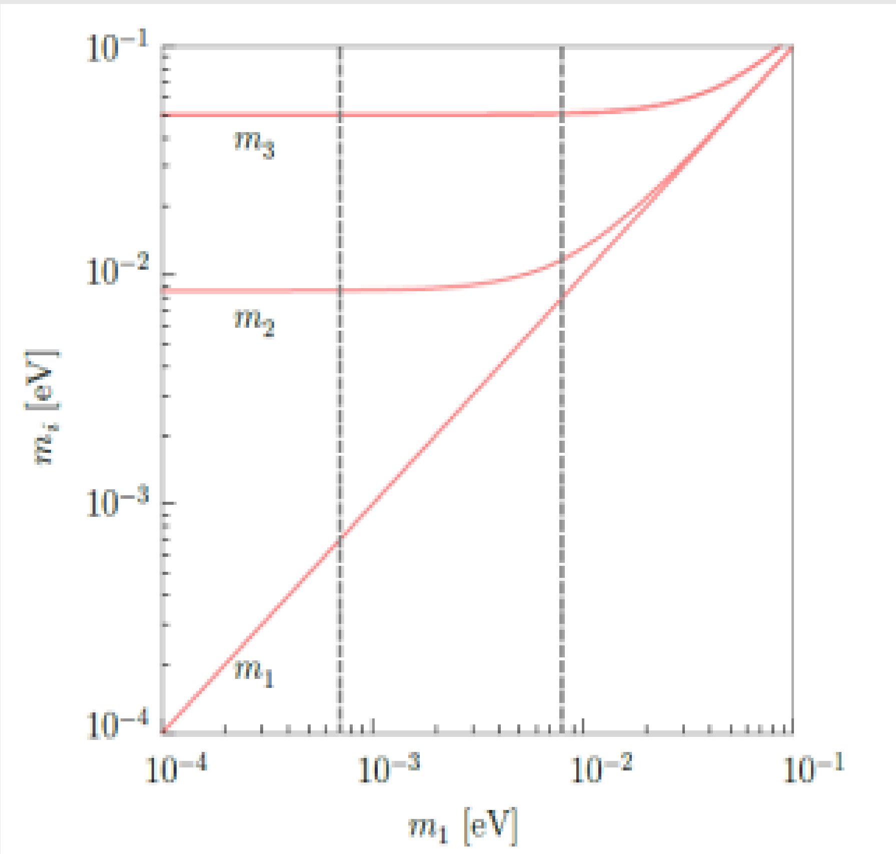


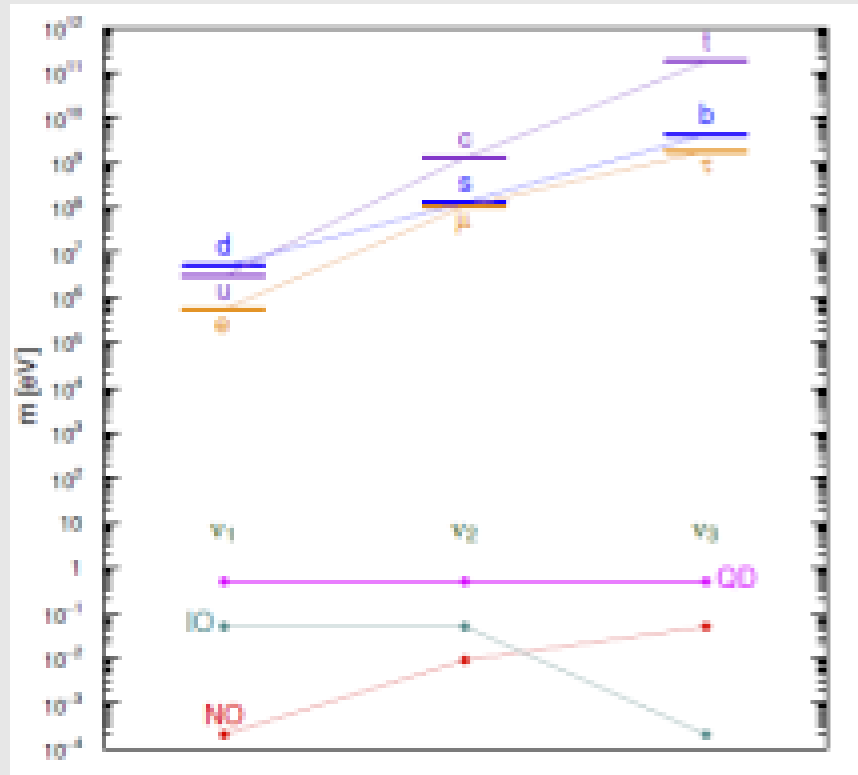
- The critical threshold point could serve as the **ultimate goal** for $0\nu2\beta$ searches.
- The possibility of **falling into the well** is very small.
- Have **unique (otherwise impossible) constraints** on non-oscillation parameters



Fixing the mass spectrum

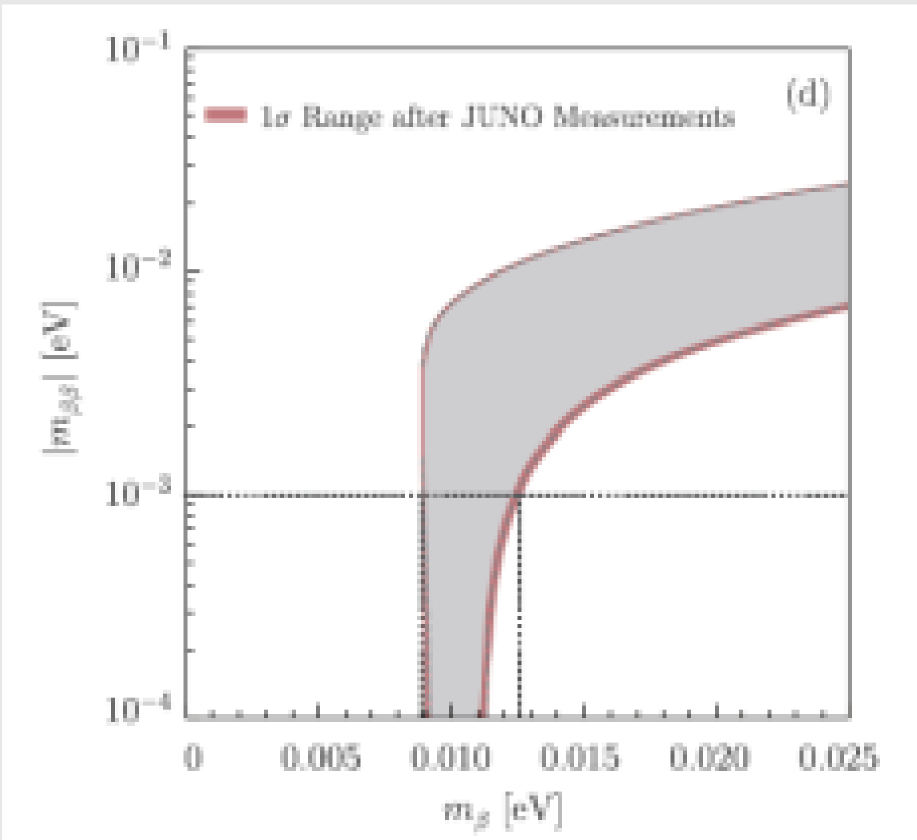
$$m_1 \in [0.7, 8]\ \text{meV}\,, \quad m_2 \in [8.6, 11.7]\ \text{meV}\,, \quad m_3 \in [50.3, 50.9]\ \text{meV}$$

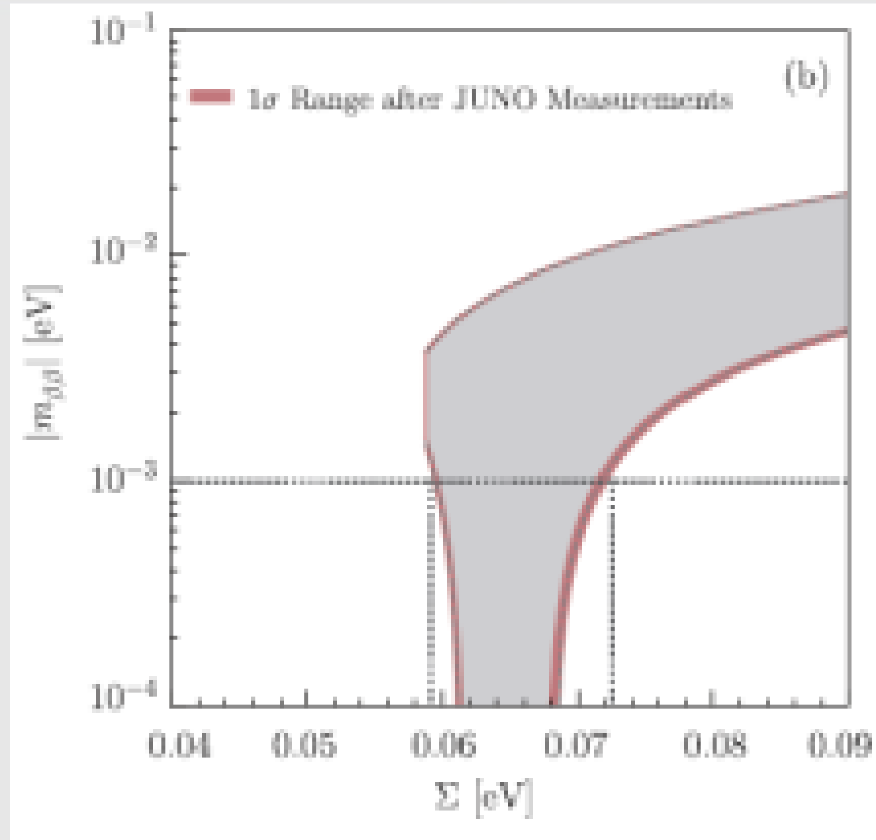




- Neutrino mass spectrum: important to understand the origin of fermion mass and mixing!

Implications for beta-decay and cosmology





$$8.9\ \text{meV} \le m_\beta \le 12.6\ \text{meV}\ , \quad 59.2\ \text{meV} \le \Sigma \le 72.6\ \text{meV}$$

- **(much) better than the projected sensitivities of future beta decay and cosmology probes!**